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A Note On the Sequence of Weakening Buffer Operator

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***Abstract*—Based on the present theories of operators, this paper give a new result on the weakening buffer operator. f is a monotonously increasing function, g is its inverse function. If d is a weakening buffer operator ,and $x(k)d$ is made of $x(k),\cdots,x(n),k=1,2,\cdots n$ .If we substitute $x(k)$ by $f(x(k)),k=1,\cdots,n$. The new result is m=[f(x(k))]d. If g(m) =e, then e is also a weakening buffer operator.**

***Keywords*—weakening Buffer operator; Grey systems theory.**

I. INTRODUCTION

Grey system is good at dealing with uncertainty problems with characteristics of "small sample" and "poor information". And therefore using inherent information to mining the law of the system is basic principal of grey system. We can seek changing law between each factor or itself according to research social, economic, ecological, and other behavioral system data. In Grey theory's opinion, although with complex representation and disorderly data, system's overall function will inevitably contain some laws and the key is how to choose the appropriate ways to mining and use it [1-3, 6-10]. In the literature [2-5], Professor Liu proposed the concept of shock disturbed operator and constructed a kind of a weakening operator that was applied widely. Based on those researches, combining with monotonic function, this paper give a new result on the weakening operator.

II. BASIC CONCEPT

Definition 2.1: Assume that the sequence of data representing a system's behavior is given, $X=(x(1),x(2),x(3),\cdots x(n))$, then

1. X is called a monotonously increasing sequence if $\forall k=2,3,4,\cdots,n$, $x(k)-x(k-1)>0$.
2. X is called a monotonously decreasing sequence, if $\forall k=2,3,4,\cdots,n$, $x(k)-x(k-1)<0$.
3. X is called a vibration sequence if $k_1,k_2\in\{2,3,4,\cdots,n\}$, $x(k_1)-x(k_1-1)>0$ and $x(k_2)-x(k_2-1)<0$,

And $M=\max_{1\le k\le n}x(k)$, $m=\min_{1\le k\le n}x(k)$, then $M-m$ is called the amplitude of X.

Definition 2.2: Assume that X is a sequence of raw data, D is an operator worked on X, and the sequence, obtained by having D worked on X, is denoted as

$$XD=(x(1)d,x(2)d,\cdots,x(n)d) \tag{1}$$

then, D is called a sequence operator, and XD the first order sequence worked on by the operator D. Sequence is referred to as operator.

A sequence operator can be applied as many times as needed. It can obtain a second order sequence, even order sequence, they can be denoted as $XD^2, \cdots, XD^r$

Axiom 2.1: Axiom of Fixed Points [4]. Assume that X is a sequence of raw data and D is a sequence operator, then D must satisfy $x(k)d = x(n)$.

Axiom 2.2: Axiom on Sufficient Usage of Information [4]. When a sequence operator is applied, all the information contained in each datum $x(k)$, $(k = 1,2,3,\cdots,n)$ of the sequence X of the raw data should be sufficiently applied, and any effect of each entry $x(k),(k = 1,2,3,\cdots,n)$ should also be directly reflected in the sequence worked on by the operator.

Axiom 2.3: Axiom of Analytic Representations [4]. For any $x(k)d,(k = 1,2,3,\cdots,n)$ can be described with a uniform and elementary analytic representation in $x(1), x(2), x(3), \cdots x(n)$.

All sequence operators, satisfying these three axioms, are called buffer operators, XD is called buffer sequence.

Definition 2.3: Assume X is a sequence of raw data, D is an operator worked on X, when X is a monotonously increasing sequence, a monotonously decreasing sequence or a vibration sequence, if the buffer sequence XD increases or decrease more slowly or vibrate with a smaller amplitude than the original sequence X, the buffer operator D is termed as a weakening operator [5, 12, 13].

Theorem 2.1

1. When X is a monotonously increasing sequence, XD is a buffer sequence, then D is a weakening operator $\Leftrightarrow x(k) \le x(k)d$. $(k = 1,2,3,...,n)$.
2. When X is a monotonously decreasing sequence, XD is a buffer sequence, then, D is a weakening operator $\Leftrightarrow x(k) \ge x(k)d$, $(k = 1,2,3,...,n)$.
3. When X is a monotonously vibration sequence and D is a weakening operator, XD is a sequence operator, then, $\max_{1\le k\le n} x(k) \ge \max_{1\le k\le n}\{x(k)d\}$, $\min_{1\le k\le n} x(k) \le \min_{1\le k\le n}\{x(k)d\}$.

That is, the data in a monotonic increasing sequence expand when a weakening operator is applied and data in a monotonously decreasing sequence shrink when a weakening operator is applied[5]..

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III. WEAKENING BUFFER OPERATOR ON STRICTLY MONOTONIC FUNCTION

Liu and Dang construct the following weakening operator in its monograph [5]. Assume $X = (x(1), x(2), x(3), \cdots x(n))$ is a sequence of raw data, . then

$$XD_1 = (x(1)d_1, \cdots, x(n)d_1),$$

and

$$x(k)d_1 = \frac{x(k) + \cdots + x(n)}{n-k+1},$$

when X is a monotonously increasing sequence, a monotonously decreasing sequence or a vibration sequence, D_1 is a weakening operator. Here, we call D_1 as the average weakening operator. And based on the operator, we construct a new weakening buffer operator through a strictly monotonic function.

Theorem 3.1: Assume $X = (x(1), x(2), x(3), \cdots x(n))$ is a sequence of raw data, $x_i > 0, f > 0$. f is a monotonously increasing function, g is its inverse function. If d is a weakening buffer operator, and $x(k)d$ is made of $x(k), \cdots, x(n), k = 1, 2, \cdots n$. If we substitute $x(k)$ by $f(x(k)), k = 1, \cdots, n$. The new results are m=[f(x(k))]d and e=g(m),then e is also a weakening buffer operator.

Proof: since d is a weakening buffer operator, then x(n)d=x(n),

$$f(x(n))d = f(x(n)),$$
$$x(n)e = g\{f(x(n))d\} = g(f(x(n))) = x(n)$$

So e satisfies the three axioms and e is a buffer operator.

1.When X is a monotonously increasing sequence, because 0< $x(k) \le \cdots \le x(n)$, f is a monotonously increasing function and $f > 0$, so

$$0 < f(x(k)) \le \cdots \le f(x(n)),$$

since d is a weakening buffer operator,

then $x(k)d \ge x(k)$,

$$f(x(k))d \ge f(x(k)),$$

g and f are inverse functions,

$$x(k)e = g(f(x(k))d) \ge g(f(x(k))) = x(k)$$

e is a weakening buffer operator.

2.When X is a monotonously decreasing sequence, because $x(k) \geq \cdots \geq x(n) > 0$, f is a monotonously increasing function and $f > 0$, so $f(x(k)) \geq \cdots \geq f(x(n)) > 0$

since d is a weakening buffer operator, then

$$x(k)d \leq x(k),$$

$$f(x(k))d \leq f(x(k)),$$

g and f are inverse functions,

$$x(k)e = g(f(x(k))d) \leq g(f(x(k))) = x(k)$$

e is a weakening buffer operator.

3.When X is a vibration sequence, let

$$x(k) = \max_{1\leq i\leq n} x(i), x(h) = \min_{1\leq i\leq n} x(i) ,$$

$$x(k) \geq x(1), \cdots, x(n) ; \; x(h) \leq x(1), \cdots, x(n)$$

Because f is a monotonously increasing function and $f > 0$,

$$f(x(k)) \geq f(x(1)), \cdots, f(x(n)) > 0 ,$$

$$0 < f(x(h)) \leq f(x(1)), \cdots, f(x(n))$$

$$f(x(k)) = \max_{1\leq i\leq n} f(x(i)), f(x(h)) = \min_{1\leq i\leq n} f(x(i))$$

since d is a weakening buffer operator, then

$$\max_{1\leq i\leq n} x(i) \geq \max_{1\leq i\leq n} x(i)d ,$$

$$\min_{1\leq i\leq n} x(i) \leq \min_{1\leq i\leq n} x(i)d ,$$

$$\max_{1\leq i\leq n} f(x(i)) \geq \max_{1\leq i\leq n} f(x(i)d) ,$$

$$\min_{1\leq i\leq n} f(x(i)) \leq \min_{1\leq i\leq n} f(x(i)d)$$

since g and f are inverse functions,

$$\max_{1\leq i\leq n} x(i) = \max_{1\leq i\leq n} g(f(x(i))) = g(\max_{1\leq i\leq n} f(x(i)))$$
$$\geq g(\max_{1\leq i\leq n} f(x(i))d) = \max_{1\leq i\leq n} g(f(x(i))d)$$
$$= \max_{1\leq i\leq n} x(i)e$$

$$\min_{1\leq i\leq n} x(i) = \min_{1\leq i\leq n} g(f(x(i))) = g(\min_{1\leq i\leq n} f(x(i)))$$
$$\leq g(\min_{1\leq i\leq n} f(x(i))d) = \min_{1\leq i\leq n} g(f(x(i))d)$$
$$= \min_{1\leq i\leq n} x(i)e$$

e is a weakening buffer operator.

If $x(k)d = x(k)d_2 = g(\frac{(n-k+1) f^2(x(k))}{f(x(k))+\cdots+f(x(n))})$, we have the following conclusion.(see [10]).

Corollary 1: Assume $X = (x(1), x(2), x(3), \cdots x(n))$ is a sequence of raw data, $x_i > 0, f > 0$. f is a monotonously increasing function, g is its inverse function.

And $XD_2 = (x(1)d_2, x(2)d_2, \cdots x(n)d_2)$. $x(k)d_2 = g(\frac{f(x(k))+\cdots+f(x(n))}{n-k+1})$,

When X is a monotonously increasing sequence, a monotonously decreasing sequence, a monotonously vibration sequence, then D_2 is a weakening buffer operator.

If f(x)=g(x)=x and

$$x(k)d = x(k)d_2 = x(k)d_3 = \frac{x(k)+\cdots+x(n)}{n-k+1},$$

we have the following conclusion.(see [5]).

Corollary 2: Assume $X = (x(1), x(2), x(3), \cdots x(n))$ is a sequence of raw data, $x_i > 0$, .

And $XD_3 = (x(1)d_3, x(2)d_3, \cdots x(n)d_3)$. $x(k)d_3 = \frac{x(k)+\cdots+x(n)}{n-k+1}$,

When X is a monotonously increasing sequence, a monotonously decreasing sequence, a monotonously vibration sequence, then D_3 is a weakening buffer operator.

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