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Robust Control of Free-floating Space Flexible Manipulator in Joint Space

Hong Zhaobin
College of Mechanical Engineering and Automation
Fuzhou University
Fuzhou, China
Benn.Hong@gmail.com

Chen Li
College of Mechanical Engineering and Automation
Fuzhou University
Fuzhou, China
Chnle@fzu.edu.cn

***Abstract*—The robust control and active vibration control for free-floating space flexible manipulator with parameter uncertainties are studied. The dynamic equations of the system are developed by using the Lagrangian assumed modes methods, it is verified that the dynamic equation can be linearly dependent on a group of inertial parameters. Based on the results and under the assumption of two-time scale, singular perturbation model of the space flexible manipulator system is obtained. The fast subsystem controller will damp out the vibration of the flexible link using optimal Linear Quadratic Regulator (LQR) method. The slow subsystem robust controller dominates the trajectory tracking in joint space. In particular, the control scheme doesn't require measuring the position, velocity nor acceleration of the base. The numerical simulation is carried out, which confirms the controller proposed is feasible and effective.**

***Keywords*—space flexible manipulator, robust control, active vibration control, singular perturbation approach, joint space**

I. INTRODUCTION

Space robot systems have been suggested for a number of important missions in space. It has been proposed to construct future space station, or to repair and serve satellites in earth orbit. As a result, the control of space robot systems has received increased attention [1-7]. Generally, space manipulator has the characteristics of light mass, long manipulator, heavy payload, etc. With the continuous development of control technology for space manipulator, it is necessary to consider the flexural effects of flexible links during the controller design and then the better performance can be obtained. Most robust control and nonlinear control schemes of the terrestrial manipulators can't be directly applied on the space flexible manipulator. From then on, some control laws are improved and used to control the space flexible manipulator. The adaptive control of controlled-attitude space robot system is investigated in [8]. An adaptive control scheme based on the augmentation approach of free-floating space robot systems is presented in [9]. The work [10] designs an augmented adaptive control of dual-arm space robot system. According to the previous researches, there are a few studies focus on the adaptive and robust control of space robot system with uncertain parameters. However, all the researches above aim at space rigid robot system. In order to ensure the operation of high accuracy, the vibration control of flexible link must be considered during the controller design.

For flexible manipulator, the dynamic model of dual-arm space manipulator is obtained by assumed modes method and a corresponding control method is presented in [11]. The nonlinear decoupling feedback control method based on the model is used to partly decouple the joint variables and elastic deformation, and the residual vibration is eliminated after the joint tracing achieved, see [12]. A methodology of stable manipulation-variable feedback control of space robots with flexible links for positioning control to a static target and continuous path tracking control is discussed in [13]. It is worth noting that the researches above are considering the terrestrial flexible manipulator or the certain space flexible manipulator. It is necessary to study the robust control and active vibration control for space flexible manipulator with uncertain parameters.

To maintain reasonable computational loading, a controller based on a reduced-order model have been proposed in [14]. In recent years, singular perturbation theory has been shown to a convenient strategy for reduced-order modeling. The high-order sliding control based on singular perturbation theory for a class of mechanical systems is proposed in [15]. Using singular perturbation theory, the dynamics of cooperating flexible manipulators based on distributed parameter model is obtained in [16]. Therefore, the paper proposes controller using singular perturbation approach for space flexible manipulator system.

In order to farthest reduce the fuel consumption of the system, space flexible manipulators are generally designed that the attitude and position of the base are not controlled. In this paper, the dynamic equations of the system are developed by using the Lagrangian assumed modes methods, it is verified that the dynamic equation can be linearly dependent on a group of inertial parameters. Based on the results and under the assumption of two-time scale, singular perturbation model of the space flexible manipulator system is obtained. The fast subsystem controller will damp out the vibration of the flexible link using optimal method. The slow subsystem robust controller dominates the trajectory tracking in joint space. The numerical simulation is carried out, which confirms the controller proposed is feasible and effective.

This paper is organized as follow: In Section II, the dynamic equation of a space flexible manipulator system is formulated. Under the assumption of two-time scale, the singular perturbation model of the space flexible manipulator system is obtained in Section III. In Section IV, the paper designs the robust controller for slow subsystem as well as the optimal LQR for fast subsystem. Simulation results are given in Section V. Finally, conclusions are presented in Section VI.

II. THE DYNAMICS OF FREE-FLOATING SPACE FLEXIBLE MANIPULATOR

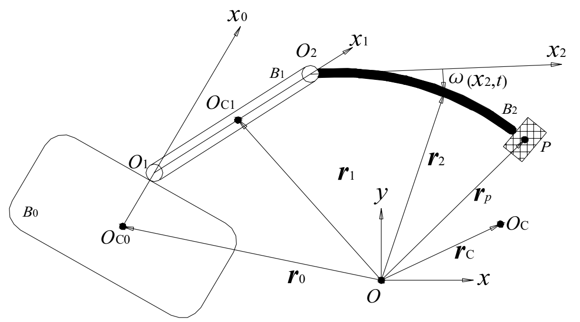

Figure 1. Free-floating space flexible manipulator

Without any loss of generality, a planar free-floating space flexible manipulator is considered here, Figure 1. The system consists of the base B_0, the rigid link B_1, the flexible link B_2, and a payload P. As the ratio between the length of link B_2 and its thickness is sufficiently large, the flexible link B_2 can be modeled as Euler-Bernoulle beams, which can only be deformed in the flexural direction. $O_{Ci}(i=0,1)$ is the mass center of B_i. O_0 coincides with O_{C0}, $O_i(i=1,2)$ is the rotational center of each joint, x_1 is the symmetry axis of body B_1, x_2 is tangent with the symmetry axis of body B_2. The other symbols are defined as follows:

$(O-xy)$	Inertial coordinate frame of system
$(O_i-x_iy_i)$	Local coordinate frame of B_i $(i=0,1,2)$
a	Distance from joint O_0 to O_1
a_1	Distance from joint O_1 to O_{C1}
l_i	Length of $B_i(i=1,2)$
m_i	Mass of $B_i(i=0,1)$
m_p	Mass of P
J_i	Inertial moment of B_i with respect to its mass center $O_{Ci}(i=0,1)$
ρ	Uniform mass density at unit length of flexible link B_2
EI	Symmetric bending rigidity of B_2
$\boldsymbol{\omega}(x_2,t)$	Flexural displacement at time t and location x_2 of B_2
θ_0	The angle between the x axis and the x_0 axis
θ_i	The angle between the x_{i-1} axis and the x_i axis $(i=1,2)$
$\boldsymbol{r}_i$	Position vector of the mass center O_{Ci} of B_i $(i=0,1,2)$
$\boldsymbol{r}_p$	Position vector of P
$\boldsymbol{r}_C$	Position vector of the mass center C of the entire system

The assumed modes method is implemented to approximate

$$\boldsymbol{\omega}(x_2,t)=\sum_{i=1}^{n}q_i(t)\varphi_i(x_2)\ . \tag{1}$$

where, $\varphi_i(x_2)$ is the spatial coordinate, $q_i(t)$ is the time-dependent generalized coordinates. In this paper, $\boldsymbol{\omega}(x_2,t)$ is dominated by the first two elastic modes, i.e. $n=2$.

From the geometric relationship of system, we can obtain

$$\begin{aligned}
\boldsymbol{r}_1&=\boldsymbol{r}_0+a\cdot\boldsymbol{e}_0+a_1\cdot\boldsymbol{e}_1\\
\boldsymbol{r}_2&=\boldsymbol{r}_0+a\cdot\boldsymbol{e}_0+l_1\cdot\boldsymbol{e}_1+x_2\cdot\boldsymbol{e}_2-\boldsymbol{\omega}\cdot\boldsymbol{e}_3\\
\boldsymbol{r}_p&=\boldsymbol{r}_0+a\cdot\boldsymbol{e}_0+l_1\cdot\boldsymbol{e}_1+l_2\cdot\boldsymbol{e}_2-\boldsymbol{\omega}(l_2,t)\cdot\boldsymbol{e}_3\ .
\end{aligned}\tag{2}$$

where $\boldsymbol{e}_3=\widetilde{\boldsymbol{I}}\cdot\boldsymbol{e}_2$, $\widetilde{\boldsymbol{I}}=[0,-1;1,0]$. In the absence of small gravity, there is not external force acted on the space flexible manipulator system, so the system is momentum-conservative in $(O-xy)$. And it satisfies

$$m_0\boldsymbol{r}_0+m_1\boldsymbol{r}_1+m_p\boldsymbol{r}_p+\int_0^{l_2}\rho\boldsymbol{r}_2\mathrm{d}x_2=M\boldsymbol{r}_C\ . \tag{3}$$

where, $M=m_0+m_1+m_p+\rho l_2$ is the total mass of the entire system. From (2) and (3), $\boldsymbol{r}_0$ can also be rewritten as

$$\boldsymbol{r}_0=\boldsymbol{r}_C+L_{00}\boldsymbol{e}_0+L_{01}\boldsymbol{e}_1+L_{02}\boldsymbol{e}_2+(L_{03}q_1+L_{04}q_2)\boldsymbol{e}_3\ . \tag{4}$$

where, $L_{00}=-(m_1+m_p+\rho l_2)a/M$,

$L_{01}=-(m_1a_1+m_pl_1+\rho l_1l_2)/M$, $L_{02}=-(\rho l_2^{\ 2}/2+m_pl_2)/M$,

$L_{03}=-\rho\int_0^{l_2}\varphi_1\mathrm{d}x_2/M$, $L_{04}=-\rho\int_0^{l_2}\varphi_2\mathrm{d}x_2/M$.

Substituting (4) into (2), we have

$$\begin{aligned}
\boldsymbol{r}_1&=\boldsymbol{r}_C+L_{10}\boldsymbol{e}_0+L_{11}\boldsymbol{e}_1+L_{12}\boldsymbol{e}_2+(L_{13}q_1+L_{14}q_2)\boldsymbol{e}_3\\
\boldsymbol{r}_2&=\boldsymbol{r}_C+L_{20}\boldsymbol{e}_0+L_{21}\boldsymbol{e}_1+L_{22}\boldsymbol{e}_2+(L_{23}q_1+L_{24}q_2)\boldsymbol{e}_3\\
\boldsymbol{r}_p&=\boldsymbol{r}_C+L_{30}\boldsymbol{e}_0+L_{31}\boldsymbol{e}_1+L_{32}\boldsymbol{e}_2+(L_{33}q_1+L_{34}q_2)\boldsymbol{e}_3\ .
\end{aligned}\tag{5}$$

where, the parameter $L_{ij}(i=1,2,3; j=0,1,2,3,4)$ is same as are the functions of the inertia parameters of the system.

Assuming that there are no mechanical restrictions, nor external forces, the momentum are conserved during the operation. Without any loss of generality, the initial momentum of the system is assumed to be zero, i.e. $\dot{\boldsymbol{r}}_C = 0$. Obviously, from (5) the velocities are linearly dependent on inertia parameters $L_{ij}(i=0,1,2,3; j=0,1,2,3,4)$ .In space flexible manipulator system, the kinetic energy of the system is given as

$$T = T_0 + T_1 + T_2 + T_p$$
$$= \sum_{i=0,1}\left(\frac{1}{2}m_i\dot{\boldsymbol{r}}_i^2 + \frac{1}{2}J_i\boldsymbol{\omega}_i^2\right) + \frac{1}{2}m_p\dot{\boldsymbol{r}}_p^2 + \frac{1}{2}\int_0^{l_2}\rho\dot{\boldsymbol{r}}_2^2\mathrm{d}x . \qquad (6)$$

where, $\boldsymbol{\omega}_i$ is the angular velocity of B_i. So the kinetic energy of the system can be linearly dependent on a group of parameters properly chosen. And the strain energy stored in the system is expressed as

$$U = \frac{1}{2}\int_0^{l_2} EI(\boldsymbol{\omega}''(x_2,t))^2\mathrm{d}x_2 . \qquad (7)$$

where, $\boldsymbol{\omega}''(x_2,t) = \dfrac{\partial^2\boldsymbol{\omega}(x_2,t)}{\partial x_2^2}$.

With Lagrangian Equation, the dynamic equation of free-floating space flexible manipulator system can be represented by the following vector equations

$$\boldsymbol{D}(\boldsymbol{\theta},\boldsymbol{q})\begin{bmatrix}\ddot{\theta}_0\\ \ddot{\boldsymbol{\theta}}_r\\ \ddot{\boldsymbol{q}}\end{bmatrix} + \boldsymbol{h}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\boldsymbol{q},\dot{\boldsymbol{q}})\begin{bmatrix}\dot{\theta}_0\\ \dot{\boldsymbol{\theta}}_r\\ \dot{\boldsymbol{q}}\end{bmatrix} + \begin{bmatrix}\boldsymbol{0}\\ \boldsymbol{Kq}\end{bmatrix} = \begin{bmatrix}0\\ \boldsymbol{\tau}\\ \boldsymbol{0}\end{bmatrix}. \qquad (8)$$

where, $\boldsymbol{\theta} = (\theta_0 \quad \boldsymbol{\theta}_r^{\mathrm{T}})^{\mathrm{T}} = (\theta_0 \quad \theta_1 \quad \theta_2)^{\mathrm{T}}$, $\boldsymbol{q} = (q_1 \quad q_2)^{\mathrm{T}}$; $\boldsymbol{D}(\boldsymbol{q}) \in \Re^{5\times5}$ is the symmetric positive-definite inertia matrix, $\boldsymbol{h}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\boldsymbol{q},\dot{\boldsymbol{q}})(\dot{\boldsymbol{\theta}}^{\mathrm{T}} \quad \dot{\boldsymbol{q}}^{\mathrm{T}})^{\mathrm{T}} \in \Re^{5\times1}$ is the vector of the Coriolis and centrifugal forces, $\boldsymbol{K} = \mathrm{diag}(k_{11},k_{22})$ is the positive-definite stiffness matrix, $\boldsymbol{\tau} = (\tau_0 \quad \tau_1 \quad \tau_2)^{\mathrm{T}}$ is the torque vector acting upon the rotational joints.

And for an arbitrary vector $\boldsymbol{z} \in \Re^5$, $\boldsymbol{h}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\boldsymbol{q},\dot{\boldsymbol{q}})$ has the following property [17]

$$\boldsymbol{z}^{\mathrm{T}}\boldsymbol{hz} = \frac{1}{2}\boldsymbol{z}^{\mathrm{T}}\dot{\boldsymbol{D}}\boldsymbol{z} . \qquad (9)$$

Although having similar form, structurally the dynamic equation of space flexible manipulator system is much more complex than that of terrestrial manipulator because of their dependence in part on base's attitude and mass properties. The coupling between the base and the arm motion is completely reflected in the new structure of $\boldsymbol{D}$ and $\boldsymbol{h}$. The $\boldsymbol{D}$ and $\boldsymbol{h}$ in (8) is nonlinearly dependent on inertial parameters of the system, but they can be linearly represented by combining function of a group selected inertial parameters. The result is good for the following design of robust control algorithm. And it is worth noting that the dynamic equation (8) doesn't include the position, velocity, and acceleration of the free-floating base, thus the control of base can be not required in the control design subsequent.

III. Singular Perturbation Approach

Because the vibration of flexible link is of relatively high frequency compared to the rigid manipulator motion, the separation of time constants between the manipulator motion and vibration control allows then to be considered separately. Therefore, we use the singular perturbation approach to derive a slow subsystem corresponding to the base's attitude and joint angles, and a fast subsystem describing the vibration of flexible link.

Let us define the inverse of the mass matrix

$$\boldsymbol{N} = \begin{bmatrix}\boldsymbol{N}_{11} & \boldsymbol{N}_{12}\\ \boldsymbol{N}_{21} & \boldsymbol{N}_{22}\end{bmatrix} = \begin{bmatrix}\boldsymbol{D}_{rr} & \boldsymbol{D}_{rf}\\ \boldsymbol{D}_{fr} & \boldsymbol{D}_{ff}\end{bmatrix}^{-1}. \qquad (10)$$

where, $\boldsymbol{D}_{rr} \in \Re^{3\times3}$, $\boldsymbol{D}_{rf} = \boldsymbol{D}_{fr}^{\mathrm{T}} \in \Re^{3\times2}$ and $\boldsymbol{D}_{ff} \in \Re^{2\times2}$ are the sub-matrices of $\boldsymbol{D}$. We multiply (9) by (10) from the left, rearrange terms, and write

$$\begin{bmatrix}\ddot{\boldsymbol{\theta}}\\ \ddot{\boldsymbol{q}}\end{bmatrix} = -\begin{bmatrix}\boldsymbol{N}_{11} & \boldsymbol{N}_{12}\\ \boldsymbol{N}_{21} & \boldsymbol{N}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{h}_{rr} & \boldsymbol{h}_{rf}\\ \boldsymbol{h}_{fr} & \boldsymbol{h}_{ff}\end{bmatrix}\begin{bmatrix}\dot{\boldsymbol{\theta}}\\ \dot{\boldsymbol{q}}\end{bmatrix} - \begin{bmatrix}\boldsymbol{N}_{11} & \boldsymbol{N}_{12}\\ \boldsymbol{N}_{21} & \boldsymbol{N}_{22}\end{bmatrix}\begin{bmatrix}0\\ \boldsymbol{Kq}\end{bmatrix} + \begin{bmatrix}\boldsymbol{N}_{11} & \boldsymbol{N}_{12}\\ \boldsymbol{N}_{21} & \boldsymbol{N}_{22}\end{bmatrix}\begin{bmatrix}\boldsymbol{\tau}'\\ 0\end{bmatrix}. \qquad (11)$$

where, $\boldsymbol{h}_{rr} \in \Re^{3\times3}$, $\boldsymbol{h}_{rf} = \boldsymbol{h}_{fr}^{\mathrm{T}} \in \Re^{3\times2}$ and $\boldsymbol{h}_{ff} \in \Re^{2\times2}$ are the sub-matrices of $\boldsymbol{h}$, $\boldsymbol{\tau}' = \boldsymbol{\Gamma\tau}$, $\boldsymbol{\Gamma} = [0 \quad \boldsymbol{I}]^{\mathrm{T}} \in \Re^{3\times2}$. Therefore, (11) can be rewrite as the following form

$$\ddot{\boldsymbol{\theta}} = -\boldsymbol{N}_{11}(\boldsymbol{h}_{rr}\dot{\boldsymbol{\theta}} + \boldsymbol{h}_{rf}\dot{\boldsymbol{q}}) - \boldsymbol{N}_{12}(\boldsymbol{h}_{fr}\dot{\boldsymbol{\theta}} + \boldsymbol{h}_{ff}\dot{\boldsymbol{q}}) - \boldsymbol{N}_{12}\boldsymbol{Kq} + \boldsymbol{N}_{11}\boldsymbol{\Gamma\tau} \quad (12)$$

$$\ddot{\boldsymbol{q}} = -\boldsymbol{N}_{21}(\boldsymbol{h}_{rr}\dot{\boldsymbol{\theta}} + \boldsymbol{h}_{rf}\dot{\boldsymbol{q}}) - \boldsymbol{N}_{22}(\boldsymbol{h}_{fr}\dot{\boldsymbol{\theta}} + \boldsymbol{h}_{ff}\dot{\boldsymbol{q}}) - \boldsymbol{N}_{22}\boldsymbol{Kq} + \boldsymbol{N}_{21}\boldsymbol{\Gamma\tau} \quad (13)$$

Assume $k = \min(k_{11},k_{22})$, then define new variable $\varepsilon^2 = 1/k$, $\varepsilon^2\xi = \boldsymbol{q}$ and $\widetilde{\boldsymbol{K}} = \varepsilon^2\boldsymbol{K}$, rewrite (12), (13) in terms of ξ and ε, we have

$$\begin{aligned}\ddot{\boldsymbol{\theta}} = &-\boldsymbol{N}_{11}(\boldsymbol{\theta},\varepsilon^2\xi)(\boldsymbol{h}_{rr}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\varepsilon^2\xi,\varepsilon^2\dot{\xi})\dot{\boldsymbol{\theta}} + \boldsymbol{h}_{rf}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\varepsilon^2\xi,\varepsilon^2\dot{\xi})\varepsilon^2\dot{\xi})\\ &-\boldsymbol{N}_{12}(\boldsymbol{\theta},\varepsilon^2\xi)(\boldsymbol{h}_{fr}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\varepsilon^2\xi,\varepsilon^2\dot{\xi})\dot{\boldsymbol{\theta}} + \boldsymbol{h}_{ff}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\varepsilon^2\xi,\varepsilon^2\dot{\xi})\varepsilon^2\dot{\xi})\\ &-\boldsymbol{N}_{12}(\boldsymbol{\theta},\varepsilon^2\xi)\widetilde{\boldsymbol{K}}\xi + \boldsymbol{N}_{11}(\boldsymbol{\theta},\varepsilon^2\xi)\boldsymbol{\Gamma\tau} ,\end{aligned} \qquad (14)$$

$$\begin{aligned}\varepsilon^2\ddot{\xi} = &-\boldsymbol{N}_{21}(\boldsymbol{\theta},\varepsilon^2\xi)(\boldsymbol{h}_{rr}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\varepsilon^2\xi,\varepsilon^2\dot{\xi})\dot{\boldsymbol{\theta}} + \boldsymbol{h}_{rf}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\varepsilon^2\xi,\varepsilon^2\dot{\xi})\varepsilon^2\dot{\xi})\\ &-\boldsymbol{N}_{22}(\boldsymbol{\theta},\varepsilon^2\xi)(\boldsymbol{h}_{fr}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\varepsilon^2\xi,\varepsilon^2\dot{\xi})\dot{\boldsymbol{\theta}} + \boldsymbol{h}_{ff}(\boldsymbol{\theta},\dot{\boldsymbol{\theta}},\varepsilon^2\xi,\varepsilon^2\dot{\xi})\varepsilon^2\dot{\xi})\\ &-\boldsymbol{N}_{22}(\boldsymbol{\theta},\varepsilon^2\xi)\widetilde{\boldsymbol{K}}\xi + \boldsymbol{N}_{21}(\boldsymbol{\theta},\varepsilon^2\xi)\boldsymbol{\Gamma\tau} .\end{aligned} \qquad (15)$$

Recall that the control objective is to determine the control input $\boldsymbol{\tau}$ to let $\boldsymbol{\theta}_r$ track desired trajectory sufficiently closely and to damp the vibration of flexible link. Therefore, a composite controller is designed based on the partially decoupled model in (14) and (15). Define the control input $\boldsymbol{\tau}$ to have two parts

$$\boldsymbol{\tau} = \boldsymbol{\tau}_s(\boldsymbol{\theta}) + \boldsymbol{\tau}_f(\boldsymbol{q}) . \tag{16}$$

where, $\boldsymbol{\tau}_s(\boldsymbol{\theta})$ is the slow control component and $\boldsymbol{\tau}_f(\boldsymbol{q})$ is a fast control component. As shown in (14) and (15), the manipulator rigid motion and vibration of flexible link are coupled only through the control input $\boldsymbol{\tau}$. Generally speaking, the given desired trajectory can be achieved by appropriate selection of $\boldsymbol{\tau}$. However, the selection based only on the desired trajectory can't guarantee a stable fast subsystem. Hence, we will design a composite controller to track the desired trajectory as well as to stabilize the flexible dynamics by singular perturbation approach.

Setting $\varepsilon = 0$, the slow manifold equation can solved from (15)

$$\boldsymbol{\xi}_s = \widetilde{\boldsymbol{K}}^{-1}\boldsymbol{N}_{22s}^{-1}[-\boldsymbol{N}_{21s}\boldsymbol{h}_{rrs}\dot{\boldsymbol{\theta}} - \boldsymbol{N}_{22s}\boldsymbol{h}_{frs}\dot{\boldsymbol{\theta}} + \boldsymbol{N}_{21s}\boldsymbol{\Gamma}\boldsymbol{\tau}_s] . \tag{17}$$

where, the superscripts s indicate that the corresponding variable is in the slow subsystem. It's noted that

$$\boldsymbol{D}_{rrs}^{-1}(\boldsymbol{\theta},0) = \boldsymbol{N}_{11s}(\boldsymbol{\theta},0) - \boldsymbol{N}_{12s}(\boldsymbol{\theta},0)\boldsymbol{N}_{22s}^{-1}(\boldsymbol{\theta},0)\boldsymbol{N}_{21s}(\boldsymbol{\theta},0) . \tag{18}$$

Using (14) and (18), we obtain the slow subsystem as

$$\boldsymbol{D}_{rrs}\ddot{\boldsymbol{\theta}} + \boldsymbol{h}_{rrs}\dot{\boldsymbol{\theta}} = \begin{bmatrix}0 & \boldsymbol{\tau}_s^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}} . \tag{19}$$

To derive the fast subsystem, select new states $\boldsymbol{\zeta}_1 = \boldsymbol{\xi} - \bar{\boldsymbol{\xi}}$, $\boldsymbol{\zeta}_2 = \varepsilon\,\dot{\boldsymbol{\xi}}$, and write (15) as

$$\begin{aligned}\varepsilon\,\dot{\boldsymbol{\zeta}}_2 &= -\boldsymbol{N}_{21}(\boldsymbol{h}_{rr}\dot{\boldsymbol{\theta}} + \boldsymbol{h}_{rf}\varepsilon\,\boldsymbol{\zeta}_2) - \boldsymbol{N}_{22}(\boldsymbol{h}_{fr}\dot{\boldsymbol{\theta}} + \boldsymbol{h}_{ff}\varepsilon\,\boldsymbol{\zeta}_2) \\ &\quad - \boldsymbol{N}_{22}\widetilde{\boldsymbol{K}}(\boldsymbol{\zeta}_1 + \bar{\boldsymbol{\xi}}) + \boldsymbol{N}_{21}\boldsymbol{\Gamma}\boldsymbol{\tau} .\end{aligned} \tag{20}$$

Moreover, we introduce a time-scale change of $\sigma = t/\varepsilon$. Setting $\varepsilon = 0$, the expression of fast subsystem can now be obtained

$$\frac{\mathrm{d}\boldsymbol{\zeta}_1}{\mathrm{d}\sigma} = \boldsymbol{\zeta}_2 , \tag{21}$$

$$\frac{\mathrm{d}\boldsymbol{\zeta}_2}{\mathrm{d}\sigma} = -\boldsymbol{N}_{22s}\widetilde{\boldsymbol{K}}\boldsymbol{\zeta}_1 + \boldsymbol{N}_{21s}\boldsymbol{\Gamma}\boldsymbol{\tau}_f . \tag{22}$$

The state-space representation of the dynamics is

$$\frac{\mathrm{d}}{\mathrm{d}\sigma}\begin{bmatrix}\boldsymbol{\zeta}_1\\ \boldsymbol{\zeta}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\boldsymbol{N}_{22s}\widetilde{\boldsymbol{K}} & 0\end{bmatrix}\begin{bmatrix}\boldsymbol{\zeta}_1\\ \boldsymbol{\zeta}_2\end{bmatrix} + \begin{bmatrix}0\\ \boldsymbol{N}_{21s}\boldsymbol{\Gamma}\end{bmatrix}\boldsymbol{\tau}_f . \tag{23}$$

IV. Controller Design

The control objective attempts to determine the input control torque $\boldsymbol{\tau}$, such that vibration of flexible link damps out as efficiently as possible while $\boldsymbol{\theta}_r$ follows the desired tracking. Using two-time scale theory, $\boldsymbol{\tau}_s(\boldsymbol{\theta})$ and $\boldsymbol{\tau}_f(\boldsymbol{q})$ are adopted in two different time scales. It is intended that $\boldsymbol{\tau}_f(\boldsymbol{q})$ affects mainly on (22), flexible vibration and $\boldsymbol{\tau}_s(\boldsymbol{\theta})$ principally on (19), decoupled rigid dynamics.

A. *Robust controller of coordinated motion*

In order to guarantee that the dynamic equation is linearly dependent on the dynamic parameters of the system, we extend output vector $\boldsymbol{\theta}_r$ to be augmented output vector $\boldsymbol{\theta} = [\theta_0 \quad \boldsymbol{\theta}_r^{\mathrm{T}}]^{\mathrm{T}}$; where θ_0 is the gesture angle of the base. The augmented output error vector $\boldsymbol{e}$ between desired augmented output $\boldsymbol{\theta}_{\mathrm{d}} = [\theta_0 \quad \theta_{1\mathrm{d}} \quad \theta_{2\mathrm{d}}]^{\mathrm{T}}$ and actual augmented output $\boldsymbol{\theta}$ can be written as

$$\boldsymbol{e} = \boldsymbol{\theta}_{\mathrm{d}} - \boldsymbol{\theta} = [0 \quad \boldsymbol{e}_r^{\mathrm{T}}]^{\mathrm{T}} . \tag{24}$$

Let an extended output error be defined by

$$\hat{\boldsymbol{s}} = \dot{\boldsymbol{e}} + \lambda\boldsymbol{e} . \tag{25}$$

where, $\lambda > 0$ is a scalar constant. Let's further define reference output velocity $\dot{\boldsymbol{\eta}}$ and acceleration $\ddot{\boldsymbol{\eta}}$ as follows

$$\dot{\boldsymbol{\eta}} = \dot{\boldsymbol{\theta}}_{\mathrm{d}} + \lambda\boldsymbol{e} ,\ \ddot{\boldsymbol{\eta}} = \ddot{\boldsymbol{\theta}}_{\mathrm{d}} + \lambda\dot{\boldsymbol{e}} . \tag{26}$$

From (25) and (26), we have

$$\hat{\boldsymbol{s}} = \dot{\boldsymbol{\eta}} - \dot{\boldsymbol{\theta}} ,\ \dot{\hat{\boldsymbol{s}}} = \ddot{\boldsymbol{\eta}} - \ddot{\boldsymbol{\theta}} . \tag{27}$$

Finally, equation (19) can be rewritten as

$$\boldsymbol{D}_{rrs}\dot{\hat{\boldsymbol{s}}} + \boldsymbol{h}_{rrs}\hat{\boldsymbol{s}} = \boldsymbol{D}_{rrs}\ddot{\boldsymbol{\eta}} + \boldsymbol{h}_{rrs}\dot{\boldsymbol{\eta}} - \begin{bmatrix}0 & \boldsymbol{\tau}_s^{\mathrm{T}}\end{bmatrix}^{\mathrm{T}} . \tag{28}$$

We now define the following robust control scheme

$$\begin{bmatrix}0\\ \boldsymbol{\tau}_s\end{bmatrix} = \widetilde{\boldsymbol{D}}_{rrs}\ddot{\boldsymbol{\eta}} + \widetilde{\boldsymbol{h}}_{rrs}\dot{\boldsymbol{\eta}} + \boldsymbol{K}_s\hat{\boldsymbol{s}} + \boldsymbol{W}_D\boldsymbol{u}_D + \boldsymbol{W}_h\boldsymbol{u}_h + \begin{bmatrix}\delta\\ \boldsymbol{0}\end{bmatrix} . \tag{29}$$

where, $\widetilde{\boldsymbol{D}}$, $\widetilde{\boldsymbol{h}}$ represent the matrixes $\boldsymbol{D}$, $\boldsymbol{h}$ in the model plant respectively, $\boldsymbol{K}_s$ is a 3×3 positive-definite, symmetric constant matrix, the parameter δ plays a key important role to let the equality of (29) be satisfied. The matrix $\boldsymbol{W}_D$ and $\boldsymbol{W}_h$ is defined respectively as: $\boldsymbol{W}_D\boldsymbol{\Phi}_D = \boldsymbol{D}_{rrs}\ddot{\boldsymbol{\eta}}$, $\boldsymbol{W}_h\boldsymbol{\Phi}_h = \boldsymbol{h}_{rrs}\dot{\boldsymbol{\eta}}$; where, $\boldsymbol{W}_D$ and $\boldsymbol{W}_h$ are the matrix functions of $\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}, \dot{\boldsymbol{\eta}}, \ddot{\boldsymbol{\eta}}$ and also independent of the inertial parameters; $\boldsymbol{\Phi}_D$ and $\boldsymbol{\Phi}_h$ are the parameter deviation vectors from the real plant. $\boldsymbol{u}_D$ and $\boldsymbol{u}_h$ can be design as

$$\boldsymbol{u}_D = \begin{cases}\rho_D\dfrac{\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}}}{\left\|\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}}\right\|} & \left\|\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}}\right\| > \varepsilon_D\\ \rho_D\dfrac{\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}}}{\varepsilon_D} & \left\|\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}}\right\| \le \varepsilon_D\end{cases} ,$$

$$\boldsymbol{u}_h = \begin{cases}\rho_h\dfrac{\boldsymbol{W}_h^{\mathrm{T}}\hat{\boldsymbol{s}}}{\left\|\boldsymbol{W}_h^{\mathrm{T}}\hat{\boldsymbol{s}}\right\|} & \left\|\boldsymbol{W}_h^{\mathrm{T}}\hat{\boldsymbol{s}}\right\| > \varepsilon_h\\ \rho_h\dfrac{\boldsymbol{W}_h^{\mathrm{T}}\hat{\boldsymbol{s}}}{\varepsilon_h} & \left\|\boldsymbol{W}_h^{\mathrm{T}}\hat{\boldsymbol{s}}\right\| \le \varepsilon_h\end{cases} . \tag{30}$$

where, ε_D and ε_h are arbitrary positive values, and ρ_D, ρ_h are chosen to satisfy

$$\rho_D \geq \left\|\overline{\boldsymbol{\Phi}}_D\right\| = \left\|\boldsymbol{\Phi}_D - \widetilde{\boldsymbol{\Phi}}_D\right\|,\ \rho_h \geq \left\|\overline{\boldsymbol{\Phi}}_h\right\| = \left\|\boldsymbol{\Phi}_h - \widetilde{\boldsymbol{\Phi}}_h\right\|. \quad (31)$$

where, $\overline{\boldsymbol{\Phi}}_D = \boldsymbol{\Phi}_D - \widetilde{\boldsymbol{\Phi}}_D$, $\overline{\boldsymbol{\Phi}}_h = \boldsymbol{\Phi}_h - \widetilde{\boldsymbol{\Phi}}_h$; $\widetilde{\boldsymbol{\Phi}}_D$ and $\widetilde{\boldsymbol{\Phi}}_h$ are the parameter deviation vectors from the model plant.

Substituting (29) into (19), the closed loop system equation is obtained as

$$\boldsymbol{D}_{rrs}\dot{\hat{\boldsymbol{s}}} + \boldsymbol{h}_{rrs}\hat{\boldsymbol{s}} + \boldsymbol{K}_s\hat{\boldsymbol{s}} = \boldsymbol{W}_D(\overline{\boldsymbol{\Phi}}_D - \boldsymbol{u}_D) + \boldsymbol{W}_h(\overline{\boldsymbol{\Phi}}_h - \boldsymbol{u}_h) + [\delta \quad \boldsymbol{0}]^{\mathrm{T}}. \quad (32)$$

Then the following theorem can be obtained

Theorem: The control scheme (29) guarantee $\lim_{t\to\infty}\hat{\boldsymbol{s}} = 0$.

Proof: Choosing the Lyapunov function candidate as

$$V = \frac{1}{2}\hat{\boldsymbol{s}}^{\mathrm{T}}\boldsymbol{D}_{rrs}\hat{\boldsymbol{s}}. \quad (33)$$

Differentiating V and using (9), (25) and (32), we have

$$\begin{aligned}\dot{V} &= \hat{\boldsymbol{s}}^{\mathrm{T}}\boldsymbol{D}_{rrs}\dot{\hat{\boldsymbol{s}}} + \frac{1}{2}\hat{\boldsymbol{s}}^{\mathrm{T}}\dot{\boldsymbol{D}}_{rrs}\hat{\boldsymbol{s}} \\ &= \hat{\boldsymbol{s}}^{\mathrm{T}}(\boldsymbol{D}_{rrs}\dot{\hat{\boldsymbol{s}}} + \boldsymbol{h}_{rrs}\hat{\boldsymbol{s}}) \\ &= \hat{\boldsymbol{s}}^{\mathrm{T}}\{\boldsymbol{W}_D(\overline{\boldsymbol{\Phi}}_D - \boldsymbol{u}_D) + \boldsymbol{W}_h(\overline{\boldsymbol{\Phi}}_h - \boldsymbol{u}_h) - \boldsymbol{K}_s\hat{\boldsymbol{s}} + [\delta \quad \boldsymbol{0}]^{\mathrm{T}}\} \\ &= -\hat{\boldsymbol{s}}^{\mathrm{T}}\boldsymbol{K}_s\hat{\boldsymbol{s}} + \hat{\boldsymbol{s}}^{\mathrm{T}}[\boldsymbol{W}_D(\overline{\boldsymbol{\Phi}}_D - \boldsymbol{u}_D) + \boldsymbol{W}_h(\overline{\boldsymbol{\Phi}}_h - \boldsymbol{u}_h)]. \end{aligned} \quad (34)$$

From the second term of the right side of (34), if $\left\|\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}}\right\| > \varepsilon_D$

$$\begin{aligned}\hat{\boldsymbol{s}}^{\mathrm{T}}\boldsymbol{W}_D(\overline{\boldsymbol{\Phi}}_D - \boldsymbol{u}_D) &= (\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}})^{\mathrm{T}}(\overline{\boldsymbol{\Phi}}_D - \boldsymbol{u}_D) \\ &= (\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}})^{\mathrm{T}}(\overline{\boldsymbol{\Phi}}_D - \rho_D\frac{\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}}}{\left\|\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}}\right\|}). \\ &= \left\|\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}}\right\|(\left\|\overline{\boldsymbol{\Phi}}_D\right\| - \rho_D) \leq 0\end{aligned} \quad (35)$$

If $\left\|\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}}\right\| \leq \varepsilon_D$

$$\begin{aligned}\hat{\boldsymbol{s}}^{\mathrm{T}}\boldsymbol{W}_D(\overline{\boldsymbol{\Phi}}_D - \boldsymbol{u}_D) &= (\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}})^{\mathrm{T}}(\overline{\boldsymbol{\Phi}}_D - \boldsymbol{u}_D) \\ &\leq (\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}})^{\mathrm{T}}(\rho_D\frac{\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}}}{\left\|\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}}\right\|} - \rho_D\frac{\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}}}{\varepsilon_D}).\end{aligned} \quad (36)$$

The last term of (36) achieve a maximum value of $\varepsilon_D\rho_D/4$ when $\left\|\boldsymbol{W}_D^{\mathrm{T}}\hat{\boldsymbol{s}}\right\| = \varepsilon_D/2$, then we have

$$\hat{\boldsymbol{s}}^{\mathrm{T}}\boldsymbol{W}_D(\overline{\boldsymbol{\Phi}}_D - \boldsymbol{u}_D) \leq \varepsilon_D\rho_D/4. \quad (37)$$

Similarly, we have

$$\hat{\boldsymbol{s}}^{\mathrm{T}}\boldsymbol{W}_h(\overline{\boldsymbol{\Phi}}_h - \boldsymbol{u}_h) \leq \varepsilon_h\rho_h/4. \quad (38)$$

Hence, by combining (34), (37) and (38), we obtain

$$\dot{V} \leq -\lambda\min(\boldsymbol{K}_s)\left\|\hat{\boldsymbol{s}}\right\|^2 + \varepsilon_D\rho_D/4 + \varepsilon_h\rho_h/4. \quad (39)$$

From (39), we have $\dot{V} \leq 0$ for $\left\|\hat{\boldsymbol{s}}\right\| \geq \omega$, where

$$\omega^2 = \frac{1}{2}(\varepsilon_D\rho_D + \varepsilon_h\rho_h)/\lambda\min(\boldsymbol{K}_s). \quad (40)$$

Furthermore, as $\varepsilon_D, \varepsilon_h \to 0$, then $\omega \to 0$, which means that $\left\|\hat{\boldsymbol{s}}\right\| \to 0$.

Thus, the control scheme (29) asymptotically stabilize the space flexible manipulator to track the desired trajectory described in terms of $\boldsymbol{\theta}_{\mathrm{d}}$, $\dot{\boldsymbol{\theta}}_{\mathrm{d}}$.

B. Controller design of fast subsystem

The fast subsystem can be written as

$$\dot{\boldsymbol{X}} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{B}\boldsymbol{\tau}_f. \quad (41)$$

where, $\boldsymbol{X} = [\boldsymbol{\zeta}_1 \quad \boldsymbol{\zeta}_2]^{\mathrm{T}}$, $\boldsymbol{A} = \begin{bmatrix} 0 & \boldsymbol{I} \\ -\boldsymbol{N}_{22s}\widetilde{\boldsymbol{K}} & 0 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} 0 \\ \boldsymbol{N}_{21s}\boldsymbol{\Gamma} \end{bmatrix}$.

To simplify the controller design for the fast subsystem, the uncertainty of the system is not considered here. Because the pair $(\boldsymbol{A}, \boldsymbol{B})$ of the fast subsystem are completely state controllable, a fast state feedback control can be devised to force its states $\boldsymbol{X}$ to zero by using optimal LQR approach control method. The cost function is chosen as

$$E = \frac{1}{2}\int_0^\infty(\boldsymbol{X}^{\mathrm{T}}\boldsymbol{Q}\boldsymbol{X} + \boldsymbol{\tau}_f^{\mathrm{T}}\boldsymbol{R}\boldsymbol{\tau}_f)\mathrm{d}t. \quad (42)$$

where, $\boldsymbol{Q}$ and $\boldsymbol{R}$ are the standard LQR weighting matrices. The fast controller can be obtained as

$$\boldsymbol{\tau}_f = -\boldsymbol{K}_{\mathrm{opt}}\boldsymbol{X}(t) = -\boldsymbol{R}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{X}(t). \quad (43)$$

where, $\boldsymbol{P}$ is the solution of the following Ricatti equation

$$\boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} - \boldsymbol{P}\boldsymbol{B}\boldsymbol{R}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{Q} = 0. \quad (44)$$

By combining the slow control (29) and fast control component (43), the system follows the desired trajectory and damps vibration of flexible link.

V. NUMERICAL SIMULATION

To show the performance of the proposed controller, a simulation is carried out on a planar space flexible manipulator system shown in Figure 1. The actual plant parameters of the system are as follow: $m_0 = 40\mathrm{kg}$, $m_1 = 2\mathrm{kg}$, $m_p = 2\mathrm{kg}$; $J_0 = 35\mathrm{kg\cdot m^2}$, $J_1 = 1.5\mathrm{kg\cdot m^2}$; $l_1 = 3.0\mathrm{m}$, $a = a_1 = 1.5\mathrm{m}$; $\rho = 1\mathrm{kg/m}$, $EI = 200\mathrm{pa}$. The values of payloads' mass m_p and flexible link's density ρ in the simulation are uncertain, and the uncertainty range is $3\mathrm{kg} \geq m_p \geq 0\mathrm{kg}$, $2\mathrm{kg/m} \geq \rho \geq 0\mathrm{kg/m}$. Their initial estimated values are $m_p = 1\mathrm{kg}$, $\rho = 0.5\mathrm{kg/m}$.

The desired trajectories of base's attitude and two joint angles are chosen as

$$\theta_{1d} = 1.57\left(\frac{t}{10} - \frac{1}{2\pi}\sin 0.2\pi t\right),$$

$$\theta_{2d} = 1.57\left(1 - \frac{t}{10} + \frac{1}{2\pi}\sin 0.2\pi t\right).$$

The initial state of the space flexible manipulator is as follow

$$\theta_0(0) = 0.1\text{rad},\ \theta_1(0) = 0.1\text{rad},\ \theta_2(0) = 1.47\text{rad}.$$

The time taking in the simulation is $t = 10\text{s}$. Fig.2 – Fig.5 show the simulation results. Fig.2 shows the angle change curves of the base; Fig.3 is the joint angles' trajectories comparison of manipulator; Fig.4 and Fig.5 show the first and second mode of flexible link. From the simulation results in Fig.2-5, it is shown that the proposed composite control scheme guarantees the system to track the desired trajectory and damp the vibration.

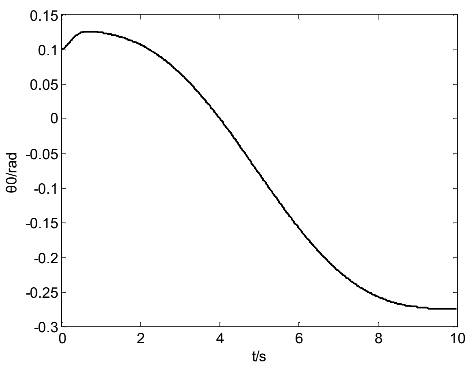

Figure 2. The angle change curves of the base

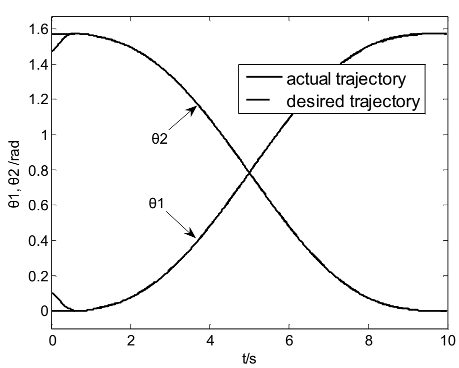

Figure 3. Trajectory comparison of the joint angles

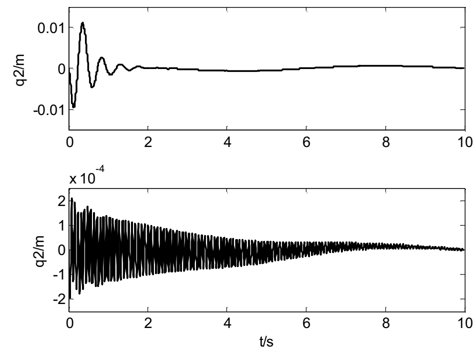

Figure 4. Mode of flexible link without vibration control

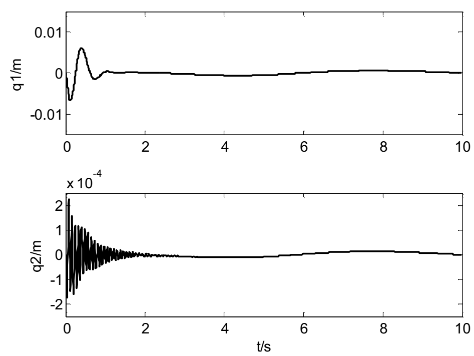

Figure 5. Mode of flexible link with vibration control

VI. Conclusion

The robust control and active vibration control of free-floating space flexible manipulator are studied in this paper. The dynamics model is divided into a slow subsystem and a fast subsystem using assumed mode method and singular perturbation theory. A robust control algorithm is applied to control the slow subsystem with consideration of model uncertainty. The fast subsystem controller will actively damp out the vibration using optimal LQR method. Simulation results confirm that the effectiveness of the proposed composite controller in the presence of parameter uncertainty.

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