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New delay-dependent conditions for $\mathscr{H}_\infty$ filtering of nonlinear systems with neutral-type time-delay

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***Abstract*— This paper focuses on the observer design for nonlinear time-delay systems of neutral type. Nonlinearities of the system under consideration are supposed to be globally Lipschitz with respect to their arguments. We assume that the system output is corrupted by a norm-bounded noise and may contain delayed states. We brought new delay-dependent conditions under which the fault-tolerant observer converges with a prescribed level of uncertainty attenuation.**

***Index Terms*— State monitoring; Network systems; Fault-tolerant observers; $\mathscr{H}_\infty$ filtering; Neutral delay; Convex optimization.**

I. Introduction

THE history of stabilization of systems with after effects goes back to the 18th century. The study of this class of systems has received substantial attention in the early 20th century in works devoted to modelling of network systems, biological, ecological and engineering systems. The subsequent developments of stability and stabilization for time-delay systems are numerous and can be traced in [1], [2], [3], [4], [5], [6] and the references therein.

Neutral-type time-delay systems is a special class of systems that appear in many engineering fields as distributed networks containing lossless transmission lines, chemical engineering reactor applications, ship stabilization and VLSI systems, see for example the references [7], [8], [9]. Even though considerable research efforts have been undertaken on various aspects of dynamical systems with time delays [1], [2], the observer design for nonlinear systems with neutral-type delays has received a little attention. In addition, the presence of nonlinearities in the dynamics of the neutral delay system increases the complexity of the observer design which is known to be quite challenging if a linear output injection term is used to handle the effects of nonlinearities. Preliminary discussions of this problem can be found in the references [10], [11], [12], [13], [14], [15]. Recent results on observation of Lipschitz nonlinear systems can be also be traced in the references [16], [17] where equivalent representation of the Lipschitz property is introduced to establish a less conservative observer design. For systems with positive-slope nonlinearities a novel design with nonlinear output injections are proposed in the references [18], [19].

The available results on filtering and observation of neural time-delay systems can be broadly classified into delay-dependent and delay-independent techniques, see for instance [20]. Despite the fact that delay-independent observer design is considered as simple and straightforward design methodologies, the delay-dependent techniques remain preferable for their robustness and highly satisfactory performances. In this paper, we lay down new matrix inequality conditions for observer/filter design of neutral-type time-delay systems subject to state-delayed nonlinearities. We show that the conditions that guarantee the existence of the observer/filter with uncertainty attenuation are almost derived without bounding the cross-terms that may involve nonlinearities; and hence, a less conservative design is obtained through the solution of set of matrix inequalities that are numerically tractable.

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Throughout this paper, we note by $\mathbb{R}$ the set of real numbers. The notation $A > 0$ (resp. $A < 0$) means that the matrix A is positive definite (resp. negative definite). A' is the matrix transpose of A. "$\star$" is used to notify an element which is induced by transposition. $\triangleq$ stands for an equality by definition. $\mathbf{0}$ stands for the null-matrix of appropriate dimensions and I represents the identity matrix of appropriate dimensions.

Before presenting the system description, let us introduce some key Lemmas that are used in the proofs of the main statements.

Lemma 1 (The Schur complement lemma): [21] Given constant matrices M, N, Q of appropriate dimensions where M and Q are symmetric, then $Q > 0$ and $M + N'Q^{-1}N < 0$ if and only if

$$\begin{bmatrix} M & N' \\ N & -Q \end{bmatrix} < 0, \text{ or equivalently } \begin{bmatrix} -Q & N \\ N' & M \end{bmatrix} < 0.$$

Lemma 2: For any constant symmetric matrix $M \in \mathbb{R}^{n\times n}$, $M = M' > 0$, scalar $\gamma > 0$, vector function $\omega : [0, \gamma] \mapsto \mathbb{R}^n$ such that the integration in the following is well defined, we have

$$\gamma \int_0^\gamma \omega'(\beta) M \omega(\beta) \mathrm{d}\beta \geq \left(\int_0^\gamma \omega(\beta)\mathrm{d}\beta\right)' M \left(\int_0^\gamma \omega(\beta)\mathrm{d}\beta\right). \tag{1}$$

Proof: See reference [1].

II. System description

Let us consider the nonlinear system

$$\begin{aligned} \dot{x}(t) - D\dot{x}(t-h) &= A\,x(t) + A_d\,x(t-h) \\ &\quad + f\Big(x(t), x(t-h), u(t)\Big), \\ y(t) &= C\,x(t) + C_d\,x(t-h) + D_1\,\xi(t), \\ x(t) &= \varphi(t),\ t \leq h. \end{aligned} \tag{2}$$

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where $x(t) \in \mathbb{R}^n$ is the state vector and $u(t)$ is the system input, and $y(t) \in \mathbb{R}^p$ is the system output. h is a known positive delay, and $\varphi(t)$ is well-defined time-dependent function. To complete the description of the neutral-type time-delay system, let us present the following assumption.

Assumption 1: The Euclidean norm of the matrix D is less than one.

Assumption 2: The system disturbance $\xi(t)$ is norm-bounded such that for all $t \geq 0$ $\|\xi(t)\| \leq C_\xi$ where $C_\xi \in \mathbb{R}_{>0}$.

Assumption 3: All the system nominal matrices A, A_d, C, C_d and D_1 are real and well-defined.

Assumption 4: The system input is globally bounded for all $t \geq 0$ and belongs to the set of bounded inputs $\mathscr{U}$ for which the system is uniformly observable.

Assumption 5: The system nonlinearity $f\Big(x(t), x(t-h), u(t)\Big)$ is globally Lipschitz with respect to $x(t)$ and $x(t-h)$, uniformly to $u(t)$. We assume that for all $x(t) \in \mathscr{M} \subset \mathbb{R}^n$ and $u(t) \in \mathscr{U} \subset \mathbb{R}^m$ there exist two set of matrices $(A_i)_{1\leq i\leq \mu}$ and $(A_i^d)_{1\leq i\leq \nu}$ such that for all $\theta \in \mathbb{R}^n$, $\phi \in \mathbb{R}^n$

$$\begin{aligned}\frac{\partial f(\theta,\phi,u)}{\partial \theta} &= F(\theta,\phi,u) \in \mathbf{Co}\left\{A_1, A_2, \cdots, A_\mu\right\},\\ \frac{\partial f(\theta,\phi,u)}{\partial \phi} &= F_d(\theta,\phi,u) \in \mathbf{Co}\left\{A_1^d, A_2^d, \cdots, A_\nu^d\right\}.\end{aligned} \tag{3}$$

The main objective of this paper is to develop a robust filter that can estimate the system states with uncertainty attenuation. The delay-dependent conditions that guarantee the existence of the observer is detailed in the following section.

III. $\mathscr{H}_\infty$-FILTER DESIGN

As shown in the previous section, the presence of both the uncertainty $\xi(t)$ and the delayed states $C_d x(t-h)$ in the equation of the system output makes the design of high-gain observers quite difficult and challenging if a prescribed level of uncertainty is imposed. This comes from the fact that the use of a high-gain output injection, that is indispensable for the stability of the observation error, increases considerably the level of uncertainty in the filter estimates. Therefore, the use of filtered-outputs becomes necessary to handle the effects of the disturbance $\xi(t)$.

If we assume that the output variable $y(t)$ is fully measured then, the $\eta(t)$ state variable, that is the solution of the differential equations,

$$\dot{\eta}(t) = \Xi\,\eta(t) + y(t), \quad \Xi \in \mathbb{R}^{p\times p} \tag{4}$$

can be also measured. Consequently, if we see the output $\eta(t)$ as *the new output of the system* then, the system uncertainty along with the delayed states $C_d\,x(t-h)$ totally disappear from the equation of the output. As a result, by adding the dynamics of $\eta(t)$ to the original system dynamics, we have

$$\begin{aligned}&\dot{\eta}(t) = \Xi\,\eta(t) + C\,x(t) + C_d\,x(t-h) + D_1\,\xi(t),\\ &\dot{x}(t) - D\dot{x}(t-h) = A\,x(t) + A_d\,x(t-h)\\ &\qquad\qquad + f\Big(x(t), x(t-h), u(t)\Big),\\ &\tilde{y}(t) = \eta(t),\end{aligned} \tag{5}$$

where $\tilde{y}$ stands for the new output and Ξ is a real Hurwitz matrix. By putting $z(t) = \begin{bmatrix}\eta(t)\\ x(t)\end{bmatrix}$ then, the last system is rewritten as

$$\begin{aligned}&\dot{z}(t) - \widetilde{D}\dot{z}(t-h) = A\,z(t) + \widetilde{A}_d\,z(t-h)\\ &\qquad + \widetilde{f}\Big(z(t), z(t-h), u(t)\Big) + \widetilde{D}_1\,\xi(t),\\ &\tilde{y}(t) = \widetilde{C}\,z(t),\end{aligned} \tag{6}$$

where

$$\begin{aligned}&\widetilde{A} = \begin{bmatrix}\Xi & C\\ \mathbf{0} & A\end{bmatrix},\ \widetilde{D} = \begin{bmatrix}\mathbf{0} & \mathbf{0}\\ \mathbf{0} & D\end{bmatrix},\\ &\widetilde{D}_1 = \begin{bmatrix}D_1\\ \mathbf{0}\end{bmatrix},\ \widetilde{C}' = \begin{bmatrix}I\\ \mathbf{0}\end{bmatrix},\ \widetilde{A}_d = \begin{bmatrix}\mathbf{0} & C_d\\ \mathbf{0} & A_d\end{bmatrix},\\ &\widetilde{f}(z(t), z(t-h), u(t)) = \begin{bmatrix}\mathbf{0}\\ f(x(t), x(t-h), u(t))\end{bmatrix}.\end{aligned} \tag{7}$$

According to the above representation, the corresponding observer is

$$\begin{aligned}&\dot{\hat{z}}(t) - \widetilde{D}\dot{\hat{z}}(t-h) = A\,\hat{z}(t) + \widetilde{A}_d\,\hat{z}(t-h)\\ &\quad + \widetilde{f}\Big(\hat{z}(t), \hat{z}(t-h), u(t)\Big) + P^{-1}Y\widetilde{C}(\hat{z}(t) - z(t)).\end{aligned} \tag{8}$$

where $P = P' > 0$ and Y are real matrices of appropriate dimensions to be determined. The design of the observer gains with uncertainty attenuation is detailed in the following statement.

Theorem 1: Consider system (6) and observer (8) under Assumptions 1-5. Based on Assumption 5, let $(\widetilde{A}_i)_{1\leq i\leq\mu}$ and $(\widetilde{A}_i^d)_{1\leq i\leq\nu}$ be the convex-hull matrices such that for all $\alpha \in \mathbb{R}^{n+p}$, $\beta \in \mathbb{R}^{n+p}$, we have

$$\begin{aligned}\frac{\partial \widetilde{f}(\alpha,\beta,u)}{\partial \alpha} &\in \mathbf{Co}\left\{\widetilde{A}_1, \widetilde{A}_2, \cdots, \widetilde{A}_\mu\right\},\\ \frac{\partial \widetilde{f}(\alpha,\beta,u)}{\partial \beta} &\in \mathbf{Co}\left\{\widetilde{A}_1^d, \widetilde{A}_2^d, \cdots, \widetilde{A}_\nu^d\right\}.\end{aligned} \tag{9}$$

If for given γ^2 there exist five symmetric and positive definite matrices P, Q, Q_1, Q_2, Z of appropriate dimensions and a real matrix Y such that the set of the following matrix inequalities hold for $1 \leq i \leq \mu$, $1 \leq j \leq \nu$

$$\begin{bmatrix}\mathscr{L}_{11} & \mathscr{L}_{12} & \mathscr{L}_{13} & \mathscr{L}_{14} & \mathscr{L}_{15} & \mathscr{L}_{16} & \mathscr{L}_{17}\\ \star & \mathscr{L}_{22} & \mathscr{L}_{23} & \mathscr{L}_{24} & \mathscr{L}_{25} & \mathscr{L}_{26} & \mathscr{L}_{27}\\ \star & \star & \mathscr{L}_{33} & \mathscr{L}_{34} & \mathscr{L}_{35} & \mathscr{L}_{36} & \mathscr{L}_{37}\\ \star & \star & \star & \mathscr{L}_{44} & \mathscr{L}_{45} & \mathscr{L}_{46} & \mathscr{L}_{47}\\ \star & \star & \star & \star & \mathscr{L}_{55} & \mathscr{L}_{56} & \mathscr{L}_{57}\\ \star & \star & \star & \star & \star & \mathscr{L}_{66} & \mathscr{L}_{67}\\ \star & \star & \star & \star & \star & \star & \mathscr{L}_{77}\end{bmatrix}(i,j) < 0 \tag{10}$$

then, the observation error is globally asymptotically stable for $\xi(t) \equiv 0$ and for the null initial conditions $e(s-h) = 0$ $\forall s \leq h$, the following integral inequality holds for all $t \geq 0$

$$\int_0^t \{e'(s)\widetilde{C}'\widetilde{C}e(s) - \gamma^2 \xi'(s)\xi(s)\} \mathrm{d}s < 0. \tag{11}$$

where $e(s) = \hat{z}(s) - z(s)$ and

$$
\begin{aligned}
\mathscr{L}_{11} &\triangleq P\left(\widetilde{A}_i^c + \widetilde{A}_d + \widetilde{A}_j^d\right) + \left(\widetilde{A}_i^{c'} + \widetilde{A}_d' + \widetilde{A}_j^{d'}\right)P + Q \\
&\quad + h\,\widetilde{A}_i^{c'} Q_1\,\widetilde{A}_i^c + \widetilde{C}'\widetilde{C}, \\
\mathscr{L}_{12} &\triangleq -\left(\widetilde{A}_i^c + \widetilde{A}_d + \widetilde{A}_j^d\right)' P\,\widetilde{D} + h\widetilde{A}_i^{c'} Q_1 \left(\widetilde{A}_d + \widetilde{A}_j^d\right), \\
\mathscr{L}_{13} &\triangleq h\widetilde{A}_i^{c'} Q_1\,\widetilde{D}, \\
\mathscr{L}_{14} &\triangleq -hP\left(\widetilde{A}_d + \widetilde{A}_j^d\right), \\
\mathscr{L}_{15} &\triangleq \mathbf{0}, \\
\mathscr{L}_{16} &\triangleq -P\,\widetilde{D}_1 - h\widetilde{A}_i^{c'} Q_1\,\widetilde{D}_1, \\
\mathscr{L}_{17} &\triangleq -h\widetilde{A}_i^{c'} Z, \\
\mathscr{L}_{22} &\triangleq -Q + h\left(\widetilde{A}_d + \widetilde{A}_j^d\right)' Q_1 \left(\widetilde{A}_d + \widetilde{A}_j^d\right), \\
\mathscr{L}_{23} &\triangleq h\left(\widetilde{A}_d + \widetilde{A}_j^d\right)' Q_1\,\widetilde{D}, \\
\mathscr{L}_{24} &\triangleq h\widetilde{D}' P\left(\widetilde{A}_d + \widetilde{A}_j^d\right), \\
\mathscr{L}_{25} &\triangleq \mathbf{0}, \\
\mathscr{L}_{26} &\triangleq \widetilde{D}' P\,\widetilde{D}_1 - h\left(\widetilde{A}_d + \widetilde{A}_j^d\right)' Q_1\,\widetilde{D}_1, \\
\mathscr{L}_{27} &\triangleq h\left(\widetilde{A}_d + \widetilde{A}_j^d\right)' Z, \\
\mathscr{L}_{33} &\triangleq -hQ_1 + hQ_2 + h\widetilde{D}' Q_1\,\widetilde{D}, \\
\mathscr{L}_{34} &\triangleq \mathbf{0}, \\
\mathscr{L}_{35} &\triangleq \mathbf{0}, \\
\mathscr{L}_{36} &\triangleq -h\widetilde{D}' Q_1\,\widetilde{D}_1, \\
\mathscr{L}_{37} &\triangleq \mathbf{0}, \\
\mathscr{L}_{44} &\triangleq -hZ, \\
\mathscr{L}_{45} &\triangleq hZ\widetilde{D}, \\
\mathscr{L}_{46} &\triangleq \mathbf{0}, \\
\mathscr{L}_{47} &\triangleq \mathbf{0}, \\
\mathscr{L}_{55} &\triangleq -h\widetilde{D}' Z\,\widetilde{D} - hQ_2, \\
\mathscr{L}_{56} &\triangleq \mathbf{0}, \\
\mathscr{L}_{57} &\triangleq \mathbf{0}, \\
\mathscr{L}_{66} &\triangleq h\widetilde{D}_1' Q_1\,\widetilde{D}_1 - \gamma^2 I, \\
\mathscr{L}_{67} &\triangleq -hD_1' Z, \\
\mathscr{L}_{77} &\triangleq -hZ, \\
\widetilde{A}_i^c &\triangleq \widetilde{A} + \widetilde{A}_i + P^{-1}Y\widetilde{C}.
\end{aligned}
$$

Proof: The equation of the observation error reads as

$$
\begin{aligned}
&\dot{e}(t) - \widetilde{D}\dot{e}(t-h) = (\widetilde{A} + P^{-1}Y\widetilde{C})\,e(t) + \widetilde{A}_d\,e(t-h) \\
&+ \widetilde{f}\Big(\hat{z}(t), \hat{z}(t-h), u(t)\Big) - \widetilde{f}\Big(z(t), z(t-h), u(t)\Big) \\
&- \widetilde{D}_1\xi(t)
\end{aligned} \tag{12}
$$

Using the mean-value Theorem, we can write that

$$
\begin{aligned}
&\widetilde{f}\Big(\hat{z}(t), \hat{z}(t-h), u(t)\Big) - \widetilde{f}\Big(z(t), z(t-h), u(t)\Big) \\
&= \int_0^1 \frac{\partial \widetilde{f}(\alpha, \beta, u(t))}{\partial \alpha}\bigg|_{\substack{\alpha = v_\lambda \\ \beta = w_\lambda}} (\hat{z}(t) - z(t))\mathrm{d}\lambda \\
&+ \int_0^1 \frac{\partial \widetilde{f}(\alpha, \beta, u(t))}{\partial \beta}\bigg|_{\substack{\alpha = v_\lambda \\ \beta = w_\lambda}} (\hat{z}(t-h) - z(t-h))\mathrm{d}\lambda \\
&= \int_0^1 \widetilde{F}(v_\lambda, w_\lambda, u(t))\,e(t)\,\mathrm{d}\lambda \\
&+ \int_0^1 \widetilde{F}_d(v_\lambda, w_\lambda, u(t))\,e(t-h)\,\mathrm{d}\lambda.
\end{aligned} \tag{13}
$$

where

$$
\begin{aligned}
v_\lambda &= \hat{z}(t) - \lambda(\hat{z}(t) - z(t)), \\
w_\lambda &= \hat{z}(t-h) - \lambda(\hat{z}(t-h) - z(t-h)).
\end{aligned} \tag{14}
$$

Consequently, Eq. (12) takes the following new form:

$$
\begin{aligned}
&\dot{e}(t) - \widetilde{D}\dot{e}(t-h) = \\
&\int_0^1 \Big(\widetilde{A} + P^{-1}Y\widetilde{C} + \widetilde{F}(v_\lambda, w_\lambda, u(t))\Big)\,e(t)\mathrm{d}\lambda \\
&+ \int_0^1 \Big(\widetilde{A}_d + \widetilde{F}_d(v_\lambda, w_\lambda, u(t))\Big)e(t-h)\,\mathrm{d}\lambda \\
&- \int_0^1 \widetilde{D}_1\xi(t)\mathrm{d}\lambda
\end{aligned} \tag{15}
$$

Let us assign the Lyapunov-Krasovskii functional $V = \sum_{i=1}^{5} V_i$ to the dynamics of the observation error (15) where

$$
\begin{aligned}
V_1 &\triangleq \Big[e(t) - \widetilde{D}\,e(t-h)\Big]' P \Big[e(t) - \widetilde{D}\,e(t-h)\Big], \\
V_2 &\triangleq \int_{t-h}^{t} e'(s)\,Q\,e(s)\,\mathrm{d}s, \\
V_3 &\triangleq \int_{-h}^{0}\int_{t+\theta}^{t} \Big[\dot{e}(v) - \widetilde{D}\,\dot{e}(v-h)\Big]' Z \Big[\dot{e}(v) - \widetilde{D}\,\dot{e}(v-h)\Big] \\
&\qquad \mathrm{d}\theta\mathrm{d}v, \\
V_4 &\triangleq h\int_{t-h}^{t} \dot{e}'(s) Q_1 \dot{e}(s)\,\mathrm{d}s, \\
V_5 &= \int_{-h}^{0}\int_{t+\theta}^{t} \dot{e}'(v-h)\,Q_2\,\dot{e}(v-h)\,\mathrm{d}\theta\mathrm{d}v,
\end{aligned} \tag{16}
$$

where P, Z Q, Q_1 and Q_2 are symmetric and positive-definite matrices of appropriate dimensions. Define

$$\zeta(t,v) \triangleq \begin{bmatrix} e(t) \\ e(t-h) \\ \dot{e}(t-h) \\ \dot{e}(v) \\ \dot{e}(v-h) \\ \xi(t) \end{bmatrix} \tag{17}$$

then, we have

$$\dot{V}_1 = 2\Big[e'(t) - e'(t-h)\widetilde{D}'\Big] P \Big[\dot{e}(t) - \widetilde{D}\,\dot{e}(t-h)\Big]. \tag{18}$$

For simplicity of notations, let us note $F_\lambda = \widetilde{F}(v_\lambda, w_\lambda, u(t))$ and $F_\lambda^d = \widetilde{F}_d(v_\lambda, w_\lambda, u(t))$. Using the fact that

$$e(t) - e(t-h) = \int_{t-h}^{t} \dot{e}(s)\,\mathrm{d}s,$$

then, we can write that

$$\begin{aligned}
\dot{V}_1 &= 2\int_0^1 \Big[e'(t) - e'(t-h)\widetilde{D}'\Big] P \\
&\times \Big[\Big(\widetilde{A} + \widetilde{A}_d + F_\lambda + F_\lambda^d + P^{-1}Y\widetilde{C}\Big)e(t) \\
&- \Big(\widetilde{A}_d + F_\lambda^d\Big)\int_{t-h}^{t} \dot{e}(s)\,\mathrm{d}s - \widetilde{D}_1\xi(t)\Big]\mathrm{d}\lambda. \\
&= \frac{1}{h}\int_0^1\int_{t-h}^{t} 2e'(t)P\Big(\widetilde{A} + \widetilde{A}_d + F_\lambda + F_\lambda^d \\
&+ P^{-1}Y\widetilde{C}\Big)\, e(t)\,\mathrm{d}v\mathrm{d}\lambda \\
&+ \frac{1}{h}\int_0^1\int_{t-h}^{t} 2e'(t)(-h\,P)\,(\widetilde{A}_d + F_\lambda^d)\,\dot{e}(v)\,\mathrm{d}v\mathrm{d}\lambda \\
&+ \frac{1}{h}\int_0^1\int_{t-h}^{t} 2e'(t)(-P\,\widetilde{D}_1)\xi(t)\,\mathrm{d}v\mathrm{d}\lambda \\
&+ \frac{1}{h}\int_0^1\int_{t-h}^{t} 2e'(t)\Big(-\widetilde{D}'P(\widetilde{A} + \widetilde{A}_d + F_\lambda + F_\lambda^d \\
&+ P^{-1}Y\widetilde{C}\Big)e(t)\,\mathrm{d}v\mathrm{d}\lambda \\
&+ \frac{1}{h}\int_0^1\int_{t-h}^{t} 2e'(t-h)\Big(h\widetilde{D}'P(\widetilde{A}_d + F_\lambda^d)\Big)\dot{e}(v)\,\mathrm{d}v\mathrm{d}\lambda \\
&+ \frac{1}{h}\int_0^1\int_{t-h}^{t} 2e'(t-h)(\widetilde{D}'P\widetilde{D}_1)\xi(t)\,\mathrm{d}v\mathrm{d}\lambda \\
&= \frac{1}{h}\int_0^1\int_{t-h}^{t} \zeta'(t,v)\Omega_1\zeta(t,v)\,\mathrm{d}v\mathrm{d}\lambda
\end{aligned} \tag{19}$$

where

$$\Omega_1 = \begin{bmatrix} \Omega_1^{11} & \Omega_1^{12} & 0 & -hP(\widetilde{A}_d + F_\lambda^d) & 0 & -P\widetilde{D}_1 \\ \star & 0 & 0 & h\widetilde{D}'P(\widetilde{A}_d + F_\lambda^d) & 0 & \widetilde{D}'P\widetilde{D}_1 \\ \star & \star & 0 & 0 & 0 & 0 \\ \star & \star & \star & 0 & 0 & 0 \\ \star & \star & \star & \star & 0 & 0 \\ \star & \star & \star & \star & \star & 0 \end{bmatrix}. \tag{20}$$

where

$$\begin{aligned}
\Omega_1^{11} &= (\widetilde{A} + \widetilde{A}_d + F_\lambda + F_\lambda^d + P^{-1}Y\widetilde{C})'P \\
&+ P(\widetilde{A} + \widetilde{A}_d + F_\lambda + F_\lambda^d + P^{-1}Y\widetilde{C}) \\
\Omega_1^{12} &= -(\widetilde{A} + \widetilde{A}_d + F_\lambda + F_\lambda^d)'P\widetilde{D}
\end{aligned} \tag{21}$$

In the other hand

$$\begin{aligned}
\dot{V}_2 &= e'(t)Qe(t) - e'(t-h)Qe(t-h) \\
&= \frac{1}{h}\int_0^1\int_{t-h}^{t} \zeta'(t,v)\,\Omega_2\,\zeta(t,v)\,\mathrm{d}v\,\mathrm{d}\lambda
\end{aligned} \tag{22}$$

where

$$\Omega_2 = \mathrm{diag}\Big(Q, -Q, 0, 0, 0, 0\Big). \tag{23}$$

We have

$$\begin{aligned}
\dot{V}_3 &= h\Big[\dot{e}(t) - \widetilde{D}\dot{e}(t-h)\Big]' Z \Big[\dot{e}(t) - \widetilde{D}\dot{e}(t-h)\Big] \\
&- \frac{1}{h}\int_{t-h}^{t}\Big[\dot{e}(v) - \widetilde{D}\dot{e}(v-h)\Big]' Z \Big[\dot{e}(v) - \widetilde{D}\dot{e}(v-h)\Big]\,\mathrm{d}v.
\end{aligned} \tag{24}$$

Using Lemma 2, we can write that

$$\begin{aligned}
\dot{V}_3 &\le \frac{1}{h}\int_0^1\int_{t-h}^{t} \zeta'(t,v) \begin{bmatrix} (\widetilde{A} + P^{-1}Y\widetilde{C} + F_\lambda)' \\ (\widetilde{A}_d + F_\lambda^d)' \\ 0 \\ 0 \\ 0 \\ -\widetilde{D}_1' \end{bmatrix} (h\,Z) \\
&\times \begin{bmatrix} \widetilde{A} + P^{-1}Y\widetilde{C} + F_\lambda & \widetilde{A}_d + F_\lambda^d & 0 & 0 & 0 & -\widetilde{D}_1 \end{bmatrix} \\
&\times \zeta(t,v)\,\mathrm{d}v\,\mathrm{d}\lambda + \frac{1}{h}\int_0^1\int_{t-h}^{t} \zeta'(t,v)\,\Omega_{32}\,\zeta(t,v)\,\mathrm{d}v\,\mathrm{d}\lambda
\end{aligned} \tag{25}$$

where

$$\Omega_{32} \triangleq \begin{bmatrix} 0 \\ 0 \\ 0 \\ I \\ -\widetilde{D}' \\ 0 \end{bmatrix} (-hZ) \begin{bmatrix} 0 & 0 & 0 & I & -\widetilde{D} & 0 \end{bmatrix}. \tag{26}$$

By differentiating V_4, we get

$$\begin{aligned}\dot{V}_4 &= h\,\dot{e}'(t)Q_1\dot{e}(t) - h\,\dot{e}'(t-h)Q_1\dot{e}(t-h)\\ &= \frac{1}{h}\int_0^1\int_{t-h}^t \dot{e}'(t)\,(h\,Q_1)\,\dot{e}(t)\,\mathrm{d}v\mathrm{d}\lambda\\ &+ \frac{1}{h}\int_0^1\int_{t-h}^t \dot{e}'(t-h)\,(-h\,Q_1)\,\dot{e}(t-h)\,\mathrm{d}v\mathrm{d}\lambda.\end{aligned} \quad (27)$$

Based on the result of Lemma 2 and by the use of system equations, we find

$$\begin{aligned}\dot{V}_4 &\le \frac{1}{h}\int_0^1\int_{t-h}^t \Big((\widetilde{A}+F_\lambda+P^{-1}Y\widetilde{C})\,e(t)\\ &+ (\widetilde{A}_d+F_\lambda^d)\,e(t-h) + \widetilde{D}\dot{e}(t-h) - \widetilde{D}_1\xi(t)\Big)'(h\,Q_1)\\ &\Big((\widetilde{A}+F_\lambda+P^{-1}Y\widetilde{C})\,e(t)\\ &+ (\widetilde{A}_d+F_\lambda^d)\,e(t-h) + \widetilde{D}\dot{e}(t-h) - \widetilde{D}_1\xi(t)\Big)\,\mathrm{d}v\mathrm{d}\lambda\\ &+ \frac{1}{h}\int_0^1\int_{t-h}^t \dot{e}'(t-h)\,(-h\,Q_1)\,\dot{e}(t-h)\,\mathrm{d}v\mathrm{d}\lambda.\end{aligned} \quad (28)$$

Finally, we write

$$\dot{V}_4 \le \frac{1}{h}\int_0^1\int_{t-h}^t \zeta'(t,v)\Omega_4\zeta(t,v)\,\mathrm{d}v\mathrm{d}\lambda, \quad (29)$$

where

$$\begin{aligned}\Omega_4 \triangleq &\begin{bmatrix}(\widetilde{A}+F_\lambda+P^{-1}Y\widetilde{C})'\\(\widetilde{A}_d+F_\lambda^d)'\\\widetilde{D}'\\0\\0\\-\widetilde{D}_1'\end{bmatrix}(h\,Q_1)\\ &\times\begin{bmatrix}\widetilde{A}+F_\lambda+P^{-1}Y\widetilde{C} & \widetilde{A}_d+F_\lambda^d & \widetilde{D} & 0 & 0 & -\widetilde{D}_1\end{bmatrix}\\ &+ \mathrm{diag}\Big(0,0,-h\,Q_1,0,0,0\Big).\end{aligned} \quad (30)$$

We have

$$\begin{aligned}\dot{V}_5 &= h\,\dot{e}'(t-h)\,Q_2\,\dot{e}(t-h) - \int_{t-h}^t \dot{e}'(v-h)\,Q_2\,\dot{e}(v-h)\\ &= \frac{1}{h}\int_0^1\int_{t-h}^t \zeta'(t,v)\Omega_5\zeta(t,v)\,\mathrm{d}v\mathrm{d}\lambda,\end{aligned} \quad (31)$$

where

$$\Omega_5 = \mathrm{diag}\Big(0,0,h\,Q_2,0,0,-h\,Q_2\Big). \quad (32)$$

As a result

$$\begin{aligned}\dot{V} &\le \frac{1}{h}\int_0^1\int_{t-h}^t \zeta'(t,v)\big(\Omega_1+\Omega_2+\Omega_{32}+\Omega_4\\ &+ \Omega_5\big)\zeta(t,v)\,\mathrm{d}v\mathrm{d}\lambda,\\ &+ \frac{1}{h}\int_0^1\int_{t-h}^t \zeta(t,v)\begin{bmatrix}(\widetilde{A}+F_\lambda+P^{-1}Y\widetilde{C})'\\(\widetilde{A}_d+F_\lambda^d)'\\0\\0\\0\\-\widetilde{D}_1'\end{bmatrix}(h\,Z)\\ &\times\begin{bmatrix}\widetilde{A}+F_\lambda+P^{-1}Y\widetilde{C} & \widetilde{A}_d+F_\lambda^d & 0 & 0 & 0 & -\widetilde{D}_1\end{bmatrix}\\ &\times\zeta(t,v)\,\mathrm{d}v + \frac{1}{h}\int_0^1\int_{t-h}^t \zeta'(t,v)\,\Omega_{32}\,\zeta(t,v)\,\mathrm{d}v\mathrm{d}\lambda.\end{aligned} \quad (33)$$

Now, the optimality condition (11) is verified if the following holds

$$\int_0^t \{e'(s)\widetilde{C}'\widetilde{C}e(s) - \gamma^2\xi'(s)\xi(s)\}\mathrm{d}s + V(t) < 0. \quad (34)$$

where $V(t) = \sum_{i=1}^5 V_i(t)$. Since we assume that all the initial conditions are null. Then, inequality (35) holds if the following holds

$$\int_0^t \{e'(s)\widetilde{C}'\widetilde{C}e(s) - \gamma^2\xi'(s)\xi(s) + \dot{V}(s)\}\mathrm{d}s < 0. \quad (35)$$

or equivalently

$$\frac{1}{h}\int_0^1\int_{t-h}^t\int_0^t \zeta'(s,v)\Omega_0\zeta(s,v) + \dot{V}(s)\}\mathrm{d}\lambda\mathrm{d}v\,\mathrm{d}s < 0. \quad (36)$$

where

$$\Omega_0 = \begin{bmatrix}\widetilde{C}'\widetilde{C} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \star & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \star & \star & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \star & \star & \star & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \star & \star & \star & \star & \mathbf{0} & \mathbf{0}\\ \star & \star & \star & \star & \star & -\gamma^2 I\end{bmatrix} \quad (37)$$

Based upon (33), we conclude that inequality (36) is verified if the following matrix inequality holds

$$\left[\begin{array}{c|c}\Pi & \begin{matrix}h(\widetilde{A}+F_\lambda+P^{-1}Y\widetilde{C})'Z\\ h\,(\widetilde{A}_d+F_\lambda^d)Z\\0\\0\\0\\-h\,\widetilde{D}_1'Z\end{matrix}\\ \hline \star & -h\,Z\end{array}\right] < 0 \quad (38)$$

where $\Pi = \Omega_0+\Omega_1+\Omega_2+\Omega_{32}+\Omega_4+\Omega_5$. Since

$$\begin{aligned}F_\lambda &\in \mathbf{Co}\{\widetilde{A}_1,\cdots,\widetilde{A}_\mu\},\\ F_\lambda^d &\in \mathbf{Co}\{\widetilde{A}_1^d,\cdots,\widetilde{A}_\nu^d\}\end{aligned} \quad (39)$$

then, (38) holds if the set of the matrix inequalities (10) are verified. This ends the proof.

Remark 1: The matrix inequalities of the main Theorem can be made convex if we put $Z = P$ and $Q_1 = P$. This restriction permits the transformation of the conditions to a set of linear matrix inequalities; however, a conservatism will be introduced in the conditions.

IV. CONCLUSION

New delay-dependent conditions for the existence of high-gain observer/filter for nonlinear neutral-type delay systems are given. By decomposition of the jacobian of the system nonlinearities to delayless and delayed matrix nonlinearities, the design becomes totally free form the information of the Lipschitz constants and allow the incorporation of the effects of the hard nonlinearities as linear perturbations to the nominal matrices. By extending the vector of the observation error, we show that we can avoid bounding the cross-terms that appears from the derivatives of the Lyapunov functions and hence, less conservative design is obtained.

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