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Interval-valued Fuzzy Control

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Abstract

In this paper, we introduce the concept of interval-valued fuzzy control. Based on the idea on the interpolation mechanism of fuzzy control, we propose the interval-valued fuzzy control model and investigate the interpolation mechanism of interval-valued fuzzy control. Finally, we use a simulation experiment of interval-valued fuzzy control to illustrate our proposed algorithm reasonable.

Keywords *Interval-valued fuzzy set; interpolation mechanism; interval-valued fuzzy inference; interval-valued fuzzy control*

1 Introduction

Since the fuzzy set was introduced by Zadeh [17], many new approaches and theories treating imprecision and uncertainty have been proposed. Some of these theories, such as the intuitionistic fuzzy sets theory pioneered by Atanassove [1, 2], the generalized theory of uncertainty (GTU) introduced by Zadeh [19], are extensions of the classic fuzzy set theory, and have allowed people to deal with uncertainty and information in a much broader perspective. Another well-known generalization of an ordinary fuzzy set is the interval-valued fuzzy set, which was first introduced by Zadeh [18]. Since this was introduced, many researchers have investigated this topic and have established some meaningful conclusions. For example, Deschrijver [9] investigated the arithmetic operators of the interval-valued fuzzy set theory, Zeng [20, 22] investigated decomposition theorem and representation theorem. Moreover, some researchers have pointed out that there is a strong connection between Atanassov's intuitionistic fuzzy sets and the interval-valued fuzzy sets. For more details, readers can refer to [8].

Fuzzy inference is an important topic in fuzzy set theory. Many researchers have investigated it from different points of view and obtained many different fuzzy inference models. These different fuzzy inference models and approaches have been applied rapidly in many areas such as fuzzy control and pattern recognition according to their feature. For example, Wang [15] proposed true-value flow inference, Zeng [21] proposed inner product truth-valued flow inference, Wu [16] investigated the principle of fuzzy inference. Applying fuzzy inference into fuzzy control, scholars have done a lot of meaningful works. For example, Li [12] proposed the interpolation mechanism of fuzzy control and applied fuzzy control in the research of inverted pendulums. Li [13, 14] investigated adaptive fuzzy controllers based on variable universe and the relationship between fuzzy controllers and PID controllers, respectively.

Since the interval-valued fuzzy set was introduced, researchers have started to investigate fuzzy inference of interval-valued fuzzy sets. For example, Gorzalczany [10, 11] proposed fuzzy inference methods of interval-valued fuzzy sets and discussed its properties. With the development of fuzzy technology and the successful application of fuzzy control, fuzzy inference of interval-valued fuzzy set has become a hot topic. Some researchers have proposed some different approximate reasoning methods. For example, Bustince [3, 4] proposed approximate reasoning models of interval-valued fuzzy sets from interval-valued fuzzy relation and indicator of inclusion grade of interval-valued fuzzy sets, respectively. Chen [5, 6] investigated the bidirectional approximate reasoning of interval-valued fuzzy sets by similarity measure of interval-valued fuzzy sets and direction index of matching function, respectively. Chun[7] applied the similarity-based bidirectional approximate reasoning method in decision-making systems. In this paper, we introduce the concept of interval-valued fuzzy control, propose the interval-valued fuzzy control model and investigate the interpolation mechanism of interval-valued fuzzy control based on the idea on the interpolation mechanism of fuzzy control. Finally, we use a simulation experiment of interval-valued fuzzy control to illustrate our proposed algorithm reasonable.

The organization of our work is as follows. In section

2, some basic notions of interval-valued fuzzy set are reviewed. In section 3, we propose the interval-valued fuzzy control model and investigate the interpolation mechanism of interval-valued fuzzy control. In section 4, we use a simulation experiment of interval-valued fuzzy control to illustrate our proposed algorithm reasonable. The conclusion is given in the last section.

2 Some notions

Throughout this paper, we use X and Y to denote the input discourse set and output discourse set, respectively, IVFSs(X) and IVFSs(Y) stand for the set of all interval-valued fuzzy subsets in X and Y, respectively. $\overline{A}$ expresses an interval-valued fuzzy set in X, the operation "c" stands for the complement operation of interval-valued fuzzy set or fuzzy set in X, $\emptyset$ stands for the empty set.

Let $L([0,1])$ be the set of all closed subintervals of the interval $[0,1]$. Then, according to Zadeh's extension principle[17], for any $\overline{a} = [a^-, a^+], \overline{b} = [b^-, b^+] \in L([0,1])$, we can popularize some operations such as $\bigvee, \bigwedge$, and c to $L([0,1])$, and have $\overline{a} \bigvee \overline{b} = [a^- \bigvee b^-, a^+ \bigvee b^+]$, $\overline{a} \bigwedge \overline{b} = [a^- \bigwedge b^-, a^+ \bigwedge b^+]$, $\overline{a}^c = [1-a^+, 1-a^-]$, $\bigvee\limits_{t\in W} \overline{a_t} = [\bigvee\limits_{t\in W} a_t^-, \bigvee\limits_{t\in W} a_t^+]$ and $\bigwedge\limits_{t\in W} \overline{a_t} = [\bigwedge\limits_{t\in W} a_t^-, \bigwedge\limits_{t\in W} a_t^+]$, where W denotes an arbitrary index set. Thus, $(L([0,1]), \bigvee, \bigwedge, c)$ is a complete lattice with a minimal element $\overline{0} = [0,0]$ and a maximal element $\overline{1} = [1,1]$. Furthermore, we have $\overline{a} = \overline{b} \Longleftrightarrow a^- = b^-, a^+ = b^+$, $\overline{a} \leqslant \overline{b} \Longleftrightarrow a^- \leqslant b^-, a^+ \leqslant b^+$ and $\overline{a} < \overline{b} \Longleftrightarrow \overline{a} \leqslant \overline{b}$ and $\overline{a} \neq \overline{b}$. Considering $L([0,1])$ is dense, therefore, $(L([0,1]), \bigvee, \bigwedge, c)$ is a superior soft algebra.

We call a mapping, $\overline{A} : X \longrightarrow L([0,1])$ an interval-valued fuzzy set in X. For every $\overline{A} \in$ IVFSs(X) and $x \in X$, then $\overline{A}(x) = [A^-(x), A^+(x)]$ is the degree of membership of an element x to the interval-valued fuzzy set $\overline{A}$. Thus, fuzzy sets $A^- : X \to [0,1]$ and $A^+ : X \to [0,1]$ are called low and upper fuzzy sets of the interval-valued fuzzy set $\overline{A}$, respectively. For simplicity, we denote $\overline{A} = [A^-, A^+]$. Therefore, some operations such as $\bigcup, \bigcap, c$ can be introduced into IVFSs(X), then (IVFSs$(X), \bigcup, \bigcap, c)$ is a superior soft algebra. Here, $\mathcal{F}(X)$ and $\mathcal{P}(X)$ stand for the set of all fuzzy sets and crisp sets in X, respectively.

Specially, if $X = \{x_1, x_2, \ldots, x_n\}$, then $\overline{A}(x_i) = [a_{i1}, a_{i2}]$, for every $x_i \in X, i = 1, 2, \cdots, n$, therefore, we have

$$\overline{A} = \{(x_1, [a_{11}, a_{12}]), (x_2, [a_{21}, a_{22}]), \ldots, (x_n, [a_{n1}, a_{n2}])\}$$

where $[a_{i1}, a_{i2}] (0 \le a_{i1} \le a_{i2} \le 1, i = 1, 2, \cdots, n)$ is the degree of membership of an element x_i to interval-valued fuzzy set $\overline{A}$.

If $\overline{A}, \overline{B} \in$IVFSs$(X)$, then the following operations can be found in Zeng [20].

$\overline{A} \subseteq \overline{B}$ if and only if $\forall x \in X, A^-(x) \le B^-(x)$ and $A^+(x) \le B^+(x)$,

$\overline{A} = \overline{B}$ if and only if $\forall x \in X, A^-(x) = B^-(x)$ and $A^+(x) = B^+(x)$,

$(\overline{A})^c = [(A^+)^c, (A^-)^c]$.

3 Interpolation mechanism of interval-valued fuzzy control

Recently, Li [12, 13, 14] pointed out that some common fuzzy inference algorithms in fuzzy control can be attributed to some kind of interpolation method, and proved that fuzzy control algorithm based on the implication operator $R(a,b) = 0 \vee (a+b-1)$ also can be attributed some kind of interpolation system.

To introduce the nomenclatures, we shall consider a reasonably simple interval-valued fuzzy control system with one-input and one-output. Let $X = [a,b]$ be the universe of input variable x, and $Y = [c,d]$ be the universe of output variable z. Denoting $\mathcal{A} = \{\overline{A}_i\}_{(1\le i\le n)}, \mathcal{B} = \{\overline{B}_i\}_{(1\le i\le n)}$, $\overline{A}_i \in$ IVFSs(X), $\overline{B}_i \in$ IVFSs(Y), and $\mathcal{A}, \mathcal{B}$are linguistic variables, thus, we can form the fuzzy inference rules as follows:

$$\text{if } x \text{ is } \overline{A}_i \text{ then } y \text{ is } \overline{B}_i$$

where $i = 1, 2, \cdots, n$; $x \in X, y \in Y$.

Given $\overline{A}'$, according to Zadeh's compositional rule of inference, the conclusion of interval-valued fuzzy inference can be determined as follow.

Model

$$\overline{B}' = \bigcup_{i=1}^{n} \overline{A}' \circ \overline{R}_i = \bigcup_{i=1}^{n} \overline{A}' \circ (\overline{A}_i \to \overline{B}_i)$$

where $\overline{R}_i(x,y) = (\overline{A}_i \to \overline{B}_i)(x,y) = [{R_i}^-(x,y), {R_i}^+(x,y)]$, ${R_i}^-(x,y) = \max(0, {A_i}^-(x) + {B_i}^-(y) - 1)$, ${R_i}^+(x,y) = {A_i}^+(x) \wedge {B_i}^+(y), i = 1, 2, \cdots, n$.

$$B'^-(y) = \bigvee_{x\in X} \bigvee_{i=1}^{n} (A'^-(x) \wedge {R_i}^-(x,y))$$

$$B'^+(y) = \bigvee_{x\in X} \bigvee_{i=1}^{n} (A'^+(x) \wedge {R_i}^+(x,y))$$

In general, the input to the fuzzy logic system is crisp or real quantity, thus, it must be fuzzified. We shall adopt the following most frequently used fuzzification system for every $x' = [x', x']$,

$$A'(x) = \begin{cases} 1, & x = x' \\ 0, & x \neq x' \end{cases}$$

Then, we will obtain an interval-valued fuzzy set $\overline{A}'$, $\overline{A}' = [A'^{-}, A'^{+}]$, where $A'^{-} = A'^{+} = A'$.

Using the above fuzzification system, we have $\overline{B}' = [B'^{-}, B'^{+}]$, where $B'^{-}(y) = \bigvee_{i=1}^{n} R_i{}^{-}(x', y)$, $B'^{+}(y) = \bigvee_{i=1}^{n} R_i{}^{+}(x', y)$, To obtain a crisp quantity, the interval-valued fuzzy set $\overline{B}'$ will be defuzzified by the use of the following centroid method:

$$y'^{-} = \frac{\int_Y yB'^{-}(y)dy}{\int_Y B'^{-}(y)dy}, \quad y'^{+} = \frac{\int_Y yB'^{+}(y)dy}{\int_Y B'^{+}(y)dy}$$

where

$$B'^{-}(y) = \bigvee_{i=1}^{n} (\max(0, A_i'^{-}(x') + B_i'^{-}(y) - 1))$$

$$B'^{+}(y) = \bigvee_{i=1}^{n} (A_i{}^{+}(x') \wedge B_i{}^{+}(y))$$

Thus, for the interval-valued fuzzy control system, we find that the interval-valued fuzzy control system can be transformed two kinds of general fuzzy control systems. Namely, they are aimed at different fuzzy inference rules, respectively. These two fuzzy inference rules are illustrated as follows.

if x is $A_i{}^{-}$, then y is $B_i{}^{-}$,$i = 1, 2, \cdots, n$

and

if x is $A_i{}^{+}$, then y is $B_i{}^{+}$,$i = 1, 2, \cdots, n$.

4 Simulation experiment of interval-valued fuzzy control

For convenience, we use the expression, $\dot{x} + x = u(t)$, to do the simulation experiment, where $x(0) = -1$, $u(t)$ is out action. Our object is to trace pulse signal,

$$r(t) = \begin{cases} 1, & t > 0, \\ 0, & \text{else} \end{cases}$$

For $u(t) \equiv 0$, we can obtain the response curve for being controlled object, then the response curve for being controlled object is listed as Fig. 1.

In our experiment, we use ID controller, $u(t) = K_I \int_0^t e(\tau)d\tau$, where $K_I = \frac{10}{3}$ is proportion coefficient, $e(t) = r(t) - x(t)$ is system error, the response curve for the system is listed as Fig. 2.

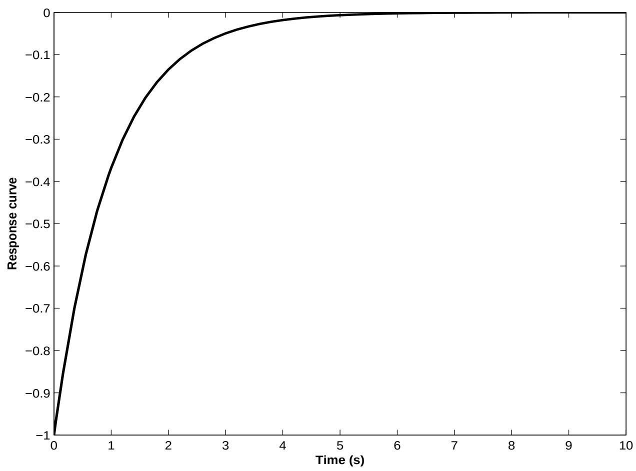

Fig. 1. The response curve for being controlled object

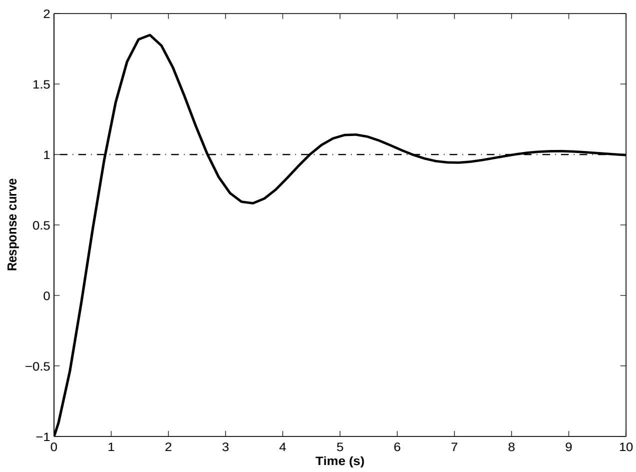

Fig. 2. The response curve for the system on ID

In the following, we give two kinds of general membership functions, their membership function curves are listed as Fig. 3. For $k > 1$,

$$w\_minus\_mf(x, [a\ b\ c]) = \begin{cases} \left(\frac{x-a}{b-a}\right)^k, & x \in [a\ b], \\ \left(\frac{x-c}{b-c}\right)^k, & x \in [b\ c], \\ 0, & \text{else} \end{cases}$$

$$trimf(x,[a\ b\ c]) = \begin{cases} \dfrac{x-a}{b-a}, & x \in [a\ b], \\ \dfrac{x-c}{b-c}, & x \in [b\ c], \\ 0, & \text{else} \end{cases}$$

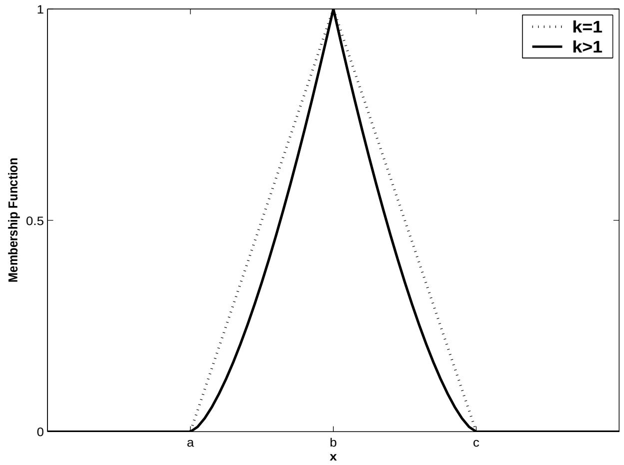

Fig. 3. Membership function curve

In the design of interval-valued fuzzy controller, we choice input variable as error integral, $x(t) = \int_0^t e(\tau)d\tau$, output variable as control quantity $u(t)$, where input variable universe $X = [-3, 3]$, output variable universe $U = [-10, 10]$. Fuzzy partition of input variable universe is saw Fig. 4. We use these five interval-valued fuzzy sets, $\overline{A}_1$, $\overline{A}_2$, $\overline{A}_3$, $\overline{A}_4$, $\overline{A}_5$ to denote such linguistic variables as "NL", "NM", "ZR", "PM", "PL", respectively. These membership functions are listed as follows, respectively.

$\overline{A}_1(x) = \left[A_1^-(x), A_1^+(x)\right]$,

$A_1^-(x) = w\_minus\_mf(x, [-3\ -3\ -1.5])$, $A_1^+(x) = trimf(x, [-3\ -3\ -1.5])$,

$\overline{A}_2(x) = \left[A_2^-(x), A_2^+(x)\right]$,

$A_2^-(x) = w\_minus\_mf(x, [-3\ -1.5\ 0])$, $A_2^+(x) = trimf(x, [-3\ -1.5\ 0])$,

$\overline{A}_3(x) = \left[A_3^-(x), A_3^+(x)\right]$,

$A_3^-(x) = w\_minus\_mf(x, [-1.5\ 0\ 1.5])$, $A_3^+(x) = trimf(x, [-1.5\ 0\ 1.5])$,

$\overline{A}_4(x) = \left[A_4^-(x), A_4^+(x)\right]$,

$A_4^-(x) = w\_minus\_mf(x, [0\ 1.5\ 3])$, $A_4^+(x) = trimf(x, [0\ 1.5\ 3])$,

$\overline{A}_5(x) = \left[A_5^-(x), A_5^+(x)\right]$,

$A_5^-(x) = w\_minus\_mf(x, [1.5\ 1.5\ 3])$, $A_5^+(x) = trimf(x, [1.5\ 1.5\ 3])$.

Similarly, for the fuzzy partition of output universe, we use these five interval-valued fuzzy sets, $\overline{B}_1$, $\overline{B}_2$, $\overline{B}_3$, $\overline{B}_4$, $\overline{B}_5$ to denote such linguistic variables as "NL", "NM", "ZR", "PM", "PL", respectively. Their membership functions are listed as follows, respectively.

$\overline{B}_1(u) = \left[B_1^-(u), B_1^+(u)\right]$,

$B_1^-(u) = w\_minus\_mf(u, [-10\ -10\ -5])$, $B_1^+(u) = trimf(u, [-10\ -10\ -5])$,

$\overline{B}_2(u) = \left[B_2^-(u), B_2^+(u)\right]$,

$B_2^-(u) = w\_minus\_mf(u, [-10\ -5\ 0])$, $B_2^+(u) = trimf(u, [-10\ -5\ 0])$,

$\overline{B}_3(u) = \left[B_3^-(u), B_3^+(u)\right]$,

$B_3^-(u) = w\_minus\_mf(u, [-5\ 0\ 5])$, $B_3^+(u) = trimf(u, [-5\ 0\ 5])$,

$\overline{B}_4(u) = \left[B_4^-(u), B_4^+(u)\right]$,

$B_4^-(u) = w\_minus\_mf(u, [0\ 5\ 10])$, $B_4^+(u) = trimf(u, [0\ 5\ 10])$,

$\overline{B}_5(u) = \left[B_5^-(u), B_5^+(u)\right]$,

$B_5^-(u) = w\_minus\_mf(u, [5\ 10\ 10])$, $B_5^+(u) = trimf(u, [5\ 10\ 10])$.

Then, based on the following 5 inference rules, we can construct the rule base for interval-valued fuzzy inference.

Rule 1: if $x(t)$ is $\overline{A}_1$, then $u(t)$ is $\overline{B}_1$;

Rule 2: if $x(t)$ is $\overline{A}_2$, then $u(t)$ is $\overline{B}_2$;

Rule 3: if $x(t)$ is $\overline{A}_3$, then $u(t)$ is $\overline{B}_3$;

Rule 4: if $x(t)$ is $\overline{A}_4$, then $u(t)$ is $\overline{B}_4$;

Rule 5: if $x(t)$ is $\overline{A}_5$, then $u(t)$ is $\overline{B}_5$.

According to the interpolation mechanism of interval-valued fuzzy control, we can obtain the control function curve for the interval-valued fuzzy inference system:

$$\overline{u} = \overline{F}(x) = \left[F^-(x), F^+(x)\right]$$

where $F^-(x) = \sum_{i=1}^{5} A_i^-(x)y_i$ and $F^+(x) = \sum_{i=1}^{5} A_i^+(x)y_i$, y_i are the peak points of fuzzy set B_i^- and B_i^+ on output universe, respectively. For convenience, we order $k = 1.5$, then we can obtain their control function curve on interval-valued fuzzy controller, see Fig.5, and the response curve for the system on interval-valued fuzzy controller, see Fig. 6.

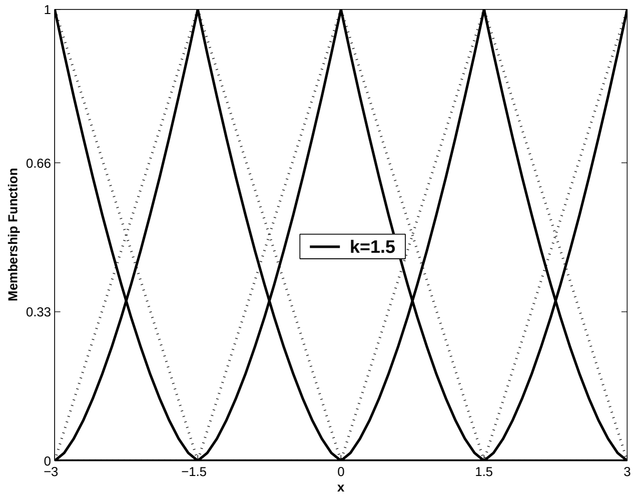

Fig. 4. Psedu-fuzzy partition of A^- and A^+ on input universe X

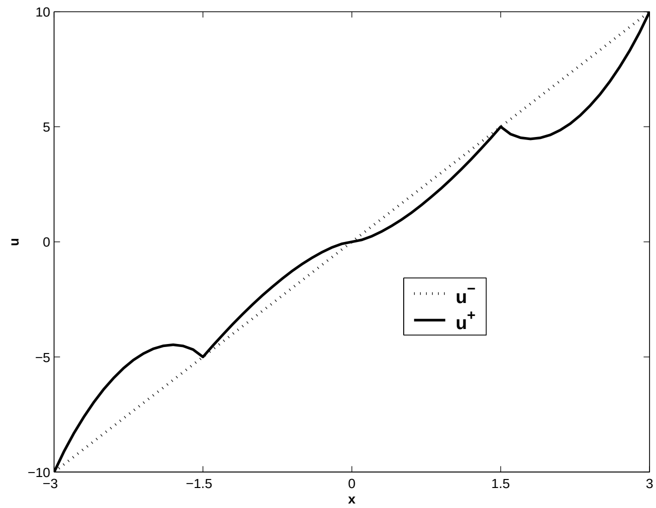

Fig. 5. Control function curve on interval-valued fuzzy controller

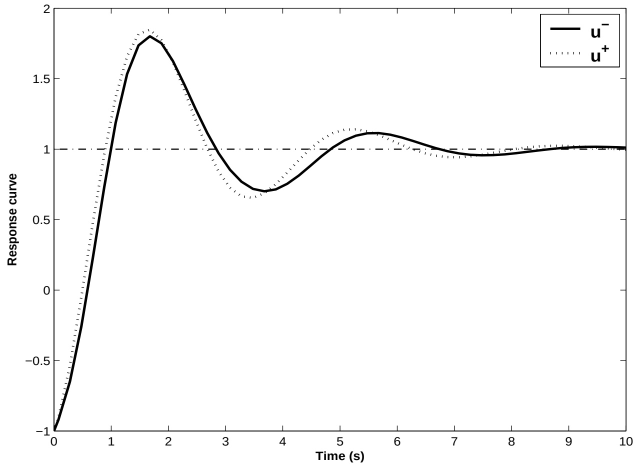

Fig. 6. The response curve for the system

5 Conclusion

Considering the importance of fuzzy inference and fuzzy control, in this paper, we introduce the concept of interval-valued fuzzy control. Based on the idea on the interpolation mechanism of fuzzy control proposed by Li [12], we propose the interval-valued fuzzy control model and investigate the interpolation mechanism of interval-valued fuzzy control. Finally, we use a simulation experiment of interval-valued fuzzy control to illustrate our proposed algorithm reasonable. We believe that the design of interval-valued fuzzy controller will induce more fuzzy technology. These technology will be extensively applied in many fields such as pattern recognition, image processing, approximate reasoning, fuzzy control, and so on.

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