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A Computer Experiment and Consideration of Rule Extraction Methods by Rough Sets Based on a Rule Space

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***Abstract*—Rough sets are often used for extracting rules from categorical data sets with condition and decision attributes. However, the conventional method used to extract these rules has difficulty in penetrating the extracting processes and in examining the validity of the results. In this paper, we first construct a simulation model, and experimentally review and discuss the conventional method in a rule space which consists of atom rules. We then propose a method that extracts insightful results in the rule space. Through review and discussion, we further propose an effective algorithm for extracting rules, and study the influences on the data arising from inconsistent and unconcerned rules from human judgment and errors by misjudgment.**

***Index Terms*—rough sets, rule extracting, lower and upper approximation, rule space, atom rules**

I. INTRODUCTION

When various objects and/or phenomena are recognized, the observer specifies some discernible categories, and allocates a label or symbol to each category. The objects and/or phenomena are then classified and/or arranged using these labels or symbols. Objects and/or phenomena generally have some characteristic attributes, the recognition of which largely depends on what kind of combination of categories they have with their attributes. For example, the "goodness" or "badness" of products can be rated based on their color, shape, function, and other similar attributes. Doctors also diagnose diseases based on combinations of symptoms such as slight fever, cough, and headache, among others. Observers consciously or unconsciously recognize objects by rating or judging rules specifying categories and their attributes. If we can extract such rules from data bases with objects or phenomena, they will be very valuable in understanding the complex structures of cognition or causality which have not yet been clarified by the sciences. The information derived will be very useful in many fields, including medical science, engineering, and marketing.

Since rough sets theory was proposed by Z. Pawlak[1], many theoretical studies concerning rough sets have been conducted, and its useful application has been reported[2]. Especially, rough sets provide a means of looking into the database and discovering the rules described above in the form of "if-then" inference rules. The means and the tools by rough sets have been summarized in a book by Mori and others[3]. This text also provides information about the software implementing algorithm of rule extraction in order to easily apply the method to various types of rule extraction problems[3]. However, examples in the literature[2], [3] are mainly trivial cases for explaining how to use the algorithm, or are in other cases only explanations of the framework for use in applied real systems. Few published studies focus on the validity or the confidence of rule extraction methods, or on the features of extracted results. As a result, some may utilize the extracted rules without paying sufficient attention to error rules or the confidence of the rules, and thus be unaware of potential dangers lying behind the extracted rules.

To address this problem, we here focus on the validity or the confidence of rules extracted by rough sets method, by experimentally studying a simulation model which contains inconsistent rules, unconcerned rules and error data. Specifically, we sought to:

1) Clarify the framework of extracting rules by the conventional method by use of a simulation model, and propose an effective algorithm to reduce rules in a rule space.
2) Individual's ratings and/or judgments commonly contain inconsistent and unconcerned rules and error data. We examine the influence of these factors on the extracted rules.
3) Examine the influence of the data set size on the extracted rules.

II. RULE EXTRACTION BY ROUGH SETS METHOD-SIMULATION EXPERIMENT

We established a simulation model, generated data sets from that model, and reviewed the conventional rough sets method by extracting rules from the generated data sets in order to consider the framework of the conventional method. The model is supposed to have condition attributes and their values $A = \{a1,\ a2\}$, $B = \{b1,\ b2\}$, $C = \{c1,\ c2\}$, $D = \{d1,\ d2,\ d3\}$,

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TABLE I
EXAMPLE OF DECISION TABLE

Sample No.	A	B	C	D	Y	Sample No.	A	B	C	D	Y
$s1$	$a1$	$b2$	$c1$	$d2$	$y2$	$s13$	$a1$	$b1$	$c2$	$d3$	$y1$
$s2$	$a1$	$b1$	$c2$	$d2$	$y2$	$s14$	$a2$	$b1$	$c2$	$d3$	$y1$
$s3$	$a1$	$b2$	$c1$	$d3$	$y2$	$s15$	$a2$	$b2$	$c2$	$d3$	$y2$
$s4$	$a1$	$b1$	$c2$	$d1$	$y1$	$s16$	$a1$	$b2$	$c2$	$d1$	$y2$
$s5$	$a2$	$b1$	$c2$	$d3$	$y1$	$s17$	$a2$	$b2$	$c1$	$d2$	$y2$
$s6$	$a1$	$b1$	$c1$	$d1$	$y1$	$s18$	$a1$	$b2$	$c1$	$d1$	$y1$
$s7$	$a2$	$b1$	$c1$	$d3$	$y1$	$s19$	$a1$	$b1$	$c1$	$d2$	$y1$
$s8$	$a1$	$b1$	$c1$	$d3$	$y1$	$s20$	$a2$	$b2$	$c2$	$d2$	$y2$
$s9$	$a1$	$b1$	$c2$	$d2$	$y2$	$s21$	$a1$	$b1$	$c2$	$d3$	$y1$
$s10$	$a1$	$b1$	$c1$	$d2$	$y1$	$s22$	$a2$	$b2$	$c1$	$d3$	$y2$
$s11$	$a2$	$b2$	$c1$	$d3$	$y2$	$s23$	$a1$	$b1$	$c2$	$d1$	$y1$
$s12$	$a1$	$b2$	$c1$	$d1$	$y1$	$s24$	$a1$	$b1$	$c2$	$d2$	$y1$

TABLE II
DECISION MATRIX TO DISCRIMINATE SET OF $Y = y1$ FROM SET OF $Y = y2$

	$s1$	$s2(s9)$	$s3$	$s11(s22)$	$s15$	$s16$	$s17$	$s20$
$s4(s23)$	$b1,\ c2,\ d1$	$d1$	$c2,\ d1$	$a1,\ b1,\ c2,\ d1$	$a1,\ b1,\ d1$	$b1$	$a1,\ b1,\ c2,\ d1$	$a1,\ b1,\ d1$
$s5(s14)$	$a2,\ b1,\ c2,\ d3$	$a2,\ d3$	$b1,\ c2$	$b1,\ c2,$	$b1,$	$a2,\ b1,\ d3$	$b1,\ c2,\ d3$	$b1,\ d3$
$s6$	$b1,\ d1$	$c1,\ d1$	$b1,\ d1$	$a1,\ b1,\ d1$	$a1,\ b1,\ c1,\ d1$	$b1,\ c1,$	$a1,\ b1,\ d1$	$a1,\ b1,\ c1d1$
$s7$	$a2,\ b1,\ d3$	$a2,\ c1,\ d3$	$a2,\ b1$	$b1$	$b2,\ c1$	$a2,\ b1,\ c1d3$	$b2,\ d2$	$b1,\ c1,\ d3$
$s8$	$b1,\ d3$	$c1,\ d3$	$b1$	$a1,\ b1$	$a1,\ b1c1$	$b1,\ c1d3$	$a1,\ b1,\ d3$	$a1,\ b1,\ c1,\ d3$
$s10(s19)$	$b1$	$c1$	$b1,\ d2$	$a1,\ b1,\ d2$	$a1,\ b1,\ c1,\ d2$	$b1,\ c1,\ d2$	$a1,\ b1$	$a1,\ b1,\ c1$
$s12(s18)$	$d1$	$b2,\ c1,\ d1$	$d1$	$a1,\ d1$	$a1,\ c1,\ d1$	$c1$	$a1d1$	$a1,\ c1,\ d1$
$s13(s21)$	$b1,\ c2,\ d3$	$d3$	$b1,\ c2$	$a1,\ b1,\ c2$	$a1,\ b1$	$b1,\ d3$	$a1,\ b1,\ c2,\ d3$	$a1,\ b1,\ d3$

and the corresponding decision attribute $Y = \{y1,\ y2\}$. The following functions which are denoted as Rule1 and Rule2 are supposed between the condition attributes and the decision:

Rule1:if ($A = a1$ and $B = b1$) or ($C = c1$ and $D = d1$) then $Y = y1$,

Rule2:if ($A = a2$ and $B = b2$) or ($C = c2$ and $D = d2$) then $Y = y2$.

The sample data of the set of the condition attribute values are randomly generated, and the corresponding decision value is decided according to Rule1 and Rule2. However, for example, in the case where $A = a1$, $B = b1$, $C = c2$, $D = d2$ (hereafter abbreviated as $a1b1c2d2$) the decision value of Y is randomly assigned, since the generated set satisfies both of the specified rules. This type of case is classed as an inconsistent case. The value of Y is also randomly assigned in the case where the set of condition attribute value, for instance, is $a1b2c2d3$. This case is an unconcerned case to which both of the specified rules cannot be applied. Both cases will usually occur in rating processes of like feelings of preference.

Table I shows the decision table of 24 samples generated according to the above procedures. The table shows:

1) $s24$ and $s2$ have the same value $a1b1c2d2$ of condition attributes and differing decision value; hence they are inconsistent because of the random decision described above.
2) $s9$ and $s2$, $s10$ and $s19$, $\cdots$ have the same condition and decision values and hence are redundant data.
3) $s1$, $s3$, $s16$, $s7$ and $s14$ are unconcerned data, and hence take their values in the table by chance.

However, in order to simply consider the conventional method, this table does not contain error data caused by misjudgment: For example, the error is in the case where $a1b1c2d3$ is judged to be $Y = y2$ instead of $Y = y1$. It should in fact be judged to be $y1$.

The decision table offers the rules by the number of the samples and the rough set method offers the method of reducing the rules in the table. The conventional method makes the decision matrix as shown in Table II, from Table I. The $(1,\ 1)$ element of Table II implies that $s4(s23)$ which takes $Y = y1$ can be discerned from $s1$, which takes $Y = y2$ if the condition attribute value is $b1$ or $c2$ or $d1$ (**Statement 1**). In general, $a(i,\ j)$, $b(i,\ j)$, $c(i,\ j)$ and $d(i,\ j)$ of $(i,\ j)$ element

TABLE III
REDUCED DECISION RULE FOR DATA SET IN TABLE I

Rule $R1$	$Y = y1$	$C.I.$	Samples
$R11$	$b1d3$	0.429	$s5$, $s7$, $s8$, $s13$, $s14$, $s21$
$R21$	$b1c1$	0.357	$s6$, $s7$, $s8$, $s10$, $s19$
$R31$	$b1d1$	0.214	$s4$, $s6$, $s23$
$R41$	$a2b1$	0.214	$s5$, $s7$, $s14$
$R51$	$c1d1$	0.214	$s6$, $s12$, $s18$
$R61$	$a1c2d3$	0.143	$s13$, $s21$

Rule $R2$	$Y = y2$	$C.I.$	Samples
$R12$	$a2b2$	0.500	$s11$, $s15$, $s17$, $s20$, $s22$
$R22$	$b2d3$	0.400	$s3$, $s11$, $s15$, $s22$
$R32$	$b2d2$	0.300	$s1$, $s17$, $s20$
$R42$	$b2c2$	0.300	$s15$, $s16$, $s20$
$R52$	$a2d2$	0.200	$s17$, $s20$

$(j = 1, \cdots, N)$ of Table II imply values of the attribute to discern $s(i)$, which takes $Y = y1$ from $s(j)$ $(j = 1, \cdots, M)$, which takes $Y = y2$. Accordingly, the following $p(i)$ is the condition of discerning $s(i)$ from all of $s(j)$:

$$p(i) = \wedge_{j=1}^{M}(a(i,\ j) \vee b(i,\ j) \vee c(i,\ j) \vee d(i,\ j)), \qquad (1)$$

where $\vee$ is disjunction and $\wedge$ is conjunction. In order to discern all of $s(i)$ from $s(j)$, The following $R1$ is required:

$$R1 = \vee_{i=1}^{N} p(i). \qquad (2)$$

The results of reduction of Table I based on (1) and (2) are shown in Table III, which also includes extracted $R2$ by the same way of extracting $R1$. Table III also contains $C.I.$ (Covering Index) which is defined as, for example, $|R11|/|Y = y1|$ ($|.|$: frequency) and represents the importance of the rule. The following can be seen:

1) The extracted rules include part of Rule1, which is specified in advance (see $R51$ and $R21$).
2) Most of the extracted rules include the part of attribute vales specified in advance in Rule1 and Rule2. However, they are doubtful if the extracted results are valid or not.

One of the reasons of 2) above is intuitively understood to be a small number of generated samples to compare with the number of all possible rules $(= 2 \times 2 \times 2 \times 3 \times 2 = 48)$. This experiment shows that the conventional rough sets method will not discover the original rules of the objects using a small number of samples, although it may discover part of the attribute values.

III. CONSIDERATION OF THE CONVENTIONAL METHOD AND A NEW EXTRACTING METHOD

The conventional method by (1), (2) is easy to understand its meanings. However, in the case where the number of attributes and their values increase greatly, the reduction cannot help, as it depends on calculations by using a computer. It is difficult to penetrate the results, and to understand the reason why these reduced results were derived from the data sets. In this section, we consider the meanings of (1), (2) and discuss them in a rule space. Table IV shows the arrangement of samples shown in Table I in the rule space Ω which contains all possible rules of the object of interest (hereafter denoted atom rules). Here, we defined four rule sets in the rule space by use of the sample data. The first one and second sets are $Y1$ and $Y2$ respectively, with the decision attributes $y1$ and $y2$. The third set is Con, the atom rules of which are contradicted in the samples. The fourth set is $Y0$, the atom rules of which are not present in the sample.

As an example we focus on Rule $r4$, corresponding to $s4(s23)$; and $r8$, corresponding to $s1$. In this table, Statement 1 in II states the following type of rules: if $*b1{*}{*}{\vee}{*}{*}c2{*}{\vee}{*}{*}{*}d1$ then $y1$ (**Statement 2**), where $*$ denote any attributes value in its position, for instance, $*b1** = a1b1c1d1,\ a2b1c1d1, \cdots$. Statement 2 inevitably includes $r4$ and excludes $r8$. Accordingly, (1) means the followings: if $\wedge_{j=1}^{M}(a(i,\ j)*** \vee *b(i,\ j)** \vee **c(i,\ j)* \vee ***d(i,\ j))$ then $y1$. This rule $r(i)$ corresponding to $s(i)$ includes $r(i)$ and excludes all of $r(j)$ corresponding to $s(j)$ $(j = 1, \cdots, M)$. Accordingly, rules derived from (2) inevitably include at least an atom rule included in $Y1 - \mathrm{Con}$ (**Statement 3**) (difference set of $Y1$ from Con) and are excluded from the rule set $\Omega - Y2$ (**Statement 4**). Table V arranges Table III in order to see what atom rules construct the rule extracted in Table III, where atom rules in bold letters belong to $Y1$, those in bold italics belong to $Y2$, and all others belong to $Y0$. Statements 3 and 4 can be confirmed in Table V. Note that their reduced rules have the Hamming distance by one between their atom rules (**Statement 5**), and the following reduction algorithm derived from Statement 3 and 4 instead of (1), (2): **Step 1**: Form Table IV. **Step 2**: Form $Y1$ and $\bar{Y}1$ (complement of $Y1$ like $Y2$). **Step 3**: Form every rule set which satisfies Statement 3 and 5. **Step 4**: Reduce every rule set formed in Step 3 by Boolean operations. This algorithm extracts rules based on the lower approximation which excludes contradicted rule samples. Rules based on the upper approximation are also obtained by inserting the following **Step 2+** just after Step 2: $Y1 \leftarrow Y1 - \mathrm{Con}$, $\bar{Y}1 \leftarrow \bar{Y}1 - \mathrm{Con}$, $Y0 \leftarrow \Omega - Y1 - \bar{Y}1$, $\mathrm{Con} \leftarrow \{\Phi\}$. Extracted rules based on the upper approximation have the possibility of finding the originally inconsistent rules, because the rules would not exclude the contradicted samples. It is clear that the proposed algorithms do not incur much calculation cost compared with the conventional method.

TABLE IV
RULE SPACE, SAMPLE NO. AND ITS FREQUENCY

Rule	a	b	c	d	$Y = y1$ Sample No.	$Y = y1$ Frequency	$Y = y2$ Sample No.	$Y = y2$ Frequency
$r1$	1	1	1	1	$s6$	1		0
$r2$	1	1	1	2	$s10$, $s19$	2		0
$r3$	1	1	1	3	$s8$	1		0
$r4$	1	1	2	1	$s4$, $s23$	2		0
$r5$	1	1	2	2	$s24$	1	$s2$, $s9$	2
$r6$	1	1	2	3	$s13$, $s21$	2		0
$r7$	1	2	1	1	$s12$, $s18$	2		0
$r8$	1	2	1	2		0	$s1$	1
$r9$	1	2	1	3		0	$s3$	1
$r10$	1	2	2	1		0	$s16$	1
$r11$	1	2	2	2		0		0
$r12$	1	2	2	3		0		0
$r13$	2	1	1	1		0		0
$r14$	2	1	1	2		0		0
$r15$	2	1	1	3	$s7$	1		0
$r16$	2	1	2	1		0		0
$r17$	2	1	2	2		0		0
$r18$	2	1	2	3	$s5$, $s15$	2		0
$r19$	2	2	1	1		0		0
$r20$	2	2	1	2		0	$s17$	1
$r21$	2	2	1	3		0	$s11$, $s22$	2
$r22$	2	2	2	1		0		0
$r23$	2	2	2	2		0	$s20$	1
$r24$	2	2	2	3		0	$s15$	1

TABLE V
REDUCED DECISION RULES AND THEIR CORRESPONDING ATOM RULES FOR THE DATA SET IN TABLE I

Rule $R1$	$Y = y1$	Corresponding Atom Rules
$R11$	$b1d3$	**r3**, **r6**, **r15**, **r18**
$R21$	$b1c1$	**r1**, **r2**, **r3**, *r13*, *r14*, **r15**
$R31$	$b1d1$	**r1**, **r4**, *r13*, *r16*
$R41$	$a2b1$	*r13*, *r14*, **r15**, *r16*, *r17*, **r18**
$R51$	$c1d1$	**r1**, **r7**, *r13*, *r19*
$R61$	$a1c2d3$	**r6**, *r12*

Rule $R2$	$Y = y2$	Corresponding Atom Rules
$R12$	$a2b2$	*r19*, ***r20***, ***r21***, *r22*, ***r23***, ***r24***
$R22$	$b2d3$	***r9***, *r12*, ***r21***, ***r24***
$R32$	$b2d2$	***r8***, *r11*, ***r20***, ***r23***, ***r24***
$R42$	$b2c2$	***r10***, *r11*, *r12*, *r22*, ***r23***, ***r24***
$R52$	$a2d2$	*r14*, *r17*, ***r20***, ***r23***

IV. EXAMPLE OF A LARGE NUMBER OF SAMPLE WITH HUMAN ERRORS

A large number of samples with human errors were generated based on the model in II and are arranged in Table VI, where the total number of samples is 300, the error rate of each sample is 0.1 and the satisfactory index ($S.I.$) defined as $S.I.(y1) = |y1|/(|y1|+|y2|)$ is shown at every atom rule in the table. This table has $\mathrm{Con} = \{r2, r3, \cdots, r16, r18, r19, r20, r22, r23\}$, $Y1 = \{r1\} + \mathrm{Con}$, $Y2 = \{r17, r21, r24\} + \mathrm{Con}$ and $Y0 = \{\Phi\}$. Generating a greater number of samples, the table will have $\mathrm{Con} = \{\Omega\}$, $Y1 = \{\Omega\}$, $Y2 = \{\Omega\}$ and $Y0 = \{\Phi\}$. For this condition, Ziarko[4] has proposed a variable precision rough set model (VP-model) which permits admissible errors β since to date the conventional method can hardly extract any valuable information from data containing contradictions and errors.

As an example, we examined the data set of Table VI using the VP-model specifying $\beta < 0.2$ (i.e. $S.I.(.) \geq 0.8$) at each atom rule. From this, $\mathrm{Con} = \{r5, r8, r9, r10, r12, r14, r15, r16, r18, r19\}$, $Y1 = \{r1, r2, r3, r4, r6, r7, r13\} + \mathrm{Con}$, $Y2 = \{r11, r17, r20, r21, r22, r23, r24\} + \mathrm{Con}$, $Y0 = \{\Phi\}$ are obtained. Applying these to the algorithm based on the upper approximation in III, the extracted rules shown in Table VII are obtained. This table shows many possibilities of the rule combination between $R1$ and $R2$. However, the combination reproducing the above Con, $Y1$, $Y2$ and $Y0$ is the

TABLE VI
RULE SPACE AND ITS FREQUENCY WITH $S.I.$ VALUES ($N = 300$)

Rule	a	b	c	d	$Y = y1$ Frequency	$Y = y2$ Frequency	$S.I.(y1)$	$S.I.(y2)$
$r1$	1	1	1	1	10	0	1.000	0.000
$r2$	1	1	1	2	15	3	0.833	0.167
$r3$	1	1	1	3	15	1	0.938	0.063
$r4$	1	1	2	1	12	2	0.857	0.143
$r5$	1	1	2	2	7	4	0.636	0.364
$r6$	1	1	2	3	15	1	0.938	0.063
$r7$	1	2	1	1	8	2	0.800	0.200
$r8$	1	2	1	2	5	5	0.500	0.500
$r9$	1	2	1	3	11	10	0.524	0.476
$r10$	1	2	2	1	6	9	0.400	0.600
$r11$	1	2	2	2	1	7	0.125	0.875
$r12$	1	2	2	3	4	3	0.571	0.429
$r13$	2	1	1	1	14	3	0.824	0.176
$r14$	2	1	1	2	2	6	0.250	0.750
$r15$	2	1	1	3	5	5	0.500	0.500
$r16$	2	1	2	1	4	8	0.333	0.667
$r17$	2	1	2	2	0	9	0.000	1.000
$r18$	2	1	2	3	5	6	0.455	0.545
$r19$	2	2	1	1	3	3	0.500	0.500
$r20$	2	2	1	2	2	8	0.200	0.800
$r21$	2	2	1	3	0	15	0.000	1.000
$r22$	2	2	2	1	1	19	0.050	0.950
$r23$	2	2	2	2	1	12	0.077	0.923
$r24$	2	2	2	3	0	13	0.000	1.000

TABLE VII
REDUCED DECISION RULES BY THE UPPER APPROXIMATION AND THEIR CORRESPONDING ATOM RULES FOR TABLE VI

Rule $R1$	$Y = y1$	Corresponding Atom Rules	Rule $R2$	$Y = y2$	Corresponding Atom Rules
$R11$	$a1b1$	**r1**, **r2**, **r3**, **r4**, *r5*, **r6**	$R12$	$a2b2$	*r19*, ***r20***, ***r21***, ***r22***, ***r23***, ***r24***
$R21$	$a1c1$	**r1**, **r2**, **r3**, **r7**, *r8*, *r9*	$R22$	$a2c2$	*r16*, ***r17***, *r18*, ***r22***, ***r23***, ***r24***
$R31$	$b1c1$	**r1**, **r2**, **r3**, **r13**, *r14*, *r15*	$R32$	$b2c2$	*r10*, ***r11***, *r12*, ***r22***, ***r23***, ***r24***
$R41$	$b1d1$	**r1**, **r4**, **r13**, *r16*	$R42$	$b2d2$	*r8*, ***r11***, ***r20***, ***r23***
$R51$	$c1d1$	**r1**, **r7**, **r13**, *r19*	$R52$	$a2d2$	*r14*, ***r17***, ***r20***, ***r23***
$R61$	$a1d1$	**r1**, **r4**, **r7**, *r10*	$R62$	$c2d2$	*r5*, ***r11***, ***r17***, ***r23***
$R71$	$b1d3$	**r3**, **r6**, *r15*, *r18*	$R72$	$a2d3$	*r15*, *r18*, ***r21***, ***r24***
$R81$	$a1d3$	**r3**, **r6**, *r9*, *r12*	$R82$	$b2d3$	*r9*, *r12*, ***r21***, ***r24***

only combination of $R11$, $R51$, $R12$ and $R62$, which is proved by their corresponding atom rules shown in the table. The combination also corresponds to the original specified rules. In this way, our proposed method has insightful abilities.

V. CONCLUSION

One of the main purposes of rough sets method is to extract the if-then inference rules from data bases. We focused on its purpose, reviewed and studied the conventional method by use of a simulation model, and proposed a new method and an effective algorithm which extracts rules in the rule space constructed by atom rules. The validity and its usefulness were confirmed by the simulation experiment.

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