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Interrelationships among Attitude-Based and Conventional Stability Concepts within the Graph Model for Conflict Resolution

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***Abstract*—Propositions are presented on interrelationships among attitude-based and conventional stability concepts within the paradigm of Graph Model for Conflict Resolution. In fact, the authors verify the following properties: if decision makers' attitudes are discrete and decision makers' preferences are complete and anti-symmetric, then i) relational Nash stability is equivalent to Nash stability, ii) relational general metarationality is equivalent to general metarationality, iii) relational symmetric metarationality is equivalent to symmetric metarationality, and iv) relational sequential stability is equivalent to sequential stability; under totally neutral attitudes of decision makers, i) relational Nash stability is equivalent to relational symmetric metarationality, and ii) relational general metarationality is equivalent to relational sequential stability.**

***Index Terms*—The Graph Model for Conflict Resolution; Attitudes; Stability Concepts**

I. Introduction

The purpose of this research is to provide propositions on interrelationships among attitude-based [19], [22], [33] and conventional stability concepts within the Graph Model for Conflict Resolution (GMCR) [4], [5]. GMCR is a flexible framework for describing and analyzing a conflict. In fact, GMCR allows one to describe and analyze a conflict with consideration of infeasibility of outcomes, irreversibility of choices of actions, countermoves against a Decision Maker (DM)'s unilateral moves, and so on [4], [5]. Consequently, this framework has been utilized to model and analyze realworld conflicts such as Cuban Missile Crisis, Garrison Diversion Unit conflict, Normandy Invasion, Fall of France, Zimbabwe conflict, Watergate Tapes controversy, Poplar River conflict, and Holston River negotiations [7]. Additionally, coalition formation [23] and strength of DMs' preferences [8] have been incorporated into the framework of GMCR. The coalition analysis framework in [23] is expanded in [17], [18] and applied to the analysis of environmental issues regarding the Kyoto Protocol [32]. Definitions of such coalition stability concepts as coalition Nash stability, coalition general metarationality, coalition symmetric metarationality, and coalition sequential stability are provided in [17], and their interrelationships to such standard stability concepts as Nash stability [26], [27], general metarationality [9], symmetric metarationality [9], and sequential stability [6], [7] within GMCR are investigated in [18]. The concept of policies, which is similar to that of strategies in game theory [28], is introduced to the graph model framework in [36], [37]. Research on the topic of incomplete preference information is also carried out [25], [29]. Cooperative decision situations are also analyzed within GMCR [34].

In a conflict, the attitudes of the various DMs can significantly influence the outcome of the conflict. Thus, analyzing the conflict as well as the accompanying attitudes is useful for better understanding a given conflict. Attitude-based analysis allows decision analysts to examine a lot of potential conflicts without needing to reevaluate DMs' preference.

Attitude-based stability concepts are furnished and applied to the analysis of the War of 1812 in [19]. Such attitude-based stability concepts in [19] as relational Nash stability, relational general metarationality, relational symmetric metarationality, and relational sequential stability are natural generalizations of the relational equilibrium concepts proposed in [15], [16] within the game theoretic paradigm and the standard stability concepts within the GMCR paradigm. Some interrelationships among attitude-based stability concepts are investigated in [22].

Attitude is defined by Krech et al. [24] as "an enduring system of positive or negative evaluations, emotional feelings and pro and con action tendencies, with respect to a social object." Following the framework in [22], the authors treat three types of attitudes, that is, positive, negative, and neutral attitudes, as in [30], [31] in which a DM's attitudes toward her/himself as well as toward others are considered. Moreover, it is assumed in this paper that positive, negative, and neutral attitudes of a DM toward others derive "altruistic," "sadistic," and "apathetic" behaviors, respectively, and those toward her/himself derive "selfish," "masochistic," and "selfless" behaviors, respectively. Table I shows these assumptions on the relationships between attitudes and DMs' behavior. These assumptions imply that a DM modeled in this paper is not "rational" in the classical game-theoretic sense, but are consistent with those of DMs' emotions in the 'soft' game theory [10], [13], [20], [21], drama theory [2], [3], [11], [12], [14], and confrontation analysis [1].

TABLE I
ASSUMPTIONS ON RELATIONSHIPS BETWEEN ATTITUDES AND BEHAVIORS [22]

Attitudes		
types	toward others	toward her/himself
positive	altruistic	selfish
negative	sadistic	masochistic
neutral	apathetic	selfless

In [22], two specific types of attitudes of DMs, that is, the kinds in which all of the DMs' attitudes are positive (see Figure 1) and negative (see Figure 2), respectively, are dealt with, and propositions on the equivalence between relational Nash stability and relational sequential stability under those types of DMs' attitudes are verified.

In this paper, two other kinds of attitudes of DMs are treated: the type in which the attitudes from one DM to her/himself are positive and those from one DM to another are neutral, and the type in which all of the DMs' attitudes are neutral. In this paper, these attitudes are called discrete and totally neutral, respectively. These types of attitudes are depicted in Figures 3 and 4, respectively, and their precise definitions are provided in Section II-C.

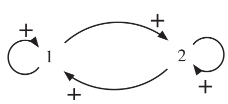

Fig. 1. Totally positive attitudes [22] for DM 1 and DM 2

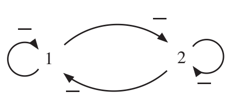

Fig. 2. Totally negative attitudes [22] for DM 1 and DM 2

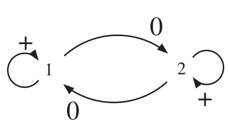

Fig. 3. Discrete attitudes [22] for DM 1 and DM 2

The objective of this paper is to verify some inclusion relationships among attitude-based and conventional stabilty concepts. More specifically, the authors verify that if decision makers' attitudes are discrete and decision makers' preferences are complete and anti-symmetric, then i) relational Nash stability is equivalent to Nash stability, ii) relational general metarationality is equivalent to general metarationality, iii) relational symmetric metarationality is equivalent to symmetric metarationality, and iv) relational sequential stability is equivalent to sequential stability. Also, the authors show that under totally neutral attitudes of decision makers, i) relational Nash stability

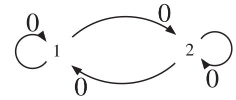

Fig. 4. Totally neutral attitudes [22] for DM 1 and DM 2

is equivalent to relational symmetric metarationality, and ii) relational general metarationality is equivalent to relational sequential stability.

The definitions of the concepts within the GMCR framework employed in this paper are presented in the next section. Then, in Section III, the propositions on interrelationships among attitude-based and conventional stability concepts are provided. The concluding remarks are furnished in the last section.

II. ATTITUDE ANALYSIS WITHIN GMCR: DEFINITIONS

This section gives the definitions of the concepts within the GMCR framework employed in this paper.

A. The Graph Models for Conflict Resolution — A General Definition

A *graph model of a conflict* is a 4-tuple $(N, S, (A_i)_{i\in N}, (\succeq_i)_{i\in N})$. N is the set of all *decision makers* (DMs), where $|N| \geq 2$. S is the set of all *states* of the focal decision making situation, where $|S| \geq 2$. For $i \in N$, (S, A_i) constitutes *DM i's graph*, for which S is the set of all vertices and $A_i \subseteq S \times S$ is the set of all arcs. It is assumed that (S, A_i) is a directed graph with no loop or multiple arcs, that is, $(s, s) \notin A_i$ for each $s \in S$, and the number of arcs between two distinct vertices are one at most. For $s, t \in S$, $(s, t) \in A_i$ means that DM i can shift the state of the conflict from state s to state t. For $i \in N$ and $s \in S$, *DM i's reachable list* from state s is defined as the set $\{t \in S \mid (s, t) \in A_i\}$, denoted by $R_i(s)$. $\succeq_i$ is the *preference* of DM $i \in N$ on S.

For $i \in N$ and $s, s' \in S$, $s \succeq_i s'$ means that s is more or equally preferred to s' by DM i. $s \sim_i s'$ means that $s \succeq_i s'$ and $s' \succeq_i s$, that is, s is equally preferred to s' by DM i. $s \succ_i s'$ means that $s \succeq_i s'$ and "not $(s' \succeq_i s)$, " that is, s is strictly more preferred to s' by DM i.

DM i's preferences $\succeq_i$ is said to be anti-symmetric, if and only if for all $s, s' \in S$, if $s \succeq_i s'$ and $s' \succeq_i s$ then $s = s'$. $\succeq_i$ is said to be complete, if and only if for all $s, s' \in S$, $s \succeq_i s'$ or $s' \succeq_i s$.

B. Stability Concepts within GMCR

Given a graph model $(N, S, (A_i)_{i\in N}, (\succeq_i)_{i\in N})$ of a conflict, one can define Nash stability [26], [27], general metarationality [9], symmetric metarationality [9], and sequential stability [6], [7] as follows:

For coalition $H \subseteq N$ and $s \in S$, the *reachable list of coalition H* from state s is defined inductively, under the restriction in which a DM can only move once at a time, as the set $R_H(s)$ that satisfies the next two conditions: (i) if $i \in H$ and $s' \in R_i(s)$, then $s' \in R_H(s)$, and (ii) if $i \in H$ and

$s' \in R_H(s)$ and $s'' \in R_i(s')$, then $s'' \in R_H(s)$. A *unilateral improvement* s' *of DM* i from state s is defined as a state that is in DM i's reachable list from s (that is, $s' \in R_i(s)$), and DM i strictly prefers state s' to state s (that is, $s' \succ_i s$). Accordingly, the set of all unilateral improvements of DM i from state s is the set $\{x \in R_i(s) \mid x \succ_i s\} \subseteq R_i(s)$, called *DM i's unilateral improvement list* from state s, and is denoted by $R_i^+(s)$. For coalition $H \subseteq N$ and $s \in S$, the *improvement list of coalition* H from state s is defined inductively, under the restriction in which a DM can only move once at a time, as the set $R_H^+(s)$ that satisfies the next two conditions: (i) if $i \in H$ and $s' \in R_i^+(s)$, then $s' \in R_H^+(s)$, and (ii) if $i \in H$ and $s' \in R_H^+(s)$ and $s'' \in R_i^+(s')$, then $s'' \in R_H^+(s)$. $\phi_i^{\simeq}(s)$ denotes the set of all states that are equally or less preferred to state s by DM i, that is, $\{x \in S \mid s \succeq_i x\}$.

Definition 1 (Nash Stability): For $i \in N$, state $s \in S$ is *Nash stable* for DM i, denoted by $s \in S_i^{\mathrm{Nash}}$, if and only if $R_i^+(s) = \emptyset$.

Definition 2 (General Metarationality): For $i \in N$, state $s \in S$ is *generally metarational* for DM i, denoted by $s \in S_i^{\mathrm{GMR}}$, if and only if for all $s' \in R_i^+(s)$, $R_{N\setminus\{i\}}(s') \cap \phi_i^{\simeq}(s) \neq \emptyset$.

Definition 3 (Symmetric Metarationality): For $i \in N$, state $s \in S$ is *symmetrically metarational* for DM i, denoted by $s \in S_i^{\mathrm{SMR}}$, if and only if for all $s' \in R_i^+(s)$, there exists $s'' \in R_{N\setminus\{i\}}(s') \cap \phi_i^{\simeq}(s)$ such that $s''' \in \phi_i^{\simeq}(s)$ for all $s''' \in R_i(s'')$.

Definition 4 (Sequential Stability): For $i \in N$, state s is *sequentially stable* for DM i, denoted by $s \in S_i^{\mathrm{SEQ}}$, if and only if for all $s' \in R_i^+(s)$, $R_{N\setminus\{i\}}^+(s') \cap \phi_i^{\simeq}(s) \neq \emptyset$.

C. Attitude-Based Stability Concepts

In this section, a framework incorporating the notion of attitudes into the GMCR is presented based on [19], [22].

1) Attitude-Based Preferences:

Definition 5 (Attitudes): For $i \in N$, the attitude of DM i is $e_i = (e_{ij})_{j\in N}$, where $e_{ij} \in \{+, 0, -\}$ for $j \in N$. e_{ij} is called the attitude of DM i to DM j.

Note that given a list $e = (e_i)_{i\in N}$ of attitudes e_i of DM i for each $i \in N$, one can specify such a valued, directed, and complete graph with loops as in Figures 1, 2, 3, and 4.

A list $e = (e_i)_{i\in N}$ of attitudes e_i of DM i for each $i \in N$ is said to be *totally positive*, if and only if $e_{ij} = +$ for all $i, j \in N$ (see Figure 1). Similarly, $e = (e_i)_{i\in N}$ is said to be *totally negative*, if and only if $e_{ij} = -$ for all $i, j \in N$ (see Figure 2). $e = (e_i)_{i\in N}$ is said to be *discrete*, if and only if $e_{ii} = +$ for all $i \in N$ and $e_{ij} = 0$ for all $i, j \in N$ such that $i \neq j$ (see Figure 3), and is said to be *totally neutral*, if and only if $e_{ij} = 0$ for all $i, j \in N$ (see Figure 4).

Definition 6 (Devoting Preference (DP) on S): The devoting preference of DM $i \in N$ to DM $j \in N$, denoted by $\mathbf{DP}_{ij}$, is defined as for $s, s' \in S$, $s\mathbf{DP}_{ij}s'$ if and only if $s \succeq_j s'$.

Definition 7 (Aggressive Preference (AP) on S): The aggressive preference of DM $i \in N$ to DM $j \in N$, denoted by $\mathbf{AP}_{ij}$, is defined as for $s, s' \in S$, $s\mathbf{AP}_{ij}s'$ if and only if $s' \succeq_j s$.

Definition 8 (Relational Preference (RP) on S): The relational preference $\mathbf{RP}(e)_{ij}$ of DM $i \in N$ to DM $j \in N$ at e is defined as follows:

$$\mathbf{RP}(e)_{ij} = \begin{cases} \mathbf{DP}_{ij} & \textit{if} \quad e_{ij} = + \\ \mathbf{AP}_{ij} & \textit{if} \quad e_{ij} = - \\ \mathbf{1} & \textit{if} \quad e_{ij} = 0, \end{cases}$$

where $\mathbf{1}$ denotes the ordering on S which is defined as for all $s, s' \in S$, $s\mathbf{1}s'$.

The definition of $\mathbf{RP}$ reflects the assumptions on the relationships between attitudes and DMs' behavior, which are shown in Table I.

Definition 9 (Totally Relational Preference (TRP) on S): The totally relational preference of DM $i \in N$ at e, denoted by $\mathbf{TRP}(e)_i$, is defined as for $s, s' \in S$, $s\mathbf{TRP}(e)_i s'$ if and only if $s\mathbf{RP}(e)_{ij}s'$ for all $j \in N$.

The totally relational preference of DM i at e is a relation on S which is consistent with the list $(\mathbf{RP}(e)_{ij})_{j\in N}$ of relational preferences $\mathbf{RP}(e)_{ij}$ of DM i to DM j at e for all $j \in N$.

2) Attitude-Based Replies and Stability Concepts:

Definition 10 (Totally Relational Reply (TRR) List): The totally relational reply list of DM $i \in N$ at e from $s \in S$ is defined as the set $\{x \in R_i(s) \cup \{s\} \mid x\mathbf{TRP}(e)_i s\}$, denoted by $\mathbf{TRR}(e)_i(s)$.

The totally relational reply list $\mathbf{TRR}(e)_i(s)$ of DM i at e from s in the attitude analysis within GMCR serves as the irreflexive reachable list $R_i(s)$ of the DM i from s in the standard analysis within GMCR.

Definition 11 (Totally Relational Reply List of Coalition): The totally relational reply list of coalition $H \subseteq N$ at e from $s \in S$ is defined inductively, under the restriction in which a DM can only move once at a time, as the set $\mathbf{TRR}(e)_H(s)$ that satisfies the next two conditions: (i) if $i \in H$ and $s' \in \mathbf{TRR}(e)_i(s)$, then $s' \in \mathbf{TRR}(e)_H(s)$, and (ii) if $i \in H$ and $s' \in \mathbf{TRR}(e)_H(s)$ and $s'' \in \mathbf{TRR}(e)_i(s')$, then $s'' \in \mathbf{TRR}(e)_H(s)$.

Definition 12 ($\mathbf{R}\phi^{\simeq}(e)_i(s)$): For $i \in N$ and $s \in S$, $\mathbf{R}\phi^{\simeq}(e)_i(s)$ is defined $\{x \in S \mid x = s \ \text{ or } \ \neg(x\mathbf{TRP}(e)_i s)\}$, where $\neg$ denotes "not."

$\mathbf{R}\phi^{\simeq}(e)_i(s)$ is the set of all states which are *not* preferred to s by DM i at e with respect to the totally relational preference $\mathbf{TRP}(e)_i$ of DM $i \in N$ at e. This serves in the attitude analysis within GMCR as $\phi_i^{\simeq}(s)$ in the standard analysis within GMCR.

Employing the foregoing definitions, attitude-based stability concepts can be defined as an extension of standard stability concepts.

Definition 13 (Relational Nash Stability): For $i \in N$, state $s \in S$ is relational Nash stable at e for DM i, denoted by $s \in S_i^{\mathrm{RNash(e)}}$, if and only if $\mathbf{TRR}(e)_i(s) = \{s\}$.

Definition 14 (Relational General Metarationality): For $i \in N$, state $s \in S$ is relational general metarational at e for DM i, denoted by $s \in S_i^{\mathrm{RGMR(e)}}$, if and only if for all $s' \in \mathbf{TRR}(e)_i(s)\setminus\{s\}$, $R_{N\setminus\{i\}}(s') \cap \mathbf{R}\phi^{\simeq}(e)_i(s) \neq \emptyset$.

Definition 15 (Relational Symmetric Metarationality): For $i \in N$, state $s \in S$ is relational symmetric metarational at

e for DM i, denoted by $s \in S_i^{\mathrm{RSMR(e)}}$, if and only if for all $s' \in \mathbf{TRR}(e)_i(s)\backslash\{s\}$, there exists $s'' \in R_{N\backslash\{i\}}(s') \cap \mathbf{R}\phi^{\simeq}(e)_i(s)$ such that $s''' \in \mathbf{R}\phi^{\simeq}(e)_i(s)$ for all $s''' \in R_i(s'')$.

Definition 16 (Relational Sequential Stability): For $i \in N$, state $s \in S$ is relational sequential stable at e for DM i, denoted by $s \in S_i^{\mathrm{RSEQ(e)}}$, if and only if for all $s' \in \mathbf{TRR}(e)_i(s)\backslash\{s\}$, $(\mathbf{TRR}(e)_{N\backslash\{i\}}(s') \setminus \{s'\}) \cap \mathbf{R}\phi^{\simeq}(e)_i(s) \neq \emptyset$.

III. Interrelationships among Stability Concepts

Consider a graph model $(N, S, (A_i)_{i\in N}, (\succeq_i)_{i\in N})$ of a conflict and a list $e = (e_i)_{i\in N}$ of attitudes e_i of DM i for $i \in N$.

A. Discrete Cases

1) RNash and Nash:

Lemma 1: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete. Then, for all $i \in N$, $\mathbf{TRP}(e)_i = \succeq_i$, that is, for all $s, s' \in S$, $s\mathbf{TRP}(e)_i s'$ if and only if $s \succeq_i s'$.

Proof: By Definition 9, we have that $s\mathbf{TRP}(e)_i s'$ if and only if $s\mathbf{RP}(e)_{ij}s'$ for all $j \in N$. That is, $s\mathbf{RP}(e)_{ii}s'$ and $s\mathbf{RP}(e)_{ij}s'$ for all $j \in N$ such that $j \neq i$. $s\mathbf{RP}(e)_{ii}s'$ is equivalent to $s\mathbf{DP}_{ii}s'$ by Definition 8 and the assumption that $e_{ii} = +$, and this is equivalent to $s \succeq_i s'$ by Definition 6. We have, moreover, that $s\mathbf{RP}(e)_{ij}s'$ for all $j \in N$ such that $j \neq i$ if and only if $s\mathbf{1}s'$, which means that $s\mathbf{RP}(e)_{ij}s'$ for all $j \in N$ such that $j \neq i$ is always true by Definition 8 and the assumption that $e_{ij} = 0$ if $j \neq i$. Thus, we have that for all $i \in N$, $s\mathbf{TRP}(e)_i s'$ if and only if $s \succeq_i s'$.

Corollary 1 (Corollary of Lemma 1): Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete. Then, $\mathbf{TRR}(e)_i(s) = R_i^+(s) \cup \{x \in R_i(s) \mid x \sim_i s\} \cup \{s\}$.

Corollary 2 (Corollary of Lemma 1): Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete and DM i's preference $\succeq_i$ is anti-symmetric for $i \in N$. Then, $\mathbf{TRR}(e)_i(s) = R_i^+(s) \cup \{s\}$.

Corollary 3 (Corollary of Corollary 1): Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete. Then, $R_i^+(s) \subseteq \mathbf{TRR}(e)_i(s)\backslash\{s\}$.

Corollary 4 (Corollary of Corollary 2): Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete and DM i's preference $\succeq_i$ is anti-symmetric for $i \in N$. Then, $\mathbf{TRR}(e)_i(s)\backslash\{s\} = R_i^+(s)$.

Proposition 1: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete. Then, for $i \in N$, $S_i^{\mathrm{RNash(e)}} \subseteq S_i^{\mathrm{Nash}}$.

Proof: Let $s \in S_i^{\mathrm{RNash(e)}}$. Then, by Definition 13, we have that $\mathbf{TRR}(e)_i(s) = \{s\}$. We also have, by Corollary 1, that $\mathbf{TRR}(e)_i(s) = R_i^+(s) \cup \{x \in R_i(s) \mid x \sim s\} \cup \{s\}$. Since $s \notin R_i^+(s)$, these imply that $R_i^+(s) = \emptyset$, which implies that $s \in S_i^{\mathrm{Nash}}$ by Definition 1.

Proposition 2: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete and DM i's preference $\succeq_i$ is anti-symmetric for $i \in N$. Then, for $i \in N$, $S_i^{\mathrm{Nash}} \subseteq S_i^{\mathrm{RNash(e)}}$.

Proof: If $s \in S_i^{\mathrm{Nash}}$, then we have that $R_i^+(s) = \emptyset$. By Corollary 2, moreover, that $\mathbf{TRR}(e)_i(s) = R_i^+(s) \cup \{s\}$. These imply that $\mathbf{TRR}(e)_i(s) = \{s\}$, which means that $s \in S_i^{\mathrm{RNash(e)}}$ by Definition 13.

Corollary 5 (Corollary of Propositions 1 and 2): Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete and DM i's preference $\succeq_i$ is anti-symmetric. Then, $S_i^{\mathrm{RNash(e)}} = S_i^{\mathrm{Nash}}$.

2) RGMR and GMR:

Lemma 2: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete and DM i's preference $\succeq_i$ is complete for $i \in N$. Then, $\mathbf{R}\phi^{\simeq}(e)_i(s) \subseteq \phi_i^{\simeq}(s)$ for all $s \in S$.

Proof: $s' \in \mathbf{R}\phi^{\simeq}(e)_i(s)$ implies that $s' = s$ or $\neg(s'\mathbf{TRP}(e)_i s)$. By Lemma 1, this is equivalent to $s' = s$ or $\neg(s' \succeq_i s)$. By the completeness of $\succeq_i$, this implies that $s' = s$ or $s \succ_i s'$, which is followed by $s \succeq_i s'$. Thus, $s' \in \{x \in S \mid s \succeq_i x\} = \phi_i^{\simeq}(s)$.

Lemma 3: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete and DM i's preference $\succeq_i$ is anti-symmetric for $i \in N$. Then, $\phi_i^{\simeq}(s) \subseteq \mathbf{R}\phi^{\simeq}(e)_i(s)$ for all $s \in S$.

Proof: $\phi_i^{\simeq}(s)$ is defined as the set $\{x \in S \mid s \succeq_i x\} = \{x \in S \mid s \sim_i x\} \cup \{x \in S \mid s \succ_i x\}$, and anti-symmetry of $\succeq_i$ implies that $\{x \in S \mid s \sim_i x\} = \{s\}$. Therefore, $s' \in \phi_i^{\simeq}(s)$ implies that $s' = s$ or $s \succ_i s'$, which means that $s' = s$ or $\neg(s' \succeq_i s)$. This implies, by Lemma 1, that $s' = s$ or $\neg(s'\mathbf{TRP}(e)_i s)$, which is equivalent to $s' \in \mathbf{R}\phi^{\simeq}(e)_i(s)$.

Corollary 6 (Corollary of Lemmas 2 and 3): Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete and DM i's preference $\succeq_i$ is complete and anti-symmetric for $i \in N$. Then, $\mathbf{R}\phi^{\simeq}(e)_i(s) = \phi_i^{\simeq}(s)$ for all $s \in S$.

Proposition 3: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete and DM i's preference $\succeq_i$ is complete for $i \in N$. Then, $S_i^{\mathrm{RGMR(e)}} \subseteq S_i^{\mathrm{GMR}}$.

Proof: Let $s \in S_i^{\mathrm{RGMR(e)}}$. Then, by Definition 14, we have that for all $s' \in \mathbf{TRR}(e)_i(s)\backslash\{s\}$, $R_{N\backslash\{i\}}(s') \cap \mathbf{R}\phi^{\simeq}(e)_i(s) \neq \emptyset$. By Corollary 3, we have that $R_i^+(s) \subseteq \mathbf{TRR}(e)_i(s)\backslash\{s\}$. Moreover, by Lemma 2, we see that $\mathbf{R}\phi^{\simeq}(e)_i(s) \subseteq \phi_i^{\simeq}(s)$, which implies that $R_{N\backslash\{i\}}(s') \cap \mathbf{R}\phi^{\simeq}(e)_i(s) \subseteq R_{N\backslash\{i\}}(s') \cap \phi_i^{\simeq}(s)$. Thus, we have that for all $s' \in R_i^+(s)$, $R_{N\backslash\{i\}}(s') \cap \phi_i^{\simeq}(s) \neq \emptyset$, which means that $s \in S_i^{\mathrm{GMR}}$.

Proposition 4: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete and DM i's preference $\succeq_i$ is anti-symmetric for $i \in N$. Then, $S_i^{\mathrm{GMR}} \subseteq S_i^{\mathrm{RGMR(e)}}$.

Proof: Let $s \in S_i^{\mathrm{GMR}}$. If $s' \in \mathbf{TRR}(e)_i(s)\backslash\{s\}$, then $s' \in S_i^+(s)$ by Corollary 4. Then, we have that $R_{N\backslash\{i\}}(s') \cap \phi_i^{\simeq}(s) \neq \emptyset$ by $s \in S_i^{\mathrm{GMR}}$, and that $\phi_i^{\simeq}(s) \subseteq \mathbf{R}\phi^{\simeq}(e)_i(s)$ by Lemma 3. Therefore, $R_{N\backslash\{i\}}(s') \cap \mathbf{R}\phi^{\simeq}(e)_i(s) \neq \emptyset$, which implies that $s \in S_i^{\mathrm{RGMR(e)}}$.

Corollary 7 (Corollary of Propositions 3 and 4): Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete and DM i's preference $\succeq_i$ is complete and anti-symmetric. Then, $S_i^{\mathrm{RGMR(e)}} = S_i^{\mathrm{GMR}}$.

3) RSMR and SMR:

Proposition 5: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete and DM i's preference $\succeq_i$ is complete for $i \in N$. Then, $R_i^{\mathrm{RSMR(e)}} \subseteq R_i^{\mathrm{SMR}}$.

Proof: Let $s \in S_i^{\mathrm{RSMR(e)}}$. Then, by Definition 15, we have that for all $s' \in \mathbf{TRR}(e)_i(s)\backslash\{s\}$, there exists $s'' \in R_{N\backslash\{i\}}(s') \cap \mathbf{R}\phi^{\simeq}(e)_i(s)$ such that $s''' \in \mathbf{R}\phi^{\simeq}(e)_i(s)$

for all $s''' \in R_i(s'')$. This is equivalent to for all $s' \in \mathbf{TRR}(e)_i(s)\backslash\{s\}$, there exists $s'' \in R_{N\backslash\{i\}}(s') \cap \mathbf{R}\phi^{\simeq}(e)_i(s)$ such that $R_i(s'') \subseteq \mathbf{R}\phi^{\simeq}(e)_i(s)$.

By Corollary 1 and $s \notin R_i^+(s)$, we have that $R_i^+(s) \subseteq \mathbf{TRR}(e)_i(s)\backslash\{s\}$. Thus, we have that for all $s' \in R_i^+(s)$, there exists $s'' \in R_{N\backslash\{i\}}(s') \cap \mathbf{R}\phi^{\simeq}(e)_i(s)$ such that $R_i(s'') \subseteq \mathbf{R}\phi^{\simeq}(e)_i(s)$. By Lemma 2, we have $\mathbf{R}\phi^{\simeq}(e)_i(s) \subseteq \phi_i^{\simeq}(s)$. So, we have that for all $s' \in R_i^+(s)$, there exists $s'' \in R_{N\backslash\{i\}}(s') \cap \phi_i^{\simeq}(s)$ such that $R_i(s'') \subseteq \mathbf{R}\phi^{\simeq}(e)_i(s)$. Again, by Lemma 2, we have that $\mathbf{R}\phi^{\simeq}(e)_i(s) \subseteq \phi_i^{\simeq}(s)$. This implies that for all $s' \in R_i^+(s)$, there exists $s'' \in R_{N\backslash\{i\}}(s') \cap \phi_i^{\simeq}(s)$ such that $R_i(s'') \subseteq \phi_i^{\simeq}(s)$, which means $s \in S_i^{\mathrm{SMR}}$.

Proposition 6: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete and DM i's preference $\succeq_i$ is anti-symmetric for $i \in N$. Then, $S_i^{\mathrm{SMR}} \subseteq S_i^{\mathrm{RSMR(e)}}$.

Proof: Let $s \in S_i^{\mathrm{SMR}}$. Then, for all $s' \in R_i^+(s)$, there exists $s'' \in R_{N\backslash\{i\}}(s') \cap \phi_i^{\simeq}(s)$ such that $R_i(s'') \subseteq \phi_i^{\simeq}(s)$. Since we have $\phi_i^{\simeq}(s) \subseteq \mathbf{R}\phi^{\simeq}(e)_i(s)$ by Lemma 3, we have that for all $s' \in R_i^+(s)$, there exists $s'' \in R_{N\backslash\{i\}}(s') \cap \mathbf{R}\phi^{\simeq}(e)_i(s)$ such that $R_i(s'') \subseteq \mathbf{R}\phi^{\simeq}(e)_i(s)$. Because $\mathbf{TRR}(e)_i(s)\backslash\{s\} \subseteq R_i^+(s)$ by Corollary 4, we see that for all $s' \in \mathbf{TRR}(e)_i(s)\backslash\{s\}$, there exists $s'' \in R_{N\backslash\{i\}}(s') \cap \mathbf{R}\phi^{\simeq}(e)_i(s)$ such that $R_i(s'') \subseteq \mathbf{R}\phi^{\simeq}(e)_i(s)$, which implies that $s \in S_i^{\mathrm{RSMR(e)}}$.

Corollary 8 (Corollary of Propositions 5 and 6): Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete and DM i's preference $\succeq_i$ is complete and anti-symmetric. Then, $S_i^{\mathrm{RSMR(e)}} = S_i^{\mathrm{SMR}}$.

4) RSEQ and SEQ:

Proposition 7: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete. Also, assume that DM i's preference $\succeq_i$ is complete for $i \in N$ and DMs' preferences $(\succeq_i)_{i\in N}$ are anti-symmetric. Then, $S_i^{\mathrm{RSEQ(e)}} \subseteq S_i^{\mathrm{SEQ}}$.

Proof: Let $s \in S_i^{\mathrm{RSEQ(e)}}$. Then, by Definition 16, we have that for all $s' \in \mathbf{TRR}(e)_i(s)\backslash\{s\}$, $(\mathbf{TRR}(e)_{N\backslash\{i\}}(s') \setminus \{s'\}) \cap \mathbf{R}\phi^{\simeq}(e)_i(s) \neq \emptyset$.

By Corollary 1 and $s \notin R_i^+(s)$, we have that $R_i^+(s) \subseteq \mathbf{TRR}(e)_i(s)\backslash\{s\}$. Thus, we have that for all $s' \in R_i^+(s)$, $(\mathbf{TRR}(e)_{N\backslash\{i\}}(s') \setminus \{s'\}) \cap \mathbf{R}\phi^{\simeq}(e)_i(s) \neq \emptyset$. By Lemma 2, we have $\mathbf{R}\phi^{\simeq}(e)_i(s) \subseteq \phi_i^{\simeq}(s)$. So, we have that for all $s' \in R_i^+(s)$, $(\mathbf{TRR}(e)_{N\backslash\{i\}}(s') \setminus \{s'\}) \cap \phi_i^{\simeq}(s) \neq \emptyset$. Moreover, since we have, by Corollary 4, that $\mathbf{TRR}(e)_j(s)\backslash\{s\} \subseteq R_j^+(s)$ for all $j \in N\backslash\{i\}$, we have that for all $s' \in R_i^+(s)$, $R^+_{N\backslash\{i\}}(s') \cap \phi_i^{\simeq}(s) \neq \emptyset$, which means that $s \in S_i^{\mathrm{SEQ}}$.

Proposition 8: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete. Also, assume that DMs' preferences $(\succeq_i)_{i\in N}$ are anti-symmetric. Then, $S_i^{\mathrm{SEQ(e)}} \subseteq S_i^{\mathrm{RSEQ}}$.

Proof: Let $s \in S_i^{\mathrm{SEQ}}$. Then, by Definition 4, we have that for all $s' \in R_i^+(s)$, $R^+_{N\backslash\{i\}}(s') \cap \phi_i^{\simeq}(s) \neq \emptyset$. Since we have, by Corollary 4, that $\mathbf{TRR}(e)_i(s)\backslash\{s\} = R_i^+(s)$, we have that for all $s' \in \mathbf{TRR}(e)_i(s)\backslash\{s\}$, $R^+_{N\backslash\{i\}}(s') \cap \phi_i^{\simeq}(s) \neq \emptyset$. We have, moreover, that $\phi_i^{\simeq}(s) \subseteq \mathbf{R}\phi^{\simeq}(e)_i(s)$ by Lemma 3, so it is satisfied that for all $s' \in \mathbf{TRR}(e)_i(s)\backslash\{s\}$, $R^+_{N\backslash\{i\}}(s') \cap \mathbf{R}\phi^{\simeq}(e)_i(s) \neq \emptyset$. Additionally, because we have, by Corollary 4, that $\mathbf{TRR}(e)_i(s)\backslash\{s\} = R_i^+(s)$, it is implied that for all $s' \in \mathbf{TRR}(e)_i(s)\backslash\{s\}$, $(\mathbf{TRR}(e)_{N\backslash\{i\}}(s') \setminus \{s'\}) \cap \mathbf{R}\phi^{\simeq}(e)_i(s) \neq \emptyset$, which means that $s \in S_i^{\mathrm{SEQ}}$.

Corollary 9 (Corollary of Propositions 7 and 8): Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are discrete and DM i's preference $\succeq_i$ is complete and DMs' preferences $(\succeq_i)_{i\in N}$ are anti-symmetric. Then, $S_i^{\mathrm{RSEQ(e)}} = S_i^{\mathrm{SEQ}}$.

B. Totally Neutral Cases

Lemma 4: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are totally neutral. Then, $\mathbf{TRR}(e)_i(s) = R_i(s)\cup\{s\}$ for all $i \in N$ and $s \in S$.

Proof: By Definition 10, $\mathbf{TRR}(e)_i(s) = \{x \in R_i(s) \cup \{s\} \mid x\mathbf{TRP}(e)_i s\}$. Since e is totally neutral, then we have, by Definition 8 and 9, that for all $s, s' \in S$, $s'\mathbf{TRP}(e)_i s$, this implies $\mathbf{TRR}(e)_i(s) = R_i(s) \cup \{s\}$.

Proposition 9: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are totally neutral. Then, for all $i \in N$ and all $s \in S$, we have that $s \in S_i^{\mathrm{RNash(e)}}$ if and only if $R_i(s) = \emptyset$.

Proof: By Definition 13, we have that $s \in S_i^{\mathrm{RNash(e)}}$ if and only if $\mathbf{TRR}(e)_i(s) = \{s\}$, which is equivalent to $R_i(s)\cup\{s\} = \{s\}$ by Lemma 4. Since $s \notin R_i(s)$, this is equivalent to $R_i(s) = \emptyset$.

Lemma 5: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are totally neutral. Then, $\mathbf{R}\phi^{\simeq}(e)_i(s) = \{s\}$ for all $i \in N$ and $s \in S$.

Proof: Let $s' \in \mathbf{R}\phi^{\simeq}(e)_i(s)$. Then, by Definition 12, we have that $s' \in \{x \in S \mid x = s \text{ or } \neg(x\mathbf{TRP}(e)_i s)\}$. Since e is totally neutral, then we have, by Definition 8 and 9, that for all $s, s' \in S$, $s'\mathbf{TRP}(e)_i s$, this implies that $s' \in \{x \in S \mid x = s\}$, which means $s' = s$.

Proposition 10: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are totally neutral. Then, for all $i \in N$ and all $s \in S$, we have that $s \in S_i^{\mathrm{RGMR(e)}}$ if and only if for all $s' \in R_i(s)$, $s \in R_{N\backslash\{i\}}(s')$.

Proof: By Definition 14, we have that $s \in S_i^{\mathrm{RGMR(e)}}$ if and only if for all $s' \in \mathbf{TRR}(e)_i(s)\backslash\{s\}$, $R_{N\backslash\{i\}}(s') \cap \mathbf{R}\phi^{\simeq}(e)_i(s) \neq \emptyset$. By Lemmas 4 and 5, we have that $\mathbf{TRR}(e)_i(s) = R_i(s) \cup \{s\}$ and $\mathbf{R}\phi^{\simeq}(e)_i(s) = \{s\}$, respectively. Therefore, we have that for all $s' \in R_i(s)$, $R_{N\backslash\{i\}}(s') \cap \{s\} \neq \emptyset$, which is equivalent to for all $s' \in R_i(s)$, $s \in R_{N\backslash\{i\}}(s')$.

Proposition 11: Assume that DMs' attitudes $e = (e_i)_{i\in N}$ are totally neutral. Then, for all $i \in N$ and all $s \in R$, we have that $s \in S_i^{\mathrm{RSMR(e)}}$ if and only if $R_i(s) = \emptyset$.

Proof: By Definition 15, we have that $s \in S_i^{\mathrm{RSMR(e)}}$ if and only if for all $s' \in \mathbf{TRR}(e)_i(s)\backslash\{s\}$, there exists $s'' \in R_{N\backslash\{i\}}(s') \cap \mathbf{R}\phi^{\simeq}(e)_i(s)$ such that $s''' \in \mathbf{R}\phi^{\simeq}(e)_i(s)$ for all $s''' \in R_i(s'')$. Since we have that $\mathbf{TRR}(e)_i(s) = R_i(s) \cup \{s\}$ and $\mathbf{R}\phi^{\simeq}(e)_i(s) = \{s\}$ by Lemmas 4 and 5, respectively, we equivalently have for all $s' \in R_i(s)$, there exists $s'' \in R_{N\backslash\{i\}}(s') \cap \{s\}$ such that $R_i(s'') \subseteq \{s\}$.

If $R_i(s) = \emptyset$, then $s \in S_i^{\mathrm{RSMR(e)}}$ is logically true. Consider the case in which $R_i(s) \neq \emptyset$ and assume that $s' \in R_i(s)$. In this situation, we have that there exists $s'' \in R_{N\backslash\{i\}}(s') \cap \{s\}$ such that $R_i(s'') \subseteq \{s\}$. $s'' \in R_{N\backslash\{i\}}(s') \cap \{s\}$ implies that $s'' = s$, and thus, we have that $R_i(s) \subseteq \{s\}$, but since $s \notin R_i(s)$, this means $R_i(s) = \emptyset$, which is a contradiction. Thus, we have that $s \in S_i^{\mathrm{RSMR(e)}}$ if and only if $R_i(s) = \emptyset$.

Proposition 12: Assume that DMs' attitudes $e = (e_i)_{i \in N}$ are totally neutral. Then, for all $i \in N$ and all $s \in R$, we have that $s \in S_i^{\mathrm{RSEQ(e)}}$ if and only if for all $s' \in R_i(s)$, $s \in R_{N \setminus \{i\}}(s')$.

Proof: By Definition 16, we have that $s \in S_i^{\mathrm{RSEQ(e)}}$ if and only if for all $s' \in \mathbf{TRR}(e)_i(s) \setminus \{s\}$, $(\mathbf{TRR}(e)_{N \setminus \{i\}}(s') \setminus \{s'\}) \cap \mathbf{R}\phi^{\simeq}(e)_i(s) \neq \emptyset$. By Lemmas 4 and 5, we have that $\mathbf{TRR}(e)_i(s) = R_i(s) \cup \{s\}$ and $\mathbf{R}\phi^{\simeq}(e)_i(s) = \{s\}$, respectively. Thus, we equivalently have that for all $s' \in R_i(s)$, $(\mathbf{TRR}(e)_{N \setminus \{i\}}(s') \setminus \{s'\}) \cap \{s\} \neq \emptyset$. Moreover, since we have, by Lemma 4, that $\mathbf{TRR}(e)_{N \setminus \{i\}}(s') \setminus \{s'\}) = R_{N \setminus \{i\}}(s')$, this is equivalent to for all $s' \in R_i(s)$, $R_{N \setminus \{i\}}(s') \cap \{s\} \neq \emptyset$, which means that for all $s' \in R_i(s)$, $s \in R_{N \setminus \{i\}}(s')$, because $s \notin R_{N \setminus \{i\}}(s')$.

Corollary 10 (Corollary of Propositions 9 and 11): Assume that DMs' attitudes $e = (e_i)_{i \in N}$ are totally neutral. Then, $S_i^{\mathrm{RNash}} = S_i^{\mathrm{RSMR}}$ for all $i \in N$.

Corollary 11 (Corollary of Propositions 10 and 12): Assume that DMs' attitudes $e = (e_i)_{i \in N}$ are totally neutral. Then, $S_i^{\mathrm{RGMR}} = S_i^{\mathrm{RSEQ}}$ for all $i \in N$.

IV. CONCLUSIONS

This paper verified some inclusion relationships among attitude-based and conventional stabilty concepts. One of the topics to be considered in future research on attitude-based stability conceps is their interrelationships with coalition stability concepts [17], [18].

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