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Frontier assignment method for sensitivity analysis of Data Envelopment Analysis

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***Abstract*—To extend the sensitivity analysis for DEA (Data Envelopment Analysis), this paper proposes FA-DEA (Frontier Assignment based DEA). The basic idea of FA-DEA is to allow decision maker to decide frontier intentionally while the fundamental DEA and super-DEA decides frontier computationally. Then FA-DEA provides chances to exclude extra-influential DMU (Decision Making Unit) and finds extra-ordinal DMU. Therefore, FA-DEA has the opportunities to find valuable information from a variety of viewpoints. Simple numerical study has shown the effectiveness of FA-DEA as a data mining tool and the difference from the traditional sensitivity analysis.**

***Keywords*—Data Envelopment Analysis, Sensitivity Analysis, Decision Making Support, Risk Finding, Data Mining, Linear Programming**

I. INTRODUCTION

DEA (Data Envelopment Analysis) is well known for evaluating the performance of DMUs (Decision Making Units). DEA assumes management activity by input and output concerning DMU. Given the inputs and outputs data of all DMUs, the performance of each DMU is evaluated by efficiency score and classified as state of efficient or inefficient. DEA calculates efficiency score of DMUs based on Pareto optimal line which is called efficiency frontier consists of efficient DMUs.

DEA sensitivity analysis which analyzes structure of remarkable DMU in detail is one of the ways to enhance practicability of DEA [1]. This traditional DEA sensitivity analysis makes calculation by eliminating an efficient DMU. Then variation of efficiency score in each DMU indicates how influential the excluded DMU is. As a result, we have chances to find influential power of the efficient DMU and reclassify efficient DMUs.

However, the traditional method has limitations. It is able to eliminate only one efficient DMU at once so that influential power of only one efficient DMU is shown. Here, let us show the possible and valuable cases in the sensitivity analysis. Each DMU has a little influential power but their multiple powers is much. The traditional method can not be applied to such a case. We may find interesting knowledge from the case. That is to say, this study can expand possibility of utilizing DEA not only for data mining and but also for multi-viewpoint analysis.

Based on this expectation, this study proposes FA-DEA (Frontier Assignment based DEA) model which treats influential power of multiple DMUs and shows effectiveness of the proposed method by numerical experiments.

II. OUTLINE OF DEA AND TRADITIONAL SENSITIVITY ANALYSIS

A. *DEA as Data mining tool*

DEA was proposed by A. Charnes et al. in 1978 as a method for management analysis [2]. It has been applied in various fields [3, 4, 5, 6]. DEA regards DMU which has smaller inputs and larger outputs as more efficient. Each DMU is evaluated relatively based on this idea. Then the efficiency score of efficient DMU is equal to one and those of inefficient DMU is less than one. DEA shows management efficiency of each DMU by calculating efficiency score. And it is valuable for a decision maker to understand and improve the situation they are facing. DEA helps us to notice the relationship and knowledge among the data [7, 8].

Efficiency score θ is calculated by dividing output by input. If there are multi-input and multi-output, DEA puts the weight to each element in order to make virtual input and output. Then DEA allows advantages of each DMU to be evaluated as much as possible by assigning not fixed weight but variable weight. Characteristics of DEA are as follows:

- Applicable for multi-input and multi-output,
- Evaluate advantageous aspect for each DMU,
- Show the performance improvement for inefficient DMUs.

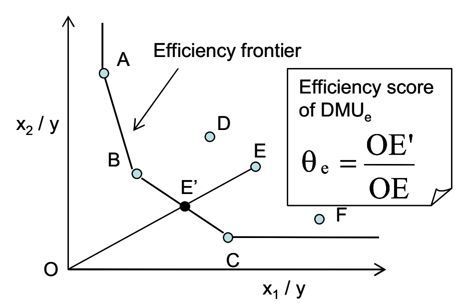

Figure. 1 Efficiency frontier and score

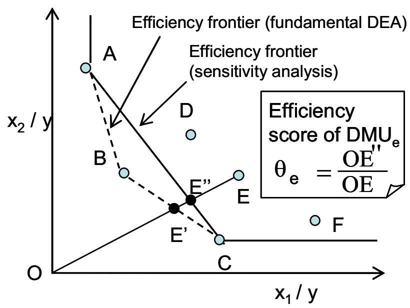

Figure. 2 Sensitivity analysis

Fig. 1 shows an efficiency frontier and an efficiency score. Here, there are six DMUs (A~F) which have 2 inputs and 1 output. In DEA, the DMU which is located near the origin is more efficient since the DMU is operating by smaller input..

There are some models in DEA for calculating efficiency score and frontier where CCR model which assumes "returns to scale" as constant and BCC model which assumes "returns to scale" as variable have been well-known[1].

Assuming that there are n DMUs ($DMU_1, DMU_2, \ldots, DMU_k, \ldots, DMU_n$) and DMU_k is characterized by m inputs ($x_{1k}, x_{2k}, \ldots, x_{mk}$) and s outputs ($y_{1k}, y_{2k}, \ldots, y_{sk}$). Then the efficiency score of DMU_k is calculated by the following linear programming.

$$
\begin{aligned}
&\min \quad \theta_k^{\{J\}} \\
&\text{s.t.} \quad \theta_k^{\{J\}} x_{ik} \geq \sum_{j=1}^{n} x_{ij}\lambda_j \quad (i = 1, 2, \ldots, m) \\
&\qquad y_{rk} \leq \sum_{j=1}^{n} y_{rj}\lambda_j \quad (r = 1, 2, \ldots, s) \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1 \\
&\theta_k^{\{J\}}: \text{free} \quad \lambda_j \geq 0 \quad \{J\} = \{DMU1, DMU2, \ldots, DMUn\}
\end{aligned}
\tag{1}
$$

λ_j is a convex combination and θ indicates efficiency score. $\theta_k^{\{J\}}$ denotes efficiency score of DMU_k in set {J}. If θ is one, the DMU is operating efficiently. On the other hand, the DMU is operating inefficiently in case θ is less than one.

Let us explain how to improve inefficient DMU. Provided that we try to improve DMU_e to E' which is located on the efficiency frontier in Fig. 1, inputs should be reduced by θ times. Practically, linear programming which includes input surplus s_x and output lack s_y is applied.

$$
\begin{aligned}
&\min \quad \theta_k^{\{J\}} \\
&\text{s.t.} \quad \theta_k^{\{J\}} x_{ik} - \sum_{j=1}^{n} x_{ij}\lambda_j - s_{xi} = 0 \quad (i = 1, 2, \ldots, m) \\
&\qquad y_{rk} - \sum_{j=1}^{n} y_{rj}\lambda_j + s_{yr} = 0 \quad (r = 1, 2, \ldots, s) \\
&\qquad \sum_{j=1}^{n} \lambda_j = 1 \\
&\qquad \theta_k^{\{J\}}: \text{free} \quad \lambda_j \geq 0 \quad s_{xi} \geq 0 \quad s_{yr} \geq 0 \\
&\qquad \{J\} = \{DMU1, DMU2, \ldots, DMUn\}
\end{aligned}
\tag{2}
$$

θ , s_x, s_y are calculated in formula (2). Then the performance improvement of input and output are proposed by the formula (3) where x_{new} and y_{new} denote one of the ideal states for inefficient DMU.

$$
\begin{aligned}
x_{new} &= \theta x - s_x \\
y_{new} &= y + s_y
\end{aligned}
\tag{3}
$$

B. Sensitivity analysis in DEA

DEA sensitivity analysis has had generally two ways. One is to change the number of DMUs and other is to change the number of input or output elements. This paper focuses on the former way as we aims for sensitivity analysis to multiple DMUs.

There has been a method, called super-DEA, by eliminating an efficient DMU one by one [9, 10]. Note that this method allows a DMU to have efficiency score over one. We can know how efficient DMU is operating its management activity beyond efficiency frontier due to this approach. Finally, influential power of the efficient DMU is signified.

To illustrate the concept of the super-DEA, let us focus on DMU_b in Fig. 2. The input and output data of efficient DMU_b are eliminated from restriction expression in formula (1). After that, the efficiency score of DMU_b is calculated. The super-DEA toward DMU_b is expressed in the following linear programming:

$$
\begin{aligned}
&\min \quad \theta_b^{\{J-b\}} \\
&\text{s.t.} \quad \theta_b^{\{J-b\}} x_{ib} \geq \sum_{j \in J-b} x_{ij}\lambda_j \quad (i = 1, 2, \ldots, m) \\
&\qquad y_{rb} \leq \sum_{j \in J-b} y_{rj}\lambda_j \quad (r = 1, 2, \ldots, s) \\
&\qquad \sum_{j \in J-b} \lambda_j = 1 \\
&\theta_b^{\{J-b\}}: \text{free} \quad \lambda_j \geq 0 \quad \{J-b\} = \{DMU_a, DMU_c, \ldots, DMU_f\}
\end{aligned}
\tag{4}
$$

As shown in Fig. 2, the efficiency score of DMU_b is more than one since DMU_b is located in more efficient place than efficiency frontier. In addition, the efficiency score of DMU_e is higher than before by changing the form of efficiency frontier. The variation rate of efficiency score is expressed as (OE''-OE') / OE. Thus, it is possible to evaluate the influential power from DMU_b to other DMUs. The Current DEA sensitivity analysis is used to eliminate one DMU. Then the degree of influential power is measured by the efficiency scores of remarkable DMU (super-DEA) and other DMUs.

C. DEA hierarchical approach

DEA hierarchical approach [11] is developed based on DEA sensitivity analysis. This method proposes not only efficiency score but also goal set of efficient DMUs and direction for inefficient DMU. Fig. 3 illustrates an outline of DEA hierarchical approach. Though the performance improvement is shown as A' in the fundamental DEA model, DEA hierarchical approach shows the detailed performance improvement step by step to be an efficient DMU (A'') [12].

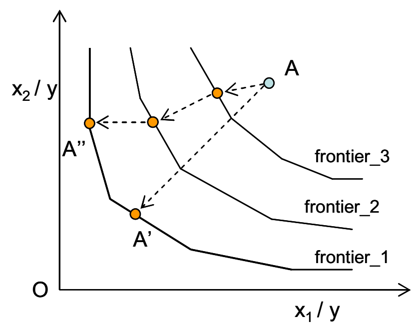

Figure. 3 DEA hierarchical approach [6]

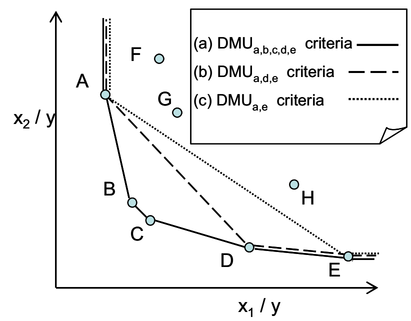

Figure. 4 Outline of FA-DEA

Let us explain how to make a hierarchical frontier. DMU whose efficiency score is one is regarded as first place group as it consists of frontier_1. DMUs of the first place group are excluded before analysis is carried out again. Then DMU whose efficiency score is one is regarded as second place group as it consists of frontier_2. Thus DMUs are hierarchized step by step in order to classify inefficient DMUs.

Moreover, when the decision makers make plan to improve the performance of DMU_a as shown in Fig. 3, they can know the way not to frontier_1 directly but to higher frontier step by step [11, 12]. That's why more beneficial improvement can be done in DEA hierarchical approach since short-term improvement is shown.

D. *Problem in the traditional sensitivity analysis*

The super-DEA eliminates data of one efficient DMU from restriction expression for the sensitivity analysis. It does not allow to eliminate more than one DMU. Therefore, the influential power of multiple DMUs can not be shown nor monitored. For instance, there are DMU_a and DMU_b. Even though independent influential power of DMU_a ($\theta_a^{\{J-a\}}$) and DMU_b ($\theta_b^{\{J-b\}}$) is a little, it is still possible that mutual influential power of DMU_a ($\theta_a^{\{J-(a,b)\}}$) and DMU_b ($\theta_b^{\{J-(a,b)\}}$) is much. It can be an interesting knowledge found from inputs and outputs of DMUs. Therefore, it is necessary to overcome the limitation of the traditional method.

While it is attractive that the fundamental DEA model is able to find an efficiency frontier computationally, sometimes the computational finding may be annoyance if outliers exist. Then supposing we make model which efficiency frontier is set intentionally, it is possible to carry out more flexible sensitivity analysis.

The DEA hierarchical approach can be complemented if these problems are solved. It means the difference of each frontier, namely each group is shown as analyst can set frontier to particular hierarchy which remarkable DMU belongs to. That's why necessary effort for the performance improvement between hierarchies shall be indicated when inefficient DMUs improve their performances.

III. Frontier Assignment based DEA

A. *Outline of proposed method*

The proposed FA-DEA (Frontier Assignment based DEA) is a new model to realize more flexible sensitivity analysis than the traditional methods. Being assigned a frontier intentionally, it measures influential power of multiple DMUs without eliminating given data. The FA-DEA is able to cover the fundamental DEA and super-DEA due to intentional setting of evaluation criteria. Fig. 4 shows the conceptual outline of the proposed method.

The frontier (a) indicates evaluation criteria based on set {J} which consists of all DMUs. This is the same efficiency frontier as CCR or BCC model. The frontier (b) is the criteria in order to measure influential power of $DMU_{b,c}$. The Sensitivity analysis is performed based on set {J-(b,c)}. At this time, the frontier is assigned intentionally. Moreover, provided that we carry out sensitivity analysis toward three DMU, or $DMU_{b,c,d}$, the frontier (c) is set intentionally and possible to measure influential power of $DMU_{b,c,d}$. Then the sensitivity analysis is done based on set {J-(b,c,d)}.

B. *Formulization and new frontier*

Let us describe the formula of FA-DEA. Suppose that there are n DMUs and p DMUs are chosen intentionally. Then DEA sensitivity analysis is carried out based on these chosen DMUs. In other words, the rest of DMUs, namely (n-p) DMUs are remarkable in the sensitivity analysis. (n-p) DMUs which are not chosen are expressed as set {J'} = {DMU_1, DMU_2, …, DMU_{n-p}} and sensitivity analysis is done based on set {J-J'} as follows:

$$
\begin{aligned}
&\min\ \theta_k^{\{J-J'\}} \\
&\text{s.t. } \theta_k^{\{J-J'\}} x_{ik} \ge \sum_{j=1}^{n} x_{ij}\lambda_j + \sum_{l=1}^{p} \alpha_l x_{il} \quad (i=1,2,\ldots,m) \\
&\quad y_{rk} \le \sum_{j=1}^{n} y_{rj}\lambda_j + \sum_{l=1}^{p} \alpha_l y_{rl} \quad (r=1,2,\ldots,s) \\
&\quad \sum_{l=1}^{p} \alpha_l = 1 \\
&\quad \theta_k^{\{J-J'\}}:\text{free} \quad \lambda_j \ge 0 \quad \alpha_l \ge 0
\end{aligned}
\tag{5}
$$

The formula (5) differs from formula (1) which is fundamental DEA linear programming. $\sum_{l=1}^{p}\alpha_l = 1$ is added as a restriction in order to form frontier by chosen DMUs. Let us note the term "frontier" in FA-DEA. While it is just a line connecting efficient DMUs, it sometimes means an evaluation criterion simply. The former example is solid line (a) in Fig. 4, the latter case is broken line (b) and dotted line (c) in Fig. 4. Regarding DMUs on the frontier as efficient, the sensitivity analysis is carried out in broken line (b) or dotted line (c) in Fig. 4.

IV. NUMERICAL EXPERIMENTS

A. Way to experiments

To evaluate the power of FA-DEA, test data were used. The test data include ten DMUs which have two inputs and one output (Table 1).

First, test data were applied to formula (5). Next the combinations of DMUs which mean targets of sensitivity analysis are changed by turn. Then the influential power of targeted DMUs is evaluated based on variation of each efficiency score.

TABLE I. Test Data for Numerical Experiments

DMU		A	B	C	D	E	F	G	H	I	J
Input	x1	4	7	8	4	2	10	3	6	7	9
	x2	3	3	1	2	4	1	7	7	6	5
Output	y	1	1	1	1	1	1	1	1	1	1

Let us denote z' an indicator which shows the influential power of DMUs in experiments. Suppose that there are n DMUs and p DMUs are chosen intentionally. Then FA-DEA is carried out based on set {J-J'}. At this time z' is defined as follow:

$$z'_{\{J'\}} = \sum_{j=1}^{n} \{\theta_j^{\{J-J'\}} - \theta_j^{J}\} \tag{6}$$

B. Result of experiments

Table 2 shows the result of experiments. The left end of Table 2 indicates DMUs set for sensitivity analysis. For example, set {J-c} consists of DMUs except for DMU_c. The number of targeted DMUs is increased from the top to down in Table 2. Note that the result in top comes not from sensitivity analysis but from the fundamental DEA (CCR model). The tracery in Table 2 signifies DMUs which form a frontier in each analysis. The targeted DMUs in FA-DEA have higher efficiency scores than 1 as evaluation criteria consists of DMUs except for targeted DMUs.

C. Discussion

The result is the same as super-DEA in case of sensitivity analysis toward single DMU. It means FA-DEA comprehends the traditional super-DEA sensitivity analysis.

TABLE II. An Experimental Result in FA-DEA

DMU set	Efficiency score of each DMU									Total score	z'
	A	B	C	D	E	F	G	I	J		
{J }	0.857	0.632	1	1	1	1	0.667	0.462	0.462	0.429	7.509
{J - c}	0.857	0.64	1.143	1	1	1	0.667	0.462	0.462	0.429	7.66
{J - d}	1	0.769	1	1.25	1	1	0.667	0.5	0.526	0.526	8.238
{J - e}	0.957	0.632	1	1	1.571	1	1	0.595	0.537	0.44	8.732
{J - f}	0.857	0.632	1	1	1	1	0.667	0.462	0.462	0.429	7.509
{J - (c,d)}	1	0.813	1.182	1.3	1	1	0.667	0.5	0.526	0.542	8.53
{J - (c,e)}	0.957	0.64	1.143	1	1.571	1	1	0.595	0.537	0.44	8.883
{J - (c,f)}	0.857	0.667	2	1	1	2	0.667	0.462	0.462	0.429	9.544
{J - (d,e)}	1	0.769	1	1.25	1.583	1	1	0.613	0.559	0.526	9.3
{J - (d,f)}	1	0.769	1	1.25	1	1	0.667	0.5	0.526	0.526	8.238
{J - (e,f)}	0.957	0.632	1	1	1.571	1	1	0.595	0.537	0.44	8.732
{J - (c,d,e)}	1	0.813	1.182	1.3	1.583	1	1	0.613	0.559	0.542	9.592
{J - (c,d,f)}	1	1	3	1.5	1	3	0.667	0.5	0.526	0.6	12.793
{J - (c,e,f)}	0.957	0.667	2	1	1.571	2	1	0.595	0.537	0.44	10.767
{J - (d,e,f)}	1	0.769	1	1.25	1.583	1	1	0.613	0.559	0.526	9.3

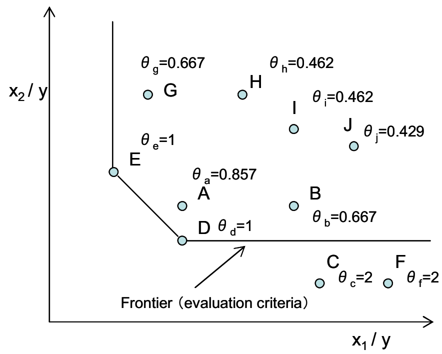

Figure. 5 Sensitivity analysis in DMU set {J－(c, f)}

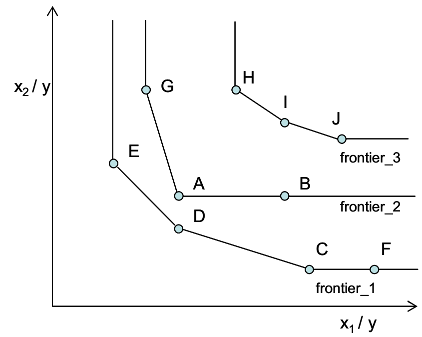

Figure. 6 Hierarchization of test data

TABLE III. Efficiency score in each hierarchy

Hierarchy	DMU	Efficiency score	Total (Average)
1st	C	5.00	15.5 (3.88)
	D	2.50	
	E	3.00	
	F	5.00	
2nd	A	1.90	5.57 (1.86)
	B	1.67	
	G	2.00	
3rd	H	1.00	3.00 (1.00)
	I	1.00	
	J	1.00	

Again, this study focuses on the sensitivity analysis toward multiple DMUs. According to Table 2, the results of sensitivity analysis toward two or three DMUs are shown. Here DMU_c and DMU_f are remarkable. That is because independent influential power z'_c=0.151, z'_f=0 is less than z'_d=0.729, z'_e=1.223, however the result in the sensitivity analysis toward $DMU_{c,f}$ (Fig. 5) is $z'_{c,f}$=2.035. It is a maximum score among the sensitivity analysis toward two DMUs. This means that FA-DEA has power to measure influential power of multiple DMUs. FA-DEA allows us to have new viewpoint for data mining capacity in DEA sensitivity analysis.

Even though independent influential power by a single DMU is a little, it can be much in case multiple DMUs exist together. It proves that FA-DEA realizes more extensive sensitivity analysis. As a result, we can find multiple particular competitors in the same industry or measure variation of self efficiency score in case of industrial reorganization. Then FA-DEA shall work for decision making support enough.

According to Table 2, DMU_a and DMU_g often become the state of efficient (its efficiency score becomes one). It means they are regarded as inefficient DMU at present, however, they have many opportunities to be the state of efficient. It is proven by changing frontier through the numerical experiments. They are evaluated better by comparison with other inefficient DMUs. Therefore, the FA-DEA enables us to measure influential power concerning inefficient DMUs by changing efficient DMUs. Thus FA-DEA also allows us to compare inefficient DMUs with each other.

D. Consideration for application to DEA hierarchical approach

The FA-DEA allows us to carry out sensitivity analysis toward multiple DMUs. Then it is possible for DEA hierarchical approach to measure difference between hierarchies. Test data in Table 1 are hierarchized as shown in Fig. 6.

Let us consider how to improve DMU_i in the third place group step by step. FA-DEA is carried out based on the third place group in order to measure the distance from the third place group (frontier_3) to the first place group (frontier_1) and the second place group (frontier_2). In other words, DMU_i, DMU_j, DMU_k are chosen intentionally to set frontier_3 as an evaluation criteria in the sensitivity analysis.

The result is shown in Table 3. The total efficiency score of the first place group is 15.5. The average of efficiency score is 3.88 by dividing 15.5 by 4 which is the number of DMUs in frontier_1. The total efficiency score of the second place group is 5.57. The average of efficiency score is 1.86 by dividing 5.57 by 3 which is the number of DMUs in frontier_2. Therefore, the difference between the first and the second place group is 2.02.

There are two steps that DMU_i improves self management to be efficient. First step is improvement from frontier_3 to frontier_2, and second step is improvement from frontier_2 to frontier_1. Then the improvement effort regarding the second step can be estimated as 2.3 times compare with the first step. Thus, FA-DEA shall be helpful to decide what scale of extent improvement should be required when an inefficient DMU is improved.

V. CONCLUSIONS

This paper has proposed FA-DEA model which allows analyst to set frontier intentionally in order to measure influential power of multiple DMUs. The paper has also shown the effectiveness of FA-DEA due to numerical experiments. More flexible sensitivity analysis can be done by applying the proposed method. Moreover, new viewpoint in sensitivity analysis is shown. It is also discussed that FA-DEA is available for decision making support on DMU's performance improvement. That is because management strategy can be thought under the complex condition that states

TABLE IV. Comparison among each method in DEA

	Fundamental model (CCR,BCC)	Traditional method (sensitivity analysis)		FA-DEA (sensitivity analysis)
DMUs for forming frontier	All	Except targeted DMU		Assign intentionally
Remarkable DMUs	—	Targeted DMU (super-DEA)	Other DMUs	All
Efficiency score	$0<\theta\leqq 1$	$1\leqq\theta$	$0<\theta\leqq 1$	$0<\theta$
Analysis for influential power	Impossible	Possible only one DMU		Possible plural DMUs

of each DMU is changing one after another. The numerical experiments have also clarified that FA-DEA includes the traditional method. Table 4 shows lists of the comparison between the traditional method and FA-DEA.

For the future work, FA-DEA shall be applied to risk management. DMU which depends on particular input or output excessively tends to be evaluated as larger efficiency score in FA-DEA. In other words, DMU which has larger efficiency score extremely signifies that it faces the risks for losing performance. Therefore, the method shall allow to find the risks of DMU by changing intentional frontiers.

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