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Enhancement of ELECTRE I using Compound Linguistic Ordinal Scale and Cognitive Pairwise Comparison

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***Abstract*—ELECTRE is one of the popular approaches of multi-criteria decision making. Setting parameters for ELECTRE is one of the essential steps. This research proposes the Cognitive Pairwise Comparison (CPC) and the Compound Linguistic Ordinal Scale (CLOS) to determine the parametric settings in ELECTRE I. The usability and applicability of the enhancement of ELECRE I are illustrated in a case of the information system selection problem.**

***Keywords*—ELECTRE, multi-criteria decision making, compound linguistic variable, rating scale, pairwise comparison**

I. Introduction

The ELECTRE, ELimination Et Choix Traduisant la REalité, (ELimination and Choice Expressing REality), initially appeared in a French operations research journal by Roy [5]. There are various versions of ELECRE models, which can be referred to in the reviewed references [1,4]. This study only focuses on the most fundamental one, ELECTRE I.

Two important notations to address the parametric settings of the decision matrix of the ELECTRE are rating scale and weight determination.

Regarding the rating scale, Miller [3] has indicated that an expert could manage a set with (7 ± 2) terms. Likert scale [2], which applies the principle of (7 ± 2), is widely used. The Compound Linguistic Ordinal Scale Model (CLOS) [9,11], which is an ordinal in ordinal scale model, is a promising alternative for the classic rating scale, Likert scale.

By breakthrough of the principle of (7 ± 2), CLOS can provide $(7\pm2)((7\pm2)-1)+1=[21,73]$ options which seem incredible for an expert being able to handle. Unlike the classical rating model which is the single step rating process, CLOS uses a Deductive Rating Strategy in which a rater chooses a 2-tuple option in two steps with a rethink process [9,11]. The advantage of CLOS is that CLOS is an ideal rating interface for addressing the problem of the rating dilemma [9,11]. One of the applications using CLOS is illustrated in [12]. CLOS is applied to the Linguistic-Possibility-Probability Aggregation Model (LPPAM) [12] in view of the direct absolute rating measurement. The details of CLOS are shown in section 2.

Regarding the weight determination, the pairwise reciprocal matrix of the Analytic Hierarchy Process (AHP) [6] is one of the methods. Yuen [10,11] has indicated two major queries on this method: cognitive misrepresentation of the pairwise reciprocal matrix using the ratio scale, and the uncertainty of the prioritization methods, which also can be found in Yuen [8]. To address these two queries, the pairwise opposite matrix (or cognitive pairwise matrix) and the cognitive prioritization method are proposed (section 3).

This paper is organized as follows. Section 2 introduces the concept of Compound Linguistic Ordinal Scale (CLOS) whilst section 3 illustrates the notion of Cognitive Pairwise Comparison (CPC) using CLOS. Section 4 presents the model of the enhanced ELECTRE I using CPC and CLOS. To demonstrate the usability and applicability, a case of the information system selection problem is illustrated in section 5. Conclusion is drawn in section 6.

II. Compound Linguistic Ordinal Scale

Compound Linguistic Ordinal Scale (CLOS) was initially developed by Yuen [9,11]. CLOS is a Deductive Rating Strategy (Rs) of the Hedge-Direction-Atom Linguistic Representation Model (HDA-LRM) with a cross reference relationship.

In the HDA-LRM, Compound Linguistic Variable (CLV) $\aleph$, a matrix of a large number of linguistic descriptors, is produced by the syntactic rule. The semantic rule, "Computing with CLV", maps CLV into representation numbers in matrix $\overline{X}_{\aleph}$ or $\overline{X}$ by Fuzzy Normal Distribution $f_{\overline{X}}(\aleph)$, and produces the numerical results meeting the different requirements of different scenario using few scalable descriptable user-defined parameters.

The Deductive Rating Strategy (Rs) is the ideal rating interface for handling the large scale of CLV. Three key concepts are presented, as follows.

A. *Syntactic rule*

Regarding the syntactic form, CLOS is established on a compound linguistic variable $\alpha\in\aleph_{mn}$ which is comprised of the structural elements from three linguistic term vectors

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respectively: hedge vector $\overrightarrow{V_h}$, directional vector $\overrightarrow{V_d}=\left[v_d^{-}, v_d^{\theta}, v_d^{+}\right]$, and atomic vector $\overrightarrow{V_a}=\left[v_{a_j}\right]$. The Compound Linguistic Variable (CLV) $\aleph_{mn}$ is built on the syntactic rule algorithm (algorithm 1), $\aleph_{mn}=G_{\aleph}\left(\overrightarrow{V_h},\overrightarrow{V_d},\overrightarrow{V_a}\right)$, and has the following form:

$$\begin{array}{c} \\ \alpha_1 \\ \vdots \\ \alpha_\eta \\ \alpha_{\eta+1} \\ \alpha_{\eta+2} \\ \vdots \\ \alpha_m \end{array} \begin{array}{c} \begin{array}{ccccc} \alpha^1 & \alpha^2 & \cdots & \alpha^{n-1} & \alpha^n \end{array} \\ \begin{bmatrix} \varnothing & v_{hd_1}\oplus v_{a_2} & \cdots & v_{hd_1}\oplus v_{a_{n-1}} & v_{hd_1}\oplus v_{a_n} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \varnothing & v_{hd_\eta}\oplus v_{a_2} & \ddots & v_{hd_\eta}\oplus v_{a_{n-1}} & v_{hd_\eta}\oplus v_{a_n} \\ v_{a_1}^{\theta} & v_{a_2}^{\theta} & \ddots & v_{a_{n-1}}^{\theta} & v_{a_n}^{\theta} \\ v_{hd_{\eta+2}}\oplus v_{a_1} & v_{hd_{\eta+2}}\oplus v_{a_2} & \ddots & v_{hd_{\eta+2}}\oplus v_{a_{n-1}} & \varnothing \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ v_{hd_m}\oplus v_{a_1} & v_{hd_m}\oplus v_{a_2} & \cdots & v_{hd_m}\oplus v_{a_{n-1}} & \varnothing \end{bmatrix} \end{array}$$

, where v_{hd} is the element of the combination of $\overrightarrow{V_h}$ and $\overrightarrow{V_d}$. (1)

Algorithm 1 (Syntactic Rule Algorithm $\aleph_{mn}=G_{\aleph}\left(\overrightarrow{V_h},\overrightarrow{V_d},\overrightarrow{V_a}\right)$

1. Input: Linguistic term sets ($\overrightarrow{V_h}$, $\overrightarrow{V_d}$, $\overrightarrow{V_a}$)
2. Proceed $G_{\overrightarrow{V_{hd}}}\left(\overrightarrow{V_h},\overrightarrow{V_d}\right)=\overrightarrow{V_{hd}}$ by

$$\left[v_{hd_i}\right]_{i=1}^{m}=\left[\left(v_{h_\eta}\oplus v_d^{-}\right),\cdots,\left(v_{h_1}\oplus v_d^{-}\right),v_d^{\theta},\left(v_{h_1}\oplus v_d^{+}\right),\cdots,\left(v_{h_\eta}\oplus v_d^{+}\right)\right]$$

3. Proceed $G_{\aleph}\left(\overrightarrow{V_{hd}},\overrightarrow{V_a}\right)$ by

$$G_{\alpha_{ij}}\left(v_{hd_i},v_{a_j}\right)\triangleq\begin{cases}\varnothing & j=1\ \&\ i\in\{1,\cdots,((m+1)/2)\} \\ v_{hd_i}\oplus v_{a_j} & j\neq 1,n\ \&\ \forall i \\ \varnothing & j=n\ \&\ i\in\{((m+1)/2),\cdots,m\}\end{cases},\ \forall i,j$$

4. Return: $\aleph_{mn}=G_{\aleph}\left(\overrightarrow{V_{hd}},\overrightarrow{V_a}\right)$, e.g. eq. (1)

B. *Semantic rule*

The numerical representation is derived by the semantic rule algorithm or Fuzzy Normal Distribution [9,11] of the form:

$$\overline{X}_{\aleph}=f_{\overline{X}}(\aleph)$$

$$=f_{\overline{X}}\left(\left\{\left(\gamma_{\alpha^j},d_{\alpha^j},\tau_{\alpha^j},\left\{\mu_{\alpha^{j\phi}}{}^{-1}\right\}^{\phi}\right)\right\}.\left[X_{\min},X_{\max}\right],\left(\varphi\left(\overrightarrow{V_h}\right),\lambda_0\right)\right) \quad (2)$$

$\overline{X}_{\aleph}$ is the numerical representation of $\aleph$ in either fuzzy or crisp value as crisp value is the special case of the fuzzy value. γ_{α^j} is the modal value, d_{α^j} is symmetric distance (by default, $d_{\alpha^1}=d_{\alpha^2}=,\ldots,=d_{\alpha^n}$), τ_{α^j} is tuning parameter of the membership function, μ_{α^j}, of α^j, and $\mu_{\alpha^{j\phi}}{}^{-1}$ is the inversed membership function, which the default setting is the inversed parabola-based membership function $PbMF_{\alpha^j}{}^{-1}$ is of the form:

$$PbMF_{\alpha^j}{}^{-1}\left(\mu_{\alpha^{j\phi}}\right)=\begin{cases}\gamma_{\alpha^j}-d_{\alpha^j}\sqrt{1-\left(\mu_{\alpha^{j\phi}}\right)^{1/\tau_{\alpha^j}}} & ,\phi='-' \\ \gamma_{\alpha^j} & ,\phi='\theta' \\ \gamma_{\alpha^j}+d_{\alpha^j}\sqrt{1-\left(\mu_{\alpha^{j\phi}}\right)^{1/\tau_{\alpha^j}}} & ,\phi='+'\end{cases} \quad (3)$$

, where $\phi='-','\theta','+'$ is determined from $\overrightarrow{V_d}$.

$\left[X_{\min},X_{\max}\right]$ is the interval of numerical representation of the scale. The 2-tuple input $\left(\varphi\left(\overrightarrow{V_h}\right),\lambda_0\right)$ determines the distribution of the $\overrightarrow{V_{hd}}$ in the membership fuzziness process (MFI). Thus, $f_{\overline{X}}(\aleph)$ is shown in algorithm 2.

Algorithm 2 (Semantic Rule Algorithm / Fuzzy Normal Distribution):

1. Get valid $\left(\left\{\left(\gamma_{\alpha^j},d_{\alpha^j},\tau_{\alpha^j},\left\{\mu_{\alpha^{j\phi}}{}^{-1}\right\}^{\phi}\right)\right\}.\left[X_{\min},X_{\max}\right],\left(\varphi\left(\overrightarrow{V_h}\right),\lambda_0\right)\right)$.
2. Calculate $MCI\left(\left[\!\left[\overrightarrow{V_h}\right]\!\right]\right)$ and $MFI\left(\left[\!\left[\overrightarrow{V_h}\right]\!\right]\right)$ by

$$MFI\left(\left[\!\left[\overrightarrow{V_h}\right]\!\right]\right)=MFI\left(\begin{bmatrix}v_{h_1}\\ \vdots \\ v_{h_\eta}\end{bmatrix}\right)=\begin{cases}\left[\mu'_{l_i}-\lambda_{\mu_{l_i}}dis\left(v_{h_i}\right),\quad 1\right]_{i=1} \\ \left[\!\left[\mu'_{l_i}-\lambda_{\mu_{l_i}}dis\left(v_{h_i}\right),\ \mu'_{u_i}+\lambda_{\mu_{l_i}}dis\left(\sigma_i\right)\right]\!\right]_{i=2}^{\eta-1} \\ \left[0\quad,\quad \mu'_{u_i}+\lambda_{\mu_{l_i}}dis\left(\sigma_i\right)\right]_{i=\eta}\end{cases}$$

,where $dis\left(v_{h_i}\right)=\dfrac{\varphi\left(v_{h_i}\right)}{\sum_{\overrightarrow{V_h}}\varphi\left(v_{h_i}\right)}$; $\lambda_{\mu_{u_i}}=\lambda_{\mu_{l_i}}=\dfrac{\lambda_0}{2}$, where $\lambda_{\mu_{u_i}},\lambda_{\mu_{l_i}}\in[0,1]$ (i.e. $\lambda_0\in[0,2]$) such that $0\le\mu_{u_i}\le 1$.

3. Calculate $MFI\left(\left[\!\left[\overrightarrow{V_h}^{+}\right]\!\right]\right)$ and $MFI\left(\left[\!\left[\overrightarrow{V_h}^{-}\right]\!\right]\right)$ by

$$MFI\left(\left[\!\left[\overrightarrow{V_h}^{-}\right]\!\right]\right)=vip\left(MFI\left(\left[\!\left[\overrightarrow{V_h}\right]\!\right]\right)\right)\equiv\overset{\mu_L^{-}\quad\mu_U^{-}}{\left\lceil\mu_{l_j},\mu_{u_j}\right\rceil_{j=1}^{\eta}}=\overset{\mu_L\quad\mu_U}{\left[\!\left[\mu_{l_j},\mu_{u_j}\right]\!\right]_{j=\eta}^{1}}$$

$$MFI\left(\left[\!\left[\overrightarrow{V_h}^{+}\right]\!\right]\right)=hrp\left(MFI\left(\left[\!\left[\overrightarrow{V_h}\right]\!\right]\right)\right)\equiv\overset{\mu_L^{+}\quad\mu_U^{+}}{\left\lfloor\mu_{l_j},\mu_{u_j}\right\rfloor_{j=1}^{\eta}}=\overset{\mu_U\quad\mu_L}{\left[\!\left[\mu_{U_j},\mu_{l_j}\right]\!\right]_{j=1}^{\eta}}$$

,where $MFI\left(\left[\!\left[\overrightarrow{V_h}\right]\!\right]\right)=MFI\left(\begin{bmatrix}v_{h_1}\\ \vdots \\ v_{h_\eta}\end{bmatrix}\right)=\overset{\mu_L\quad\mu_U}{\left[\!\left[\mu_{l_j},\mu_{u_j}\right]\!\right]_{j=1}^{\eta}}$

4. Calculate $FI\left(\left[\!\left[\widehat{\alpha^j}\right]\!\right]\right),\forall j$ by

$$FI\left(\left[\!\left[\widehat{\alpha^j}\right]\!\right]\right)=\begin{cases}\left[\mu_{\alpha^{j-}}{}^{-1}\left(\mu_{l_i}^{-}\right),\ \mu_{\alpha^{j-}}{}^{-1}\left(\mu_{l_i}^{-}\right)\right]_{i=1}^{\eta} & ,j\in\{2,\ldots,n\}\ \&\ \phi='-' \\ \left[\mu_{\alpha^j}{}^{-1}\left(\mu_{l_\theta}\right),\ \mu_{\alpha^j}{}^{-1}\left(\mu_{l_\theta}\right)\right]_{i=\eta+1} & ,\forall j\ \&\ \phi='\theta' \\ \left[\mu_{\alpha^{j+}}{}^{-1}\left(\mu_{l_i}^{+}\right),\ \mu_{\alpha^{j+}}{}^{-1}\left(\mu_{l_i}^{+}\right)\right]_{i=\eta+2}^{m} & ,j\in\{1,\ldots,n-1\}\ \&\ \phi='+'\end{cases}$$

5. If $\overline{X}_{\aleph}$ is in crisp number, then $\overline{X}_{\aleph}=mean\left(FI\left(\left[\!\left[\widehat{\alpha^j}\right]\!\right]\right)\right)$.
6. Return $\overline{X}$;.

C. *Deductive Rating Strategy*

It seems incredible that an expert can handle $\left|\aleph_{7\pm2,7\pm2}\right|=[21,73]$ linguistic terms although CLV can produce a large scale of compound linguistic terms. Thus deductive rating strategy is proposed. Algorithm 3 shows the rating steps whilst figure 1 shows an example of the rating interface.

Algorithm 3 (Deductive Rating Strategy : $\left(\overrightarrow{V_{hd}}_j, \overrightarrow{V_a}, Rs\right)$:

1. Observe external information;
2. Understand the problem which needs to be classified;
3. Understand the CLOS model;
4. Choose v_{a_j} in $\overrightarrow{V_a} \equiv \left[v_{a_j}\right]_{j=1}^{n}$ (first rating step);
5. Computer shows second options by
$$\overrightarrow{V_{hd}}_j = Rs\left(v_{a_j}\right) = \begin{cases} \left[v_{hd_i}\right]_{i=1}^{\eta} & \text{if } j=1 \\ \left[v_{hd_i}\right]_{i=1}^{m} & \text{if } j \neq 1, n \\ \left[v_{hd_i}\right]_{i=\eta+2}^{m} & \text{if } j=n \end{cases}.$$
6. Select the second option with revision of first option (second rating step);
   6.1 If first option is confirmed, then the rater chooses v_{hd_i} in $\overrightarrow{V_{hd}}_j$;
   6.2 Else go to Step 4;
7. Return $\alpha_{ij} = \left(v_{hd_i}, v_{a_j}\right)$

III. Cognitive Pairwise Comparison

The Cognitive Pairwise Comparison (CPC) was initially proposed by Yuen [10,11]. Three key notions are as follows.

A. *Cognitive pairwise matrix*

The Cognitive Pairwise Matrix (Pairwise Opposite Matrix or Cognitive Comparison Matrix) B of the objective O with respect to the criteria $\{c_i\}$, i.e. $Clst\left(O,\{c_i\}\right)$, is of the form.

$$B_O = \varphi\left(Clst\left(O,\{c_i\}\right)\right) = \begin{array}{c} \\ c_1 \\ c_2 \\ \vdots \\ c_n \end{array}\begin{array}{c} \begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array} \\ \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix} \end{array} \tag{4}$$

, $b_{ij} = v_i - v_j$, $\forall i, j \in (1,\ldots,n)$. b_{ij} is the comparison score from the cognitive rating scale $\aleph$. $Clst$ is a cluster. φ is the Cogntive Assessment Function (CAF) performed by the expert.

A Cognitive Pairwise Matrix B is validated by the Accordant Index of the form:

$$AI = \frac{1}{n^2}\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} d_{ij}}\,,$$

$$d_{ij} = \sqrt{Mean\left(\left(\frac{1}{\kappa}\left(B_i + B_j^T - b_{ij}\right)\right)^2\right)},\ \forall i, j \in (1,\ldots,n)$$

, where $AI \geq 0$, and κ is the normal utility such that

$$\left(b_{ik} + b_{kj}\right) \in [-\kappa, \kappa],\ \forall i, j, k \in (1,\ldots,n) \tag{5}$$

The maximal value of the numerical rating scale is the default setting of κ if v_i is a non-negative value. Otherwise, κ is increased to meet the above constraint.

If $AI = 0$, then B is perfectly accordant; If $0 < AI \leq 0.1$, then B is satisfactory, then. If $AI > 0.1$, then B is unsatisfactory .

B. *Cognitive prioritization operator*

Yuen [10,11] has proposed several cognitive prioritization operators. Two operators are recommended: Least Penalty Squares (LPS) and the Row Average plus the normal Utility (RAU). The comprehensive numerical analyses conclude that, in most cases, if $\text{AI} \leq 1$, the results of RAU and LPS are the same or very closed [10,11].

If $\text{AI} \leq 1$, RAU has better computational efficiency, and has the form:

$$v_i = \left(\frac{1}{n}\sum_{j=1}^{n} b_{ij}\right) + \kappa, \forall i \in \{1,\ldots,n\} \tag{6}$$

To normalize the $\{v_i\}$, then

$$W = \left\{w_i : w_i = \frac{v_i}{n\kappa}, \forall i \in \{1,\ldots,n\}\right\}, \sum_{i\in\{1,\ldots,n\}} v_i = n\kappa \tag{7}$$

C. *Cognitive rating scale*

The cognitive rating scale is the interval scale, which applies the concept of CLOS. Thus the cognitive rating scale is the Compound Linguistic Interval Scale (CLIS).

Let the comparison interval scale schema of the Hedge-Direction-Atom Linguistic Representation Model be $\left(\aleph, \bar{X}_{\aleph} = \left\{\bar{X}_{\aleph}^-, \bar{X}_{\aleph}^+\right\}, f_{\bar{X}}(\aleph)\right)$. To construct the labels of the comparison interval scale $\aleph$, let

$\overrightarrow{V_a} = [\text{Equal,Slight,Moderate,Strong,Essential}]$,

$\overrightarrow{V_h} = [\text{Little,Quite,Much}]$,

$\overrightarrow{V_d} = [\text{Below,Absolutely,Above}]$.

By algorithm 1, then

$$\aleph = \begin{bmatrix} \varnothing & MB\text{-}Sl & MB\text{-}Mo & MB\text{-}St & MB\text{-}Es \\ \varnothing & QB\text{-}Sl & QB\text{-}Mo & QB\text{-}St & QB\text{-}Es \\ \varnothing & LB\text{-}Sl & LB\text{-}Mo & LB\text{-}St & LB\text{-}Es \\ A\text{-}Eq & A\text{-}Sl & A\text{-}Mo & A\text{-}St & A\text{-}Es \\ LA\text{-}Eq & LA-Sl & LA\text{-}Mo & LA\text{-}St & \varnothing \\ QA\text{-}Eq & QA\text{-}Sl & QA\text{-}Mo & QA\text{-}St & \varnothing \\ MA\text{-}Eq & MA-Sl & MA\text{-}Mo & MA\text{-}St & \varnothing \end{bmatrix}$$

Regarding the representation values, let $\left[X_{\min}, X_{\max}\right] = [0,\kappa]=[0,8]$, $d_{\alpha^{1,\dots,5}}=2$, $\vec{\gamma}=[0,2,4,6,8]$, $\tau_{\alpha^{1,\dots,5}}=2$, $\left\{\mu_{\alpha^{j\varphi}}{}^{-1}\right\}=PbMF^{-1}$, $\varphi\left(\overline{V_h}\right)=[1,1,1]$, $\lambda_0=0.5$. By algorithm 2, $\overline{X}_{\aleph}^{+}$ is of the form:

$$\overline{X}_{\aleph}^{+}=\begin{bmatrix} \varnothing & 0.65 & 2.65 & 4.65 & 6.65 \\ \varnothing & 1.37 & 3.37 & 5.37 & 7.37 \\ \varnothing & 1.76 & 3.76 & 5.76 & 7.76 \\ 0 & 2.00 & 4.00 & 6.00 & 8.00 \\ 0.24 & 2.24 & 4.24 & 6.24 & \varnothing \\ 0.63 & 2.63 & 4.63 & 6.63 & \varnothing \\ 1.36 & 3.36 & 5.36 & 7.36 & \varnothing \end{bmatrix}$$

, and the opposite matrix of $\overline{X}_{\aleph}^{+}$ is $\overline{X}_{\aleph}^{-}=-\overline{X}_{\aleph}^{+}$.

If an object A performs better than an object B, then the score of the rating scale is chosen from $\overline{X}_{\aleph}^{+}$. Otherwise, it is from $\overline{X}_{\aleph}^{-}$.

As there are many rating options, a deducted rating strategy of CLIS is needed. Fig. 1 illustrates the interface to show the rating process is performed by algorithm 3.

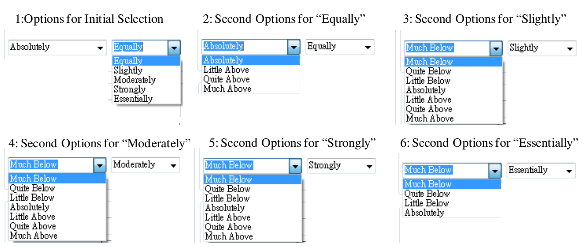

Figure 1: Deductive rating strategy for the compound interval scale

IV. THE ENHANCED ELECTRE I USING CLOS AND CPC

The enhanced ELECTRE I using CLOS and CPC comprises four steps, as follows:

Step 1: Forming decision matrix

A typical *m* by *n* decision matrix O is shown as below:

$$O=\begin{matrix} & \begin{pmatrix} w_1 & \cdots & w_j & \cdots & w_n \end{pmatrix} \\ & \begin{matrix} c_1 & \cdots & c_j & \cdots & c_n \end{matrix} \\ \begin{matrix} T_1 \\ \vdots \\ T_i \\ \vdots \\ T_m \end{matrix} & \begin{pmatrix} & & & & \\ & & & & \\ & & r_{ij} & & \\ & & & & \\ & & & & \end{pmatrix} \end{matrix} \quad (10)$$

$c_j \in C$ is the positive criterion. $T_i \in T$ is the alternative. T^* is the ideal alternative from *T*.

$r_{ij} \in r$ is the utility value from either the normalized qualitative data or normalized quantitative data. If the data are quantitative, the data are supplied directly and then normalized. If the utility value is qualitative, data are uncertain and evaluated by the normalized cognitive prioritization of the pairwise opposite matrix using CLOS.

$w_j \in W$ is the weight of the criterion c_j such that $\sum_{j=1}^{n} w_j = 1$. The cognitive pairwise comparison using CLOS is also the ideal method to determine the weights.

Step 2: Computing concordance and discordance

Considering O, let the preference relation R be $\succsim$, where $c_j \succsim c_k$ means c_j outranks c_k (that is, "c_j is at least as good as c_k"). The concordance index β and the discordance index δ are two essential concepts in ELECTRE [1].

For an ordered pair of granular information $\left(c_j, c_k\right)$, the concordance index is of the form:

$$\beta_{jk}=\beta\left(c_j \succsim c_k\right)=\sum_{\{i: r_{ij} \geq r_{ik}\}} w_i \quad (8)$$

, where $\sum_{i=1}^{z} w_i = 1$, and $\left\{i: r_{ij} \geq r_{ik}\right\}$ is the set of indices for all criteria belonging to the concordant coalition with the outranking relation, i.e. $c_j \succeq c_k$.

The value of the concordance index β_{jk} must be greater than or equal to a given concordance level, *s*, whose value generally falls within the range $\left[0.5, 1-\min\left(\left\{w_j\right\}\right)\right]$, i.e., $\beta_{jk} \geq s$.

The discordance is measured by a discordance level defined as follows:

$$\delta_{jk}=\delta\left(c_j \succsim c_k\right)=\max_{\{i: r_{ij} < r_{ik}\}}\left\{r_{ik}-r_{ij}\right\} \quad (9)$$

The power of the discordance means that if its value surpasses a given level, ν, the assertion is no longer valid. Discordant coalition expects no power whenever $\delta_{jk} \leq \nu$.

A weakness of ELECTRE might lie in its use of the critical threshold values as these values are rather arbitrary, although their impact upon the ultimate result may be significant [7]. This research defines the threshold values, as follows:

$$s=\frac{1}{n \cdot (n-1)} \sum_{j=1}^{n} \sum_{k=1, k \neq j}^{n} \beta_{jk} \quad (10)$$

$$\nu=\frac{1}{n \cdot (n-1)} \sum_{j=1}^{n} \sum_{k=1, k \neq j}^{n} \delta_{jk} \quad (11)$$

Also this research proposes the Concordance Relation (CR) matrix and Discordance Relation (DR) matrix respectively as follows:

$$CR=\left\{cr_{jk}:cr_{jk}=\begin{cases}1, \beta_{jk}\geq s\\ 0, \beta_{jk}<s\end{cases}, \forall j,k\in\{1,\ldots,n\}\ \&\ j\neq k\right\} \quad (12)$$

$$DR=\left\{dr_{jk}:dr_{jk}=\begin{cases}1, \delta_{jk}\leq v\\ 0, \delta_{jk}>v\end{cases}, \forall j,k\in\{1,\ldots,n\}\ \&\ j\neq k\right\} \quad (13)$$

Step 3: Exploiting the outranking relation

The exploit of this outranking relation is to identify a small as possible subset of actions, from which the best compromise action could be selected. Let $\overline{c}$ be the partition. Each class on $\overline{c}=[\overline{c}_1,\overline{c}_2,\ldots]$ is composed of a set of (considerate) equivalent actions. The new preference relation, $\succ'$, is defined on $\overline{c}$, which is of the form [1]:

$$\overline{c}_p \succ' \overline{c}_q \Leftrightarrow \{\exists c_j \in \overline{c}_p \quad \& \quad \exists c_k \in \overline{c}_q \mid c_j \succsim c_k, for\ \overline{c}_p \neq' \overline{c}_q\} \quad (14)$$

Explicitly, this research proposes the outranking relation (OR) matrix to explore the preference relations, as follows:

$$OR=\left\{or_{jk}:or_{jk}=\begin{cases}1, cr_{jk}=dr_{jk}\\ 0, cr_{jk}\neq dr_{jk}\end{cases}, \forall j,k\in\{1,\ldots,n\}\ \&\ j\neq k\right\} \quad (15)$$

Step 4: Performing complementary analysis

ELECTRE I does not indicate the rank of the preference. Thus the complementary analysis using net outranking relationship is proposed in [7] as follows.

The net concordance index is of the form:

$$\beta_j=\sum_{\substack{k=1\\k\neq j}}^{n}\beta_{jk}-\sum_{\substack{k=1\\k\neq j}}^{n}\beta_{kj}, \ \forall j\in\{1,\ldots,n\} \quad (16)$$

The net discordance index is of the form:

$$\delta_j=\sum_{\substack{k=1\\k\neq j}}^{n}\delta_{jk}-\sum_{\substack{k=1\\k\neq j}}^{n}\delta_{kj}, \forall j\in\{1,\ldots,n\} \quad (17)$$

However, the rank of net concordance index is not always consistent with the rank of net discordance index. Thus, this research proposes the net balance index as follows:

$$\phi_j=\beta_j-\delta_j, \forall j\in\{1,\ldots,n\} \quad (18)$$

V. A Numerical Example

A SME trading company would like to purchase a customer relationship management information system to streamline its business. Four candidates, T_1,T_2,T_3,T_4, of the information systems are screened initially. The company would like to choose the best one on the basis of five criteria: competitive strategy (c_1), business process (c_2), usability (c_3), technical support (c_4), and price attractiveness (c_5). The enhanced ELECTRE I is employed as follows.

Step 1: Forming the decision matrix

The cognitive pairwise comparisons are used to determinate the decision matrix. The scale for CPCs is shown in part c of section 3.

Let $B_j=\varphi(\langle c_j,\{T_i\}\rangle)$. The cognitive pairwise matrices of the four alternatives with respect to each criterion are shown in table 1.

To derive the weights of the five criteria, the pairwise opposite matrix is formed as follows.

$$\varphi(\langle O,\{c_i\}\rangle)=\begin{array}{c} \\ c_1\\ c_2\\ c_3\\ c_4\\ c_5\end{array}\begin{array}{c}\begin{array}{ccccc} c_1 & c_2 & c_3 & c_4 & c_5\end{array}\\ \begin{bmatrix}0 & -3.76 & -1.76 & 2.24 & 0.24\\ 3.76 & 0 & 2.24 & 6.24 & 4\\ 1.76 & -2.24 & 0 & 4 & 2.24\\ -2.24 & -6.24 & -4 & 0 & -2\\ -0.24 & -4 & -2.24 & 2 & 0\end{bmatrix}\end{array}$$

AI=0.017< 1 and it is acceptable.

By using RAU, the vectors of the normalize weights from the above six pairwise opposite matrices are formed in following decision matrix.

$$\begin{pmatrix}0.185 & 0.281 & 0.229 & 0.128 & 0.178\end{pmatrix}$$

$$O=\begin{array}{c} \\ T_1\\ T_2\\ T_3\\ T_4\end{array}\begin{array}{c}\begin{array}{ccccc} c_1 & c_2 & c_3 & c_4 & c_5\end{array}\\ \begin{pmatrix}0.362 & 0.283 & 0.201 & 0.219 & 0.158\\ 0.285 & 0.391 & 0.346 & 0.232 & 0.21\\ 0.234 & 0.144 & 0.3 & 0.261 & 0.271\\ 0.118 & 0.183 & 0.153 & 0.288 & 0.361\end{pmatrix}\end{array}$$

TABLE 1: PAIRWISE OPPOSITE MATRICES, $B_j, j=1,\ldots,5$

	T_1	T_2	T_3	T_4	T_1	T_2	T_3	T_4
	B_1, AI=0.015				B_2, AI=0.050			
T_1	0	2.63	4	7.76	0	-3.76	4.24	3.76
T_2	-2.63	0	1.76	5.36	3.76	0	8	6.24
T_3	-4	-1.76	0	3.76	-4.24	-8	0	-1.37
T_4	-7.76	-5.36	-3.76	0	-3.76	-6.24	1.37	0
	B_3,AI=0.074				B_4, AI=0.040			
T_1	0	-4.24	-3.76	1.76	0	-0.63	-1.37	-2
T_2	4.24	0	1.36	6.63	0.63	0	-0.63	-2.24
T_3	3.76	-1.36	0	4	1.37	0.63	0	-0.63
T_4	-1.76	-6.63	-4	0	2	2.24	0.63	0
	B_5, AI= 0.042							
T_1	0	-1.76	-3.76	-6.24				
T_2	1.76	0	-2.24	-4.63				
T_3	3.76	2.24	0	-3.37				
T_4	6.24	4.63	3.37	0				

TABLE 2: CONCORDANCE INDEX AND DISCORDANCE INDEX MATRICES

	$\{\beta_{jk}\}$				$\{\delta_{jk}\}$			
	T_1	T_2	T_3	T_4	T_1	T_2	T_3	T_4
T_1		0.185	0.466	0.695		0.144	0.112	0.203
T_2	0.815		0.695	0.695	0.077		0.060	0.151
T_3	0.534	0.305		0.414	0.139	0.247		0.091
T_4	0.305	0.305	0.586		0.244	0.208	0.147	

TABLE 3: RELATION MATRICES

Concordance relation CR				Discordance relation DR			
	0	0	1		1	1	0
1		1	1	1		1	1
1	0		0	1	0		1
0	0	1		0	0	1	

TABLE 4: OUTRANKING RELATION MATRIX

	T_1	T_2	T_3	T_4
T_1		0	0	0
T_2	1		1	1
T_3	1	0		0
T_4	0	0	1	

TABLE 5: β_j, δ_j ϕ_j, AND RANK WITH RESPECT TO T_1,T_2,T_3,T_4

	β_j	δ_j	ϕ_j	*Rank*
T_1	-0.309	-0.001	-0.308	3
T_2	1.410	-0.310	1.720	4
T_3	-0.494	0.157	-0.652	2
T_4	-0.606	0.154	-0.761	1

Step 2: Computing concordance and discordance

$\{\beta_{jk}\}$ and $\{\delta_{jk}\}$ are shown in table 2 by eqs. 8-9. These follow s=0.5 and v=0.152 which are computed by eqs. 10-11. The relation matrices of concordance index and discordance index computed by eqs. 12-13 are shown in table 3.

Step 3: Exploiting the outranking relation

From table 4 calculated by eq. 15, obviously, T_2 is the best choice. However, ELECTRE I cannot tell the ranks of the other candidates. The next step explores this issue.

Step 4: Performing complementary analysis

Table 5 shows the results of the complementary analysis by eqs. 16-18. The rank is (3, 4, 2,1) for (T_1,T_2,T_3,T_4).

VI. CONCLUSION

This paper demonstrates the concepts of the enhancement of ELECTRE I based on the Compound Linguistic Ordinal Scale and the Cognitive pairwise Comparison. An example of the selection of customer relationship management information systems in a SME trading company illustrates the usability and applicability of the enhanced ELECTRE I. Regarding contributions, the enhanced ELECTRE I can be applied to other application areas with supplying more rating scale items for the cognitive pairwise comparisons to determine the parameter settings for a decision matrix. Similarly to ELECTRE I, the CLOS and CPC can also be applied to other ELECTRE types.

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