Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

Fuzzy Portfolio Selection based on Value-at-Risk

Bo Wang[1], Shuming Wang[2], Junzo Watada[3],
Department of Management Engineering, Graduate School of Production,
Waseda University, Kitakyushu, Japan
E-mail: [1]bwangips@gmail.com, [2]smwangips@gmail.com, [3]junzow@osb.att.ne.jp,

***Abstract*—In this paper, using Value-at-Risk, a new fuzzy portfolio selection model named VaR-FPSM is proposed. The Value-at-Risk is the measure of risk, which describes the greatest loss of an investment with some confidence level. When security returns are same kind of fuzzy variable, we derive two crisp equivalent forms of the VaR-FPSM. Furthermore, in general situations, we designed a fuzzy simulation based particle swarm optimization (PSO) algorithm to find an approximately optimal result. To illustrate the proposed model and hybrid PSO algorithm, a numerical example is provided and some discussions on the results are given.**

***Keywords*—Value-at-Risk, fuzzy variable, portfolio selection model, hybrid particle swarm optimization, fuzzy simulation.**

I. INTRODUCTION

Value-at-Risk (VaR) of an investment is the likelihood of the greatest loss with some confidence level [9]. It is defined with respect to a specific portfolio of financial assets, at a specified probability and time horizon. The probability that the mark-to-market loss on the portfolio over the time horizon is greater than VaR, assuming normal markets and no trading, is the specified probability level [2]. Actually, the term “Value at risk” is used both for a risk measure and a risk metric which are two different concepts. If you want to know more about the difference, please refer to [1].

In the past few decades, many researchers paid great attentions on the topic of risk measure. An early user was Markowitz [3], in his groundbreaking paper ‘Portfolio Selection’, he adopted a VaR metric of single period variance of return and used this to develop techniques of portfolio optimization. Followed his step, Benati et al [6] considered an extension of the Markowitz model, in which the variance has been replaced with the VaR. Using this concept, a new portfolio optimization problem is formulated, and they prove that the problem can be formulated as an integer programming instance. Go through recent papers, various theories are combined with portfolio selections problems. Watada [16] extended Markowitz’s mean-variance idea in the fuzzy environment in a different way. Simonelli [7] used entropy which is introduced by Shannon [8] as an index of indeterminacy and considered the indeterminacy an increase of the risk from a portfolio. Compared with other models, their research proved the excellence of entropy. Huang [4] built the mean-entropy models for fuzzy portfolio selections. In their paper, entropy is used as a measure of risk, the smaller the entropy is, the safer the portfolio is. And the models they built are solved by Genetic Algorithm (GA).

In our short paper, we utilize the VaR metric [9], [10], a widely accepted measure of risk reported across market participants and industry segments, as the objective function to model the fuzzy portfolio selection problem. Compared with variance and entropy, fuzzy value at risk we used can directly show the loss of one selection case under fixed confidence level. Base on the properties of our model, we give two theorems as tools to solve some special cases. In the ordinary course of events, we provide the particle swarm optimization (PSO) algorithm as the solution.

The following of our paper is organized as: Section 2 introduces the fuzzy variable; In Section 3, we describe the fuzzy Value-at-Risk and provide an example; In Section 4, the portfolio selection model based on fuzzy VaR is proposed, in this part, we introduce two theorems which can solve some special cases by linear programming method; For normal situation, the problem could be solved by the hybrid PSO algorithm introduced in Section 5; In Section 6, we use the proposed algorithm to solve an numerical example; Finally Section 7 presents conclusions.

II. FUZZY VARIABLE

Considering the uncertain fluctuate of a portfolio, in our short paper, we use fuzzy variable describe a possible portfolio value after one period. Fuzzy variable is a fundamental mathematical tool to describe fuzzy uncertainty. Before introducing the fuzzy VaR, we briefly review some information about fuzzy variables.

Suppose ξ be a fuzzy variable whose membership function is μ_ξ, and r be a real number. The possibility and credibility of event $\xi \le r$ are expressed respectively as follows:

$$\mathrm{Pos}\{\xi \le r\} = \sup_{t \le r} \mu_\xi(t),$$

$$\mathrm{Nec}\{\xi \le r\} = 1 - \sup_{t > r} \mu_\xi(t). \quad (1)$$

And,

$$\mathrm{Cr}\{\xi \le r\} = \frac{1}{2}[\mathrm{Pos}\{\xi \le r\} + \mathrm{Nec}\{\xi \le r\}]. \quad (2)$$

The credibility measure is formed on a basis of the possibility and necessity measures and in the simplest case it is taken as their average. The credibility measure is a self-dual

978-1-4244-2794-9/09/$25.00

set function [12], i.e $\mathrm{Cr}\{\xi \le r\} = 1 - \mathrm{Cr}\{\xi > r\}$. From equation (1), we get,

$$\mathrm{Cr}\{\xi \le r\} = \frac{1}{2}[\sup_{t \le r} \mu_\xi(t) + 1 - \sup_{t > r} \mu_\xi(t)]. \quad (3)$$

For further discussion, we notice the following:

For an n-ary fuzzy vector $\xi = (\xi_1, \xi_2, ..., \xi_n)$, ξ_k is a fuzzy variable for $k = 1, 2, ..., n$, the membership function of ξ is given by taking the minimum of the individual coordinates, that is

$$\mu_\xi(t) = \min\{\mu_{\xi_1}(t_1), \mu_{\xi_2}(t_2), ..., \mu_{\xi_n}(t_n)\}, \quad (4)$$

where $t = (t_1, t_2, ..., t_n) \in \Re^n$. For more knowledge on fuzzy variable, you may refer to [12], [13], [14].

III. FUZZY VALUE-AT-RISK

In our paper, we use L to describe the fuzzy loss variable of portfolio investment. According to the definition in [9], the VaR of L with a confidence of $(1-\beta)$ can be expressed by:

$$\mathrm{VaR}_\beta = \sup\{\lambda \mid \mathrm{Cr}\{L \ge \lambda\} \ge \beta\}, \quad (5)$$

where $\beta \in (0,1)$.

Example 1. *For a normal distribution fuzzy variable* L=FN$(1.0, 2.0)$, *calculate* $\mathrm{VaR}_{0.1}$.

At first, we need to calculate the credibility function of $L \ge r$. *The membership function of this fuzzy variable is:*

$$\mu_L(t) = \mathrm{Exp}\{-[(t-1.0)/2]^2\} \quad (6)$$

From equation (1), we can compute the possibility and the credibility of $\xi \ge r$,

$$\mathrm{Pos}(L \ge r) = \begin{cases} 1 & r \le 1.0 \\ \mathrm{Exp}\{-[(r-1.0)/2]^2\} & r > 1.0 \end{cases}$$

$$\mathrm{Nec}\{L \le r\} = \begin{cases} 1\text{-}\mathrm{Exp}\{-[(r-1.0)/2]^2\} & r \le 1.0 \\ 0 & r > 1.0 \end{cases} \quad (7)$$

Hence, according to equation (2), the credibility of $L \ge r$ *is expressed as:*

$$\mathrm{Cr}\{L \ge r\} = \begin{cases} 1 - \mathrm{Exp}\{-[(r-1.0)/2]^2\}/2 & r \le 1.0 \\ \mathrm{Exp}\{-[(r-1.0)/2]^2\}/2 & r > 1.0 \end{cases} \quad (8)$$

Therefore, making use of equation (5), we have,

$$\mathrm{VaR}_{0.1} = \sup\{r \mid \mathrm{Cr}\{L \ge r\} \ge 0.1\} = 3.54$$

IV. FUZZY VAR PORTFOLIO SELECTION MODEL

A. Mathematics modelling

Markowitz's principle for portfolio selection is widely used by researchers for many years. In his groundbreaking papers [3], [15], the situations were considered as: maximizing investment return for an appointed level of risk or minimizing investment risk for an appointed level of return. In his papers, the investment return was expressed by the expected return of a portfolio and the investment risk was decided by the variance. Our paper retains Markowitz's selection principle, however, we use fuzzy variable to describe the possible portfolio return and the fuzzy VaR to characterize the risk degree.

Supposing p_i' is the estimated closing prices of the securities i in the future, p_i is the closing prices at present; d_i is the estimated dividends of the securities i from now to the future time, then the return of security i in our paper is described by fuzzy variable ξ_i as:

$$\xi_i = (p_i' + d_i - p_i)/p_i, i = 1, 2..., n,$$

Our model is built based on the assumption that: Some brave investors will first require the expected return will arrive at a high enough level, then consider about the risk (the smaller the better). In this way, the following model is given:

$$\begin{cases} \min \mathrm{VaR}_\beta = \mathrm{Sup}\{\lambda \mid \mathrm{Cr}\{L \ge \lambda\} \ge \beta\} \\ \text{subject to:} \\ E[x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n] \ge R \\ x_1 + x_2 + \cdots + x_n = 1 \\ x_i \ge 0, \ i = 1, 2, ..., n \end{cases} \quad (9)$$

Respectively, R is the lowest return one investor can accept. L is the fuzzy loss variable.

B. Some Properties of VaR-FPSM

Suppose those portfolios' returns are independent trapezoidal fuzzy variables, we give the following theorem:

Theorem 1. *Suppose those portfolios' returns are independent trapezoidal fuzzy variables as* $\xi_i = (a_i, b_i, c_i, d_i)$, *and denote* x_i *as the investment proportions in securities i, then for any selection case,* $\beta \le 0.5$, *the VaR-FPSM becomes*

$$\begin{cases} \min \sum_{i=1}^{n} x_i[(2\beta-1)a_i - 2\beta b_i] \\ \text{subject to:} \\ \sum_{i=1}^{n} x_i \frac{(a_i+b_i+c_i+d_i)}{4} \geq R \\ x_1+x_2+\cdots+x_n=1 \\ x_i \geq 0,\ i=1,2,\ldots,n \end{cases}; \tag{10}$$

While $\beta > 0.5$ *, the VaR-FPS model becomes*

$$\begin{cases} \min \sum_{i=1}^{n} x_i[(2\beta-2)c_i - (2\beta-1)d_i] \\ \text{subject to:} \\ \sum_{i=1}^{n} x_i \frac{(a_i+b_i+c_i+d_i)}{4} \geq R \\ x_1+x_2+\cdots+x_n=1 \\ x_i \geq 0,\ i=1,2,\ldots,n \end{cases}. \tag{11}$$

Proof. Considering the portfolio returns are independent, then,

$$\sum_{i=1}^{n} x_i\xi_i = \left(\sum_{i=1}^{n} x_i a_i, \sum_{i=1}^{n} x_i b_i, \sum_{i=1}^{n} x_i c_i, \sum_{i=1}^{n} x_i d_i\right). \tag{12}$$

For any trapezoidal fuzzy variable, the expected value can be calculated as:

$$E[\xi] = \frac{(a+b+c+d)}{4}.$$

Therefore, we have:

$$\mathrm{E}\left[\sum_{i=1}^{n} x_i\xi_i\right] = \sum_{i=1}^{n} x_i \frac{(a_i+b_i+c_i+d_i)}{4}. \tag{13}$$

Now we compute the VaR_{β}. The conclusion is given directly because of the space limitation. But this result can be obtained considering the proof of theorem 2.

According to equation (12), the Valu-at-Risk function of $L = -(x_1\xi_1 + x_2\xi_2 + \cdots + x_n\xi_n)$ can be described as:

$$\mathrm{VaR}_{\beta} = \begin{cases} \sum_{i=1}^{n} x_i[(2\beta-1)a_i - 2\beta b_i] & 0 < \beta \leq 0.5 \\ \sum_{i=1}^{n} x_i[(2\beta-2)c_i - (2\beta-1)d_i] & \beta > 0.5 \end{cases}$$

Considering the different cases of β as above, the VaR-FPSM has the forms (10) and (11), respectively.

The proof is complete.

Update our VaR-FPSM, we provided theorem 1.

Remark: In theorem 1, when $b_i = c_i$ *, these trapezoidal independent fuzzy variables change to triangle. Therefore for any triangle independent fuzzy values* (a_i, b_i, c_i) *when* $\beta \leq 0.5$ *our model becomes*

$$\begin{cases} \min \sum_{i=1}^{n} x_i[(2\beta-1)a_i - 2\beta b_i] \\ \text{subject to:} \\ \sum_{i=1}^{n} x_i \frac{(a_i+2b_i+c_i)}{4} \geq R \\ x_1+x_2+\cdots+x_n=1 \\ x_i \geq 0,\ i=1,2,\ldots,n \end{cases}, \tag{14}$$

Otherwise, while $\beta > 0.5$ *we have:*

$$\begin{cases} \min \sum_{i=1}^{n} x_i[(2\beta-2)b_i - (2\beta-1)c_i] \\ \text{subject to:} \\ \sum_{i=1}^{n} x_i \frac{(a_i+2b_i+c_i)}{4} \geq R \\ x_1+x_2+\cdots+x_n=1 \\ x_i \geq 0,\ i=1,2,\ldots,n \end{cases}. \tag{15}$$

Theorem 2. *Suppose the returns of security are independent normal distribution fuzzy variables expressed as* ξ_i=FN(a_i, σ_i), *the membership function of i is* $\mu_{\xi_i}(x) = \mathrm{Exp}\{-[(x_i - a_i)/\sigma_i]^2\}$ *, when* $\beta \leq 0.5$ *the VaR-FPSM becomes*

$$\begin{cases} \min \sum_{i=1}^{n} x_i[a_i + \sigma_i\sqrt{\ln(1/2\beta)}] \\ \text{subject to:} \\ \sum_{i=1}^{n} x_i a_i \geq R \\ x_1+x_2+\cdots+x_n=1 \\ x_i \geq 0,\ i=1,2,\ldots,n \end{cases}; \tag{16}$$

While $\beta > 0.5$ *, the VaR-FPSM becomes*

$$\begin{cases} \min \sum_{i=1}^{n} x_i \{a_i + \sigma_i \sqrt{\ln[1/(2-2\beta)]}\} \\ \text{subject to:} \\ \sum_{i=1}^{n} x_i a_i \geq R \\ x_1 + x_2 + \cdots + x_n = 1 \\ x_i \geq 0, \ i = 1, 2, ..., n \end{cases} . \quad (17)$$

Proof. Calculate the possibility and credibility of this fuzzy variable.

$$\mathrm{Pos}(x \geq r) = \begin{cases} 1 & r \leq a \\ \mathrm{Exp}\{-[(r-a)/\sigma]^2\} & r > a \end{cases},$$

$$\mathrm{Pos}(x < r) = \begin{cases} \mathrm{Exp}\{-[(r-a)/\sigma]^2\} & r \leq a \\ 1 & r > a \end{cases}.$$

Therefore, we obtain

$$\begin{aligned} \mathrm{Cr}\{\xi \geq r\} &= \frac{1}{2}(\mathrm{Pos}(x \geq r) + 1 - \mathrm{Pos}(x < r)) \\ &= \begin{cases} 1 - \mathrm{Exp}\{-[(r-a)/\sigma]^2\}/2 & r \leq a \\ \mathrm{Exp}\{-[(r-a)/\sigma]^2\}/2 & r > a \end{cases}. \end{aligned} \quad (18)$$

Similarly,

$$\mathrm{Cr}(\xi \leq r) = \begin{cases} \mathrm{Exp}\{-[(r-a)/\sigma]^2\}/2 & r \leq a \\ 1 - \mathrm{Exp}\{-[(r-a)/\sigma]^2\}/2 & r > a \end{cases}.$$

The expected value of $\xi = \mathrm{FN}(a, \sigma)$ can be calculated by:

$$E(\xi) = \int_0^{\infty} \mathrm{Cr}\{\xi \geq R\} \mathrm{d}r - \int_{-\infty}^{0} \mathrm{Cr}\{\xi \leq R\} \mathrm{d}r .$$

Suppose $a > 0$, that is,

$$\begin{aligned} E(\xi) &= a + \int_a^{\infty} \mathrm{Exp}\{-[(r-a)/\sigma]^2\}/2\mathrm{d}r - \int_{-\infty}^{a} \mathrm{Exp}\{-[(r-a)/\sigma]^2\}/2\mathrm{d}r \\ &= a. \end{aligned}$$

If $a < 0$, the result is a as well, according to equation (18), the VaR_{β} can be computed:

$$\mathrm{VaR}_{\beta} = \begin{cases} a + \sigma \sqrt{\ln(1/2\beta)} & 0 < \beta \leq 0.5 \\ a + \sigma \sqrt{\ln[1/(2-2\beta)]} & 0.5 < \beta < 1 \end{cases} \quad (19)$$

From fuzzy set theory, for nth independent normal distribution fuzzy variables, the following conclusions are given:

$$\sum_{i=1}^{n} x_i \mathrm{FN}(a_i, \sigma_i) = \mathrm{FN}(\sum_{i=1}^{n} x_i a_i, \sum_{i=1}^{n} x_i \sigma_i).$$

Therefore, we conclude that:

$$E\left[\sum_{i=1}^{n} x_i \mathrm{FN}(a_i, \sigma_i)\right] = \sum_{i=1}^{n} x_i a_i$$

$$\begin{aligned} &\mathrm{VaR}_{\beta}(\sum_{i=1}^{n} x_i \mathrm{FN}(a_i, \sigma_i)) \\ &= \begin{cases} \sum_{i=1}^{n} x_i [a_i + \sigma_i \sqrt{\ln(1/2\beta)}] & 0 < \beta \leq 0.5 \\ \sum_{i=1}^{n} x_i \{a_i + \sigma_i \sqrt{\ln[1/(2-2\beta)]}\} & 0.5 < \beta < 1 \end{cases} \end{aligned}$$

Considering the different cases of β as mentioned above, the VaR-FPSM has the forms (16) and (17), respectively.

The proof is complete.

Based on the result proved above, we give theorem 2.

The updated models (10), (11), (14), (15), (16) and (17) are linear programming problems which can be solved directly through Simplex Algorithm.

V. ALGORITHM

Generally speaking, when the security returns are fuzzy variables with different distributions, we cannot calculate the VaR and find the optimal solution using the Theorems 1 and 2. Therefore, we use fuzzy simulation to approximate the VaR which will be incorporated into a PSO algorithm.

A. Fuzzy Simulation

The fuzzy simulation was developed by Liu and Iwamura [13], Liu and Liu [14]. In this paper, we will use this technique to calculate the fuzzy VaR, and then integrate it into PSO [17], after iteration and comparison we can find the optimal solution.

B. Particle Swarm Optimization

PSO algorithm was initialized by Kennedy and Eberhart [11] in 1995. This method uses collaboration among a population of simple search agents to find optimal solution in possible space. Many researchers have made use of this algorithm which has been proved effectively in a variety of fields. The PSO algorithm finds optimal solution by comparing the fitness value, if the position of one particle can produce a better result, the other particles will get close to this one. In our models, VaR calculated by fuzzy simulation is considered as the fitness value. We summarize the hybrid PSO algorithm as follows:

[Step1]. Suppose the swarm has n particles, we have k securities. Initialize each particle which contains the information that how much each security should be bought.

Diferent from traditional PSO algorithm, particle initialization here is more limited, we need to improve some part. In this step, we first generate a group of random values $(a_1, a_2, ..., a_k)$, get the summation as sum and let the random values divide the sum , as a result, we obtain a list of values $(x_1, x_2, ..., x_k)$ which are computed as $x_i = a_i / sum$. Therefore, x_i is always belong to (0,1) and the summation of x_i is absolutely 1. In our portfolio selection models, this qualification is necessary. However, these new produced values may still not competent initializing the particle of model 2: We need to calculate the expected value of the security returns $E(x_1\xi_1 + x_2\xi_2 ... + x_k\xi_k)$, if the result is smaller than our hope, we will transfer the random() function again until the satisfied coefficients are created. We use array particle[i][j] to record these legal coefficients, and i means the ith particle of n, while j means the jth security of k.

[Step2]. Initialize the personal best (pbest[i][j]) of each particle, global best (gbest[][j]) of the swarm and the best result of VaR.

After initializing each particle, we order pbest[i][j]=particle[i][j]. Then, calculate the VaR of each particle by fuzzy simulation and save these values in array VaR[0][i]. As we have mentioned before, we use the loss function compute the VaR. Find the smallest VaR and the particle 'b' which bring the value, initialize gbest[0][j]=particle[b][j]. After generating some reasonable values as the begin speed v[i][j], our initialization process is accomplished.

[Step3]. Update each particle by the following equation:

v[i][j]=v[i][j]+c1*random(0,1)*(pbest[i][j]-particle[i][j])
+c2*random(0,1)*(gbest[0][j]-particle[i][j]), (20)

particle[i][j]=particle[i][j]+v[i][j]. (21)

The particles' positions are mainly updated by their speeds which determined by how far their present positions are from the personal best and global best positions. In equation (20) c1 and c2 are learning rates which is normally considered as 2 both. Therefore, if the present position of particle 'a' is in the left of personal best or global best's, the speed will become larger, consequently, the particle probability arrive at a better position next step.

[Step4]. After updating, we should still make some changes to particle[i][j] as the initialization job in step 1: Get the summation sum of particle[i][j], we order $\text{particle[i][j]} = \text{particle[i][j]} / sum$ and then check out whether the expected value is acceptable. If the expected value is reasonable, compute the VaR and compare them (the smaller the better), transverse comparison can update the pbest[i][j] while total comparison would modify gbest[t][j]. t means the program has iterative for t times.

[Step5]. Iterative the particles for T times, return the position of particle[i][j] which brings the smallest VaR, that is the optimal solution. The k elements of particle[i][j] show how much each security should be bought.

VI. NUMERICAL EXAMPLE

In order to illustrate the proposed method, we consider the following numerical example.

Suppose there are ten securities whose returns are triangle or normal distribution fuzzy variables, table 1 below describe these data in detail. For brave investors, we set the expect value of the investment is no less than 0.6 and confidence level is 0.9. Consequently, using model (9) mentioned above, the following specific case is formed:

$$\begin{cases} \min \mathrm{VaR}_{0.1} = \mathrm{Sup}\{\lambda \mid \mathrm{Cr}\{L \geq \lambda\} \geq 0.1\} \\ \text{subject to:} \\ E[x_1\xi_1 + x_2\xi_2 + \cdots + x_{10}\xi_{10}] \geq 0.6 \\ x_1 + x_2 + \cdots + x_{10} = 1 \\ x_i \geq 0, \ \ i = 1, 2, ..., 10 \end{cases} .$$

$L = -(x_1\xi_1 + x_2\xi_2 + \cdots + x_{10}\xi_{10})$, confidence level is 0.9, Respectively.

TABLE I. SECURITY RETURNS

Security No.	Return	Security No.	Return
1	(-0.7, 0.6, 1.8)	6	$N(0.8, 1.4^2)$
2	(-0.8, 0.4, 2.0)	7	$N(0.6, 1.0^2)$
3	(-1.1, 0.6, 2.8)	8	$N(0.9, 1.7^2)$
4	(-1.6, 0.9, 3.2)	9	$N(0.7, 1.1^2)$
5	(-1.4, 0.8, 3.9)	10	$N(1.1, 1.9^2)$

Obviously, the returns are different fuzzy variable which cannot be solved by linear programming method. Therefore, we use the hybrid PSO algorithm and the results are obtained as follow:

After 500 times iterative, the greatest loss under confidence 0.9 is 0.468 and the invest money is allocated as Table II.

TABLE II. OPTIMAL SOLUTION ($\beta = 0.1$)

Security No.	Money Distribution	Security No.	Money Distribution
1	0.770	6	0.000
2	0.000	7	0.000
3	0.000	8	0.000
4	0.000	9	0.187
5	0.043	10	0.000

To illustrate the relationship between expected value and VaR, we let the expected value change from 0.5 to 0.85, the following figure 1 describes the response of VaR.

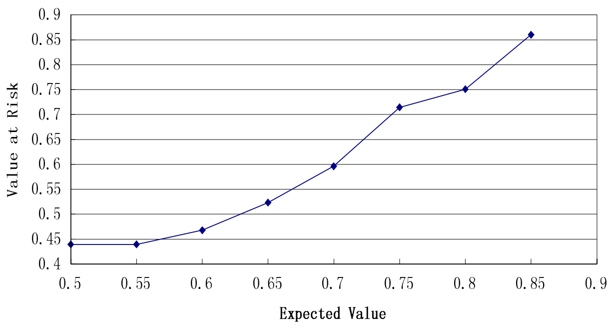

Figure 1. The Sensitivity of VaR to Expected Value

1. From Figure 1, as we increase the expected value, the VaR becomes larger. The curve in Figure 1 is similar with some hyperbola, compared with the expected value changes from 0.5 to 0.65, when the expected value is larger than 0.65, the difference of VaR is quite distinctness. This is reasonable: The return of the ten securities in our numerical example is almost larger than 0.6, in order to minimize the VaR, our algorithm can find similar optimal solutions while the expected value is not very lager. However, as we use a high expected level, the final optimal solutions must focus on some larger return securities which always include high risk, and the higher the expected value is, the faster the increase of VaR will be.

2. Another point is: While the expected value becomes larger and larger, money allocation of the optimal solution trends to gather on some high risk securities, this condition should be avoided by decision-maker, anyway, dispersive investment is always better than the focused one.

Based on the above analysis, the investor should set an appropriate expected value which is satisfied and not causing too much potential loss. In our numerical example, 0.7 could be an acceptable value.

VII. CONCLUSION

In this paper, we built the portfolio selection model using fuzzy value-at-risk. The smaller the VaR is, the better the selection is. When these security returns are all independent trapezoidal or normal distribution fuzzy variables, we proposed two theorems as the solutions. For the common situations, we provided the fuzzy simulation integrated PSO algorithm to find the optimal solution. After analyzing the proposed numerical example, we suggest that the expected value of an investment would better be determined from the sensitivity of VaR. In the reasonable scope, the expected value we adopted should not cause large VaR. In the future, we will use fuzzy random variable in order to describe the uncertain security returns better, make use of the proposed hybrid algorithm, more accurate result will be obtained. Also, the VaR-FPSM model proposed in this paper can be applicable to various invest problems.

REFERENCES

[1] G. A. Holton, Value-at-Risk: Theory and Practice, Academic Press, 2003.

[2] P. Jorion, Value at Risk: The New Benchmark for Managing Financial Risk, McGraw-Hill, 2006.

[3] H. Markowitz, “Portfolio selection”, Journal of Finance, vol. 7, no. 1, pp. 77-91, 1952.

[4] X.Huang, “Mean-entropy models for fuzzy portfolio selection”, IEEE Transactions on Fuzzy Systems, vol. 16, no. 4, pp. 1096-1101, 2008.

[5] P. C. Lin and P. C. Ko, “Portfolio value-at-risk forecasting with GA-based extreme value theory”, Expert Systems with Applications, vol. 36, no. 2, pp. 2503-2512, 2009.

[6] S. Benati and R. Rizzi, “A mixed integer linear programming formulation of the optimal mean/Value-at-Risk portfolio problem”, European Journal of Operational Research, vol. 176, no. 1, pp. 423-434, 2007.

[7] M. R. Simonolli, “Indeterminacy in portfolio selection”, Eur. J. Oper. Res., 163, 2005, 170-176.

[8] C. E. Shannon, The Mathematical Theory of Communication. Urbana, IL: Univ. of Illinois Press, 1949.

[9] D.Duffie and J.Pan, “An overviewof value-at-risk”, Journal of Derivatives, vol. 4, no. 3, , 1997, 7-49.

[10] P.Jorion, Value at Risk: The New Benchmark for Controlling Market Risk, McGraw-Hill, 2000.

[11] J. Kennedy, R. C. Eberhaart, “Particle swarm optimization”, *In: Proceedings of the 1995 IEEE International Conference on Neual Network,* IV, 1995, pp. 1942-1948.

[12] B. Liu and Y. K. Liu, “Expected value of fuzzy variable and fuzzy expected value models”, IEEE Transaction on Fuzzy Systems, vol. 10, no. 4, pp. 445-450, 2002.

[13] S. Nahmias, “Fuzzy variable”, Fuzzy Sets and Systems, vol. 1, no. 2, pp. 97-101, 1978.

[14] S. Wang, Y. Liu and X. Dai, “On the continuity and absolute continuity of credibility functions”, Journal of Uncertain System, vol. 1, no. 2, pp. 185-200, 2007.

[15] H. M. Markowitz, “Portfolio selection: Efficient Diversification of Investment”, New York: Wiley, 1959.

[16] J. Watada, “Fuzzy portfolio selection and its application to decision making”, Tatra Mountains Math. Publication, Vol, 13, pp. 219-248, 1997.

[17] S. Wang, J. Watada and W. Pedrycz, “Value-at-Risk based two-satge fuzzy facility location problems”, Working Paper, Waseda University, ISME, ISME20081201, 30 pages, (2008).