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Charged Active Contour Model

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Abstract— **A novel, physical-based force, called as Lorentz force inspired by classical electrodynamics is introduced for the active contour models, and the truncation computing method of Lorentz Force is detailed. Several comparative results indicate that the new force field has similar effects with the Gradient Vector Flow (GVF) field, but it has a remarkable promotion of evaluation speed. The new force is more suitable to be applied to real-time image processing applications.**

***Keywords*—Active contour, Lorentz Force, Charged Particle Model**

I. INTRODUCTION

Active contour models (or snake models) are widely used in computer vision and image processing applications, particularly to image segmentation. One difficult with snake model is their poor convergence to the true boundary concavities. Many authors manage to overcome it, and many new force models are brought foreword. One significant improvement of the force models is the Gradient Vector Flow (GVF) presented by Chenyang Xu and Jerry L. Prince[1-3]. Particular advantages of the GVF snake over a traditional snake are its insensitivity to initialize and its ability to move into boundary concavities. Its initialization can be inside, outside, or across the object's boundary. Unlike the snake with pressure forces, the GVF snake does not need prior knowledge about whether to shrink or to expand towards the boundary. The GVF snake also has a large capturing range, which means that, barring interference from other objects, it can be initialized far away from the boundary. However, the GVF also has its drawbacks. The capturing range is almost proportional to the iterations of solving the Euler equations[1, 2]. In order to gain a larger capturing range, at least dozens of iterations are needed. It is time consuming.

Recently, a new Active Contour model: charged particle model (CPM) was proposed by Jalba[4]. The CPM is inspired by classical electrodynamics and consists of a system of charged particles moving in an electrostatic field[5]. The force of the CPM includes two parts: Lorentz force and Coulomb force. Lorentz force is an attracting force between the free positive charged particles(points in the active contours) and the fixed negative charged particles(the edge of the map), which reflects the strength of the edge map. Coulomb force is a repulsive force due to the interaction of the free positive charged particle with other free positive charged particles. The Coulomb force is deemed as the internal force, while the Lorentz force is deemed as the external force. According to Particle Dynamics, the active contour made of free positive charged particles (free points) will move under the resultant force of the internal and the external forces until it is balanced. The curve is then reconstructed through ordering the free particles into a sequence which describes a closed contour along the boundary of the object. Unlike other active contour models (e.g. Mentioned in paragraph 1), the CPM is a non-parameters active contour, which means that there is no geometric relationship among the points in the active contour before they are ordered. Furthermore, the CPM has other demerits. It can not guarantee continuous and closed final contours; it does not stabilize as there is no effective stopping term; worst of all, it is computationally intensive[6]. When there are a large amount of free positive charged particles, it is very time-consuming to compute the Coulomb force in every iteration step. However, the CPM has at least one prominent merit over the GVF: its computing speed of external force field is much faster than the GVF's.

A charged active contour is proposed in [6] by Rong Yang etc., which incorporating the particle based electrostatic interactions into geometric active contour framework. The force field includes two parts: stationary boundary attraction force field and constantly changed boundary competition force field. This model has the merits of both the geometric active contour and the CPM. However, the computing speed is still slow due to constantly computing the boundary competition force field in every iteration step.

This paper will also devote to incorporating the particle based electrostatic interactions into geometric active contour framework. However, only the Lorentz force field is considered. Lorentz force field has similar effects with GVF field, but it has a much faster speed than GVF field. Consequently, it is more suitable to be applied to Real-time image processing applications.

II. BACKGROUNDS

A. traditional active contours

A traditional 2-D parametric deformable model or deformable contour is a curve $C(s)=[x(s), y(s)]$, $s\in[0, 1]$ that moves in an image domain to minimize the energy function [1]

$$E = \int_0^1 \frac{1}{2}\alpha(C'(s))^2 + \frac{1}{2}\beta(C''(s))^2 + E_{ext}(\mathrm{C}(s))ds \tag{1}$$

where α and β are weighting parameters that control the deformable contour's tension and rigidity respectively; $C'(s)$ and $C''(s)$ denote the first and second derivatives of $C(s)$ with respect to s. The external potential function $E_{ext}(C(s))$ is derived from the image so that it takes on its smaller values at the features of interest, such as boundaries. Solved by the variation method, the minimum of E has to satisfy the following Euler-Lagrange equation

$$-(\alpha C'(s))'+(\beta C''(s))''+\nabla E_{ext}(C(s))=0 \qquad (2)$$

Discretization of (2) by finite difference gives a linear system.

$$KQ = F_{ext} \qquad (3)$$

where K is a stiffness matrix depending on α and β; Q and F_{ext} denote the discrete point vectors on the active contours and the corresponding external forces respectively.

A general and elegant approach to fit the deformable curve to data, especially when the data are time-varying, is to make the models dynamic. A dynamic formulation imposes a natural temporal continuity on the model, thereby permitting a smoothly animated display of the data fitting process. The discrete equations of motion are

$$M\ddot{Q} + C\dot{Q} + KQ = F_{ext} \qquad (4)$$

where M is imaginary mass matrix, C is imaginary damp matrix. Traditionarily, let $M=0$ and $C=I$ (identity matrix). Using forward finite difference to discretize this equation in time, we can get the following iterative formula [6]

$$\begin{cases} \dfrac{Q^{t+1}-Q^{t}}{\tau} + KQ^{t} = F_{ext}^{t} \\ Q^{0} = Q_{0} \quad \text{initial} \quad \text{estimation} \end{cases} \qquad (5)$$

where τ is the time step. Equation (5) can be written as

$$Q^{t+1} = (I - \tau K)Q^{t} + \tau F_{ext}^{t} \qquad (6)$$

with an initial estimation.

B. GVF field

The choice of F_{ext} can have a profound impact on both implementations and the behaviors of a snake. Several types of dynamic external forces have been invented to try to improve upon the standard snake potential forces. Chenyang Xu proposed an external force which is called as gradient vector flow (GVF) fields. The GVF deformable model has a large capture range, and is insensitive to initialization and able to move into boundary concavities. This increased capturing range is achieved through a spatially varying diffusion process. The gradient vector flow field is defined to be the vector field $g(x,y)=(u(x,y),v(x,y))$ that minimizes the energy functional

$$\varepsilon = \iint \mu[(\frac{\partial u}{\partial x})^2 + (\frac{\partial u}{\partial y})^2 + (\frac{\partial v}{\partial x})^2 + (\frac{\partial v}{\partial y})^2] + |\nabla f|^2 \, |g - \nabla f|^2 \, dxdy \qquad (7)$$

where f is an edge map functional derived from the image having the property that it is larger near the image edges, μ is a regularization parameter governing the tradeoff between the smoothing term and the data term. The numeral solution to this energy functional is by treating u and v as functions of time and solving the difference equation by iterative method.

C. Charged Particle model

Assuming that all edges in a map are composed of negatively charged particles p_i (pixel points) which are fixed and generated by input image at position $r_{pi}=[x_{pi}, y_{pi}]$, $i=1,\ldots n$, and that active contours are composed of positively charged particles q_i which are manually placed at position $r_{qi}=[x_{qi}, y_{qi}]$, $j=1,\ldots m$, and can freely move in the map, the free particles q_i will move under the resulting force of Lorentz force and Coulomb force. According to the particle dynamics, the resulting force of free particles q_i at position $r_{qi}=[x_{qi}, y_{qi}]$ is

$$F(r_{qi}) = F_C(r_{qi}) + F_L(r_{qi}) \qquad (8)$$

where F_C is the Coulomb force, and F_L is the Lorentz force. Assuming all positive particles have the same electric charge e_1, and all negative particles have the same electric charge e_2, the Lorentz force and Coulomb force can be respectively expressed as

$$F_L(r_{qi}) = -\sum_{\substack{k=1 \\ k\neq i}}^{n} \omega_1 \frac{r_{qi} - r_{pk}}{\left| r_{qi} - r_{pk} \right|^3} \qquad (9)$$

and

$$F_C(r_{qi}) = \sum_{\substack{k=1 \\ k\neq i}}^{m} \omega_2 \frac{r_{qi} - r_{qk}}{\left| r_{qi} - r_{qk} \right|^3} \qquad (10)$$

where $\omega_1=ce_1e_2$, $\omega_2=ce_1^2$, and c is a constant. Comparing (9) with (10), the Lorentz force and Coulomb force have the similar expressions. However, the evaluation methods are quite different. The Lorentz force reflects the strength of the edge map, an attracting force between the free positive charged particles (points in the active contours) and the fixed negative charged particles (the edge of the map). It is independent of the time and only changeable along with the position of the moving charged particle. Hence, this force field can be computed beforehand like the GVF field (the evaluation method will be discussed in section IV). On the contrary, the Coulomb force is dependent of time. The movement of every free positive charged particle will have an influence on the force field. Therefore, the force field should be constantly updated. It is time-consuming and expensive in computing the force field of Coulomb force in deformable models, especially when there are large amount of free positive charged particles. A rough evaluation of computing the force field of Coulomb force requires $O(m^2)$ operations, where m is the number of the free charged particles.

III. PROPOSED METHOD

This paper will devote to introduce the CPM into the active contour model. In this model, we treat CPM as an external force field. The force of CPM includes two parts:

Lorentz force and Coulomb force. As stated in the former section, the force field of Coulomb force is dependent of time and should be constantly recomputed. In every iteration of computing, $O(m^2)$ operations are required. For a large input image which usually requires a large number of free particles, the force evaluation in (10) will become prohibitively expensive. In fact, it is unnecessary to take the Coulomb force into account in deformed models. When the moving particles are kept far away from each other, the Coulomb force will approach its minimum. In other words, the main function of the Coulomb force is to make the moving particles to repel each other so that the particles are uniformly distributed along the edges. In active contours, the distance between the moving particles is regulated through inserting or deleting some particles by some rules, which usually led the particles to a uniform distribution. Furthermore, considering the condition that e_2 is much larger than e_1, the Coulomb force is small enough to be ignored against Lorentz force. Hence, the proposed method will only introduce the Lorentz force into active contour models. In this paper, the external force of active contour models in (6) is replaced with the Lorentz force in (9), and we call this model as charged active model.

IV. Truncation Computing Method Of Lorentz Force

Lorentz force is independent of the time. It can be computed beforehand. Although Lorentz force field computes only once, for the large input image, the evaluation is also expensive. Direct computing the Lorentz force field requires $O(n\times L)$ operations, where L is the total pixel number in the input image, n is the total pixel number of edge in the map. In active contours, the main function of the external force is to attract the contours to the edge, so high precision of the force field is inessential. Furthermore, although a larger capture range is desired in order to an easy and free initialization, too much larger capture range will do little help to the active contour models. Therefore, in order to fast computing the force field, truncation schemes is used. A squared cutoff is introduced in this paper. A particle (pixel) on the edge of the map will influence or interact with pixels inside a fictitious box of $(2s+1)\times(2s+1)$ centered at the particle, and the pixels outside the squared box are excluded from interacting with the particle. Fig. 1 illustrates this squared cutoff method. In this figure, the centered pixel (lattice) filled with dark is deemed as one pixel on the edge in a map. If s is set to 3, the gray pixels (lattices) are within the interaction of the pixel, and the white pixels are excluded from the interacting.

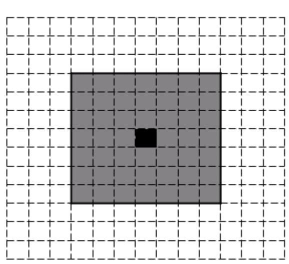

Figure 1. Squared cutoff method

Considering a sample pixel (m, n) in a map, only the pixels ($m+k$,$n+l$) on the edges can interact with the sample pixel, where $k,l\in[-s, s]$. The Lorentz force in (9) can be rewritten in this truncation schemes as

$$F_L(m,n) = -\sum_{\substack{k=-s\\k\neq 0}}^{s}\sum_{\substack{l=-s\\l\neq 0}}^{s} ce_1e_2\left(\frac{k}{\left(k^2+l^2\right)^{\frac{3}{2}}}+\frac{l}{\left(k^2+l^2\right)^{\frac{3}{2}}}\mathbf{i}\right)\times h(m+k,n+l) \tag{11}$$

where $\mathbf{i}$ is a unit vector for y-direction, and $h(x, y)$ is defined as

$$h(x,y)=\begin{cases}1 & \text{for pixel}(x,y)\text{ on edges}\\ 0 & \text{otherwise}\end{cases} \tag{12}$$

Equation (11) is equivalent to discrete convolution with the kernel g, which is called a convolution mask. The coefficients vector $g(x, y)$ is given

$$g(x,y)=\begin{cases}-\dfrac{x}{\left(x^2+y^2\right)^{\frac{3}{2}}}-\dfrac{y}{\left(x^2+y^2\right)^{\frac{3}{2}}}\mathbf{i} & \text{otherwise}\\ 0 & \text{for } x=0 \text{ and } y=0\end{cases} \tag{12}$$

The kernel g can be computed beforehand. If we set ce_1e_2 to 1, the evaluation of Lorentz force in (11) for the sample pixel (m, n) at the most requires $2(2s+1)\times(2s+1)$ addition operations .

V. Experiments And Results

In this section, several comparative results are presented on pictures with different sizes. The comparisons are carried on between the GVF field and the Lorentz Force field. In this paper, the iteration number of GVF evaluation is set to 30, and the regularization parameter μ governing the tradeoff between the smoothing term and the data term is set to 0.1. The region parameter s in Lorentz Force field is set to 12. According to these settings, the capturing ranges are approximated between the GVF field and the Lorentz force field. Fig. 2(a) is a 64×64 U-shape, which is often used in papers to test the arithmetic of active contours [2,7,8]. The corresponding GVF field and Lorentz Force field are shown in Fig. 2(b) and Fig. 2(c) respectively. Theses two force fields are unified. Comparing these two figures, the distributions of the force around the edge are almost the same, and all the force vectors point to the edge of the map which is desired in active contours. However, the discrepancy of evaluation time is much large. The evaluation times of GVF field and Lorentz force field on a notebook PC (IBM T60, CPU: dual core 2.0GHz, Memory: 2G) are 0.066 second and 0.003 second respectively. The evaluation time of the GVF field is about 20 times longer than that of the Lorentz Force. The convergence

TABLE 1 COMPARATIVE RESULTS OF EVALUATION TIME

Image of different size (Pixel)	***GVF field (second)***	***Lorentz Force field (second)***	***Saving***
U-shape(64×64)	0.066	0.003	95%
MRI(230×206)	0.4	0.016	96%
Hand-drawing(1024×768)	29	0.5	98%

of active contours based upon the GVF field and the Lorentz force field are displayed in Fig. 2(d) and Fig. 2(e-f) respectively. From Fig. 2(d) and Fig. 2(e-f), it goes without saying that Lorentz Force field approximately has the same effects with the GVF field. In fact, the Lorentz Force field has almost all the merit of GVF field including insensitivity to initialization, capability of moving into boundary concavities and large capture ranges.

Another example is a MRI(Magnetic Resonance Image) with the size of 230×206 pixels shown in Fig. 3(a), and its edge map is shown in Fig. 3(b) . From Fig. 3(c) and Fig.3(d), the GVF and Lorentz Force also have the similar force field. The corresponding evaluation times are 0.4 second and 0.016 second.

A larger hand-drawing picture with the size of 1024× 768 pixels is tested (not shown in this paper). The evaluation times of the GVF field and the Lorentz Force field are 29 seconds and 0.5 second respectively. For the convenience of comparison, we list the evaluation time in table 1. From table 1, we can see that with the increasing of the picture size, the saving time is more obviously if we adopt the Lorentz force field rather than the GVF field.

VI. CONCLUSIONS

The GVF snake is popular in the image segmentation in computer vision and image processing applications due to the robust and the large capture-range characters. However, the procedure is a rather slow. A charged active contour is introduced. The force field comes from the CPM or the classical electrodynamics. However, only the Lorentz Force is considered in this paper as the external force of the active contour models. The new external force has the similar effects with the GVF, but it has a much faster evaluation speed than the GVF. Consequently, it is more suitable to be applied to Real-time image processing applications.

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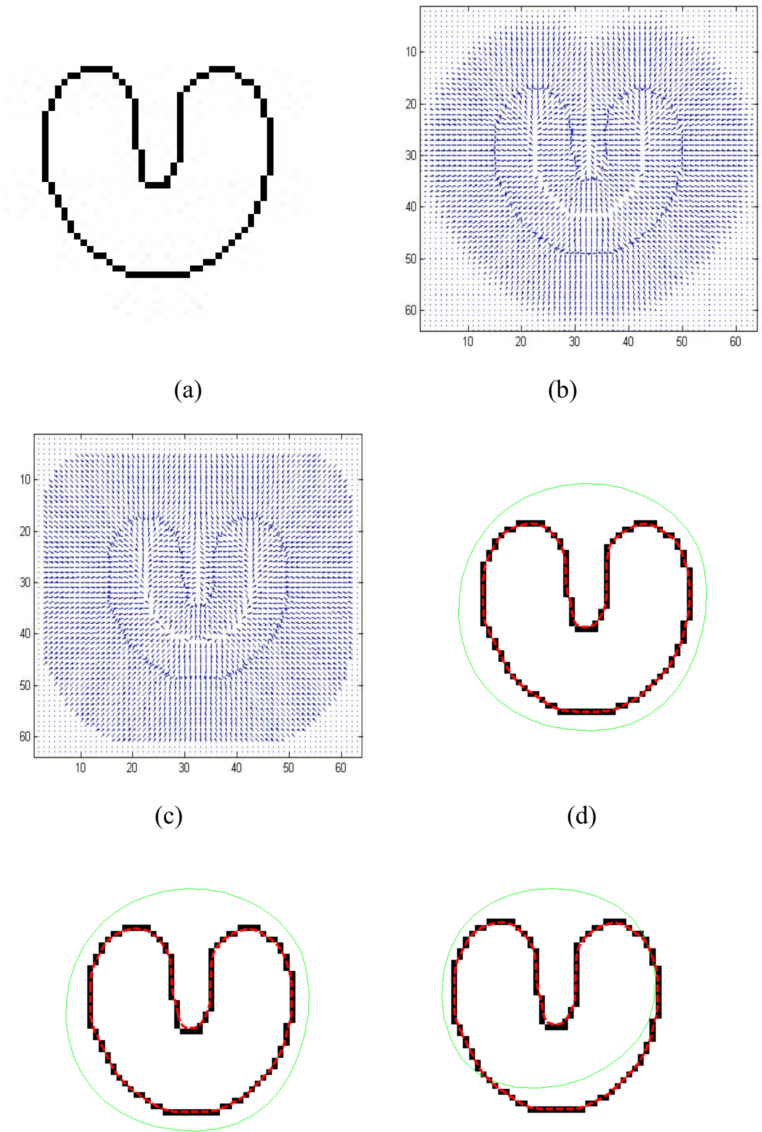
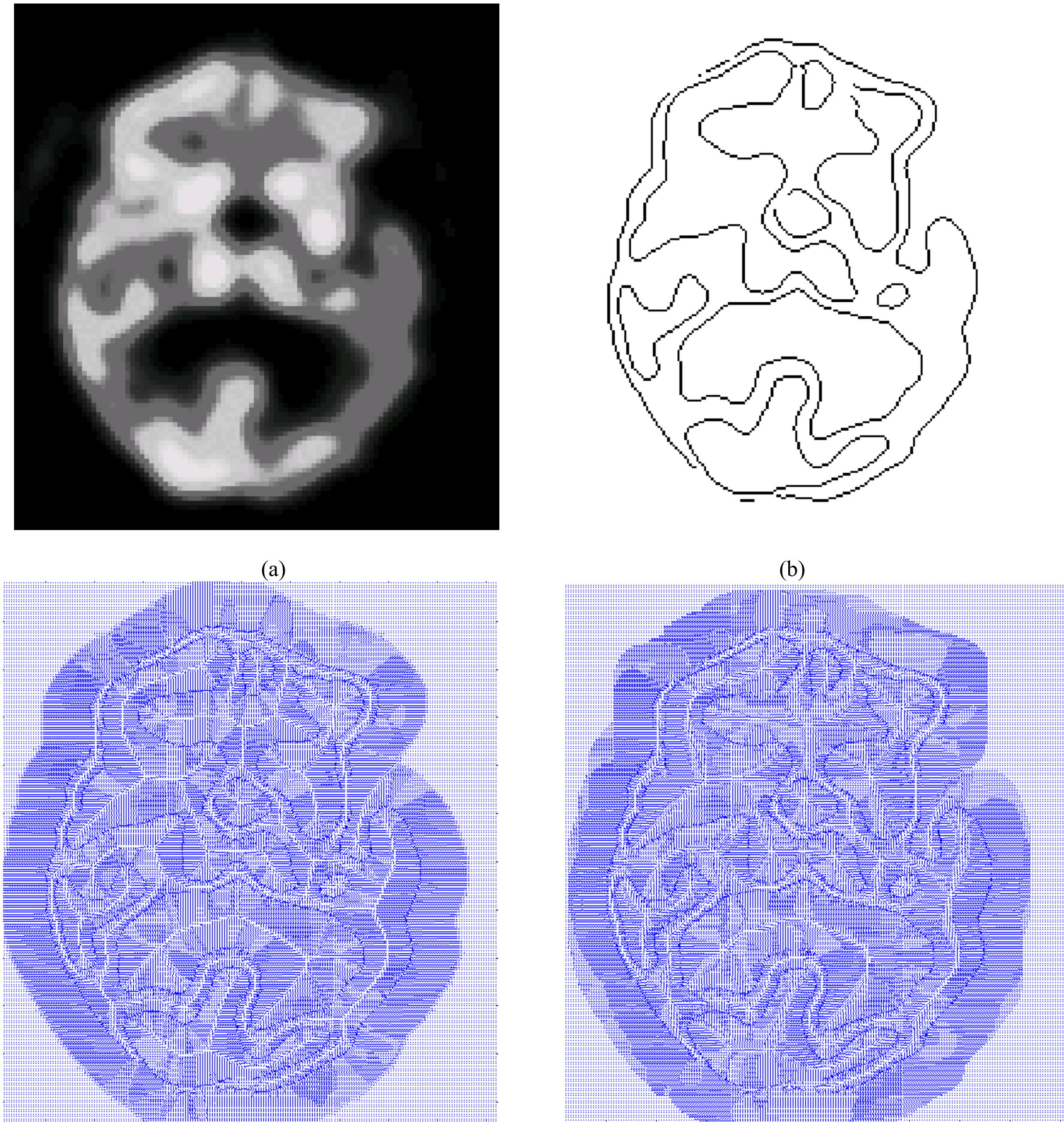


Figure 2. Comparisons between GVF and Lorentz Force field under U-shape.
(a) U-shape , (b) its GVF field, (c) its Lorentz Force Field,
(d) its active contour under GVF, and its active contour under Lorentz Force field in (e) and (f).



(a) (b) (c) (d)

Figure 3. Comparisons between GVF and Lorentz Force field under MRI.

(a) MRI, (b) its egde map, (c) its GVF field and (d) its Lorentz force field