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Designing Type-2 Fuzzy Logic System Controllers via Fuzzy Lyapunov Synthesis for the Output Regulator of a Servomechanism with Nonlinear Backlash

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***Abstract*—Fuzzy Lyapunov Synthesis is extended to the design of Type-2 Fuzzy Logic System Controllers for the output regulation problem for a servomechanism with nonlinear backlash. The problem in question is to design a feedback controller so as to obtain the closed-loop system in which all trajectories are bounded and the load of the driver is regulated to a desired position while also attenuating the influence of external disturbances. The servomotor position is the only measurement available for feedback; the proposed extension is far from trivial because of nonminimum phase properties of the system. Performance issues of the Type-2 Fuzzy Logic Regulator constructed are illustrated in a simulation study.**

***Index Terms*—Type-2 Fuzzy Control, Fuzzy Lyapunov Synthesis, Backlash.**

I. Introduction

A major problem in control engineering is a robust feedback design that asymptotically stabilizes a nonminimal plant while also attenuating the influence of parameter variations and external disturbances. In the last decade, this problem was heavily studied and considerable research efforts have resulted in the development of systematic design methodology for nonlinear feedback systems. A survey of these methods, fundamental in this respect is given in [7].

In the present paper the output regulator problem is studied for an electrical actuator consisting of a motor part driven by a DC motor and a reducer part (load) operating under uncertainty conditions in the presence of nonlinear backlash effects. The objective is to drive the load to a desired position while providing the roundedness of the system motion and attenuating external disturbances. Because of practical requirements [10], the motor angular position is assumed to be the only information available for feedback.

This problem was first reported in [1], where the problem of control nonminimum phase systems was solved by using nonlinear H_∞ control, but the reported results do not provide robustness evidence. In [3], authors report a solution to the regulation problem using a Type-1 Fuzzy Logic System Controller. In [4] and [5] were reported solutions using Type-2 Fuzzy Logic Systems Controllers, and making a genetic optimization of the membership function's parameters, but do not specify the criteria used in the optimization process and Genetic Algorithm design. In [6], a comparison of the use of Genetic Algorithms to optimize Type-1 and Type-2 Fuzzy Logic Systems Controllers is reported, but a method to achieve this optimization is not provided.

The solution that we propose in this paper is to design a Type-2 Fuzzy Logic System Controller extending the Fuzzy Lyapunov Synthesis [13], a concept that is based on the Computing with Words [16][20] approach of the Lyapunov Synthesis [9], this approach was reported in [2] for the design of stable Type-2 Fuzzy Logic Systems Controllers.

The paper is organized as follows. The dynamic model of the servomechanism is presented in Section II. The problem statement is in Section III. Section IV address Type-2 Fuzzy Sets and Systems theory. The design of the Type-2 Fuzzy Logic Controller is described in Section V. The numerical simulations for the designed Type-2 Fuzzy Logic Controller are presented in Section VI. Conclusions are presented in Section VII.

II. Dynamic model

The dynamic model of the angular position $q_i(t)$ of the DC motor and $q_o(t)$ of the load are given according to

$$\begin{aligned} J_0 N^{-1}\ddot{q}_0 + f_0 N^{-1}\dot{q}_0 &= T + w_0 \\ J_i\ddot{q}_i + f_i\dot{q}_i + T &= \tau_m + w_i, \end{aligned} \tag{1}$$

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hereafter, J_0, f_0, $\ddot{q}_0$ and $\dot{q}_0$ are, respectively, the inertia of the load and the reducer, the viscous output friction, the output acceleration, and the output velocity. The inertia of the motor, the viscous motor friction, the motor acceleration, and the motor velocity denoted by J_i, f_i, $\ddot{q}_i$ and $\dot{q}_i$, respectively. The input torque τ_m serves as a control action, and T stands for the transmitted torque. The external disturbances $w_i(t)$, $w_0(t)$ have been introduced into the driver equation (1) to account for destabilizing model discrepancies due to hard-to-model nonlinear phenomena, such as friction and backlash.

The transmitted torque T through a backlash with an amplitude j is typically modeled by a dead-zone characteristic [19, p. 7]:

$$T(\Delta q) = \begin{cases} 0 & |\Delta q| \leq j \\ K\Delta q - Kj\,\mathrm{sign}(\Delta q) & \text{otherwise} \end{cases} \tag{2}$$

with

$$\Delta q = q_i - Nq_0, \tag{3}$$

where K is the stiffness, and N is the reducer ratio. Such a model is depicted in Fig. 1. Provided the servomotor position $q_i(t)$ is the only available measurement on the system, the above model (1)-(3) appears to be non-minimum phase because along with the origin the unforced system possesses a multi-valued set of equilibria (q_i, q_0) with $q_i = 0$ and $q_0 \in [-j, j]$.

To avoid dealing with non-minimum phase system, we replace the backlash model (2) with its monotonic approximation:

$$T = K\Delta q - K\eta(\Delta q) \tag{4}$$

where

$$\eta = -2j\frac{1 - \exp\left\{-\frac{\Delta q}{j}\right\}}{1 + \exp\left\{-\frac{\Delta q}{j}\right\}}. \tag{5}$$

The present backlash approximation is inspired from [18]. Coupled to the drive system (1) subject to motor position measurements, it is subsequently shown to continue a minimum phase approximation of the underlying servomotor, operating under uncertainties $w_i(t)$, $w_0(t)$ to be attenuated. As a matter of fact, these uncertainties involve discrepancies between the physical backlash model (2) and its approximation (4) and (5).

III. Problem Statement

To formally state the problem, let us introduce the state deviation vector $x = [x_1, x_2, x_3, x_4]^T$ with

$$x_1 = q_0 - q_d,$$

$$x_2 = \dot{q}_0,$$

$$x_3 = q_i - Nq_d,$$

$$x_4 = \dot{q}_i,$$

where x_1 is the load position error, x_2 is the load velocity, x_3 is the motor position deviation from its nominal value, and

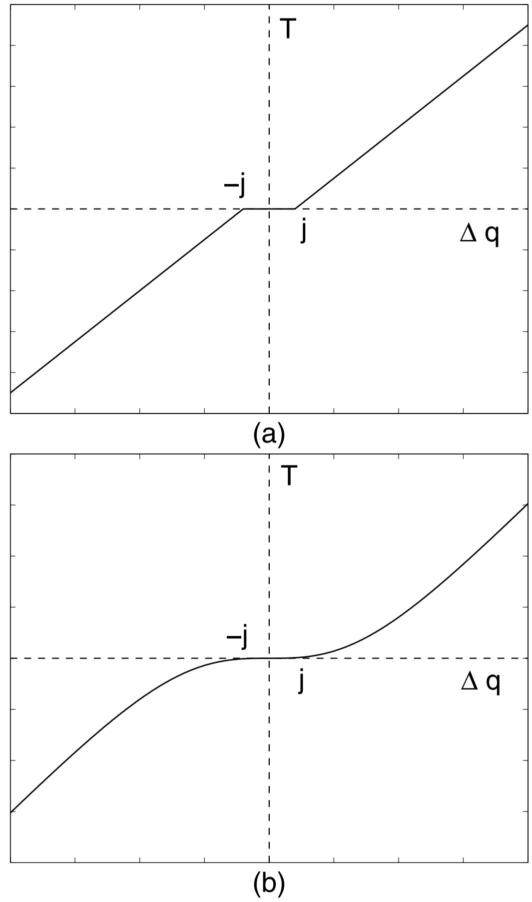

Fig. 1. a) The dead-zone model of backlash; b) The monotonic approximation of the dead-zone model

x_4 is the motor velocity. The nominal motor position Nq_d has been pre-specified in such a way to guarantee that $\Delta q = \Delta x$, where

$$\Delta x = x_3 - Nx_1.$$

Then, system (1)-(5), represented in terms of the deviation vector x, takes the form

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= J_0^{-1}[KNx_3 - KN^2x_1 - f_0x_2 + KN\eta(\Delta q) + w_o], \\ \dot{x}_3 &= x_4, \\ \dot{x}_4 &= J_i^{-1}[\tau_m + KNx_1 - Kx_3 - f_ix_4 + K\eta(\Delta q) + w_i]. \end{aligned} \tag{6}$$

The zero dynamics

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= J_0^{-1}[-KN^2x_1 - f_0x_2 + KN\eta(-Nx_1)], \end{aligned} \tag{7}$$

of the undisturbed version of system (6) with respect to the output

$$y = x_3 \tag{8}$$

is formally obtained (see [8] for details) by specifying the control law that maintains the output identically zero.

The objective of the Fuzzy Control output regulation of the nonlinear driver system (1) with backlash (4) and (5), is thus to design a Fuzzy Controller so as to obtain the closed-loop system in which all these trajectories are bounded and the output $q_0(t)$ asymptotically decays to a desired position q_d as $t \to \infty$ while also attenuating the influence of the external disturbances $w_i(t)$ and $w_0(t)$.

IV. Type-2 Fuzzy Sets and Systems

The concept of Type-2 Fuzzy Set was introduced by Zadeh [21],[22],[23] as an extension of the concept of an ordinary Fuzzy Set.

A Type-2 Fuzzy Set, denoted as $\tilde{A}$ is characterized by a Type-2 Membership Function $\mu_{\tilde{A}}(z, \mu(z))$ [17], where $z \in Z$ and $\mu \in J_z \subseteq [0,1]$, i.e,

$$\tilde{A} = \{((z, \mu(z)), \mu_{\tilde{A}}(z, \mu(z)))|\forall z \in Z, \forall \mu(z) \in J_z \subseteq [0,1]\} \tag{9}$$

in which $0 \leq \mu_{\tilde{A}}(z, \mu(z)) \leq 1$. The set $\tilde{A}$ can also be expressed as follows [17]:

$$\tilde{A} = \int_{z \in Z} \int_{\mu(z) \in J} \mu_{\tilde{A}}(z, \mu(z))/(z, \mu(z)) \tag{10}$$

where $J_z \subseteq [0,1]$ and $\int\int$ denotes union over all admissible z and $\mu(z)$ [17].

J_z is called primary membership of z, where $J_z \subseteq [0,1]$ for $\forall z \in Z$ [17]. The uncertainty in the primary memberships of a Type-2 Fuzzy Set $\tilde{A}$, consists of a bounded region that is called the *footprint of uncertainty* (FOU) [17]. It is the union of all primary memberships [17].

An Interval Type-2 Fuzzy Set $\tilde{A}$ is to date the most widely used kind of Type-2 Fuzzy Sets, and is the only kind of Type-2 Fuzzy Sets that is considered in this paper. It is described as

$$\tilde{A} = \int_{z \in Z} \int_{\mu(z) \in J_z} 1/(z, \mu(z)) = \int_{z \in Z} \left[\int_{\mu(z) \in J_z} 1/\mu(z) \right] /z, \tag{11}$$

where z is the *primary variable*, $J_z \subseteq [0,1]$ is the *primary membership* of z, $\mu(z)$ is the *secondary variable*, and $\int_{\mu(z) \in J_z} 1/\mu(z)$ is the *secondary membership function* at z. Note that (11) means $\tilde{A} : Z \to \{[a,b] : 0 \leq a \leq b \leq 1\}$. Uncertainty about $\tilde{A}$ is conveyed by the union of all of the primary memberships, called the FOU of $\tilde{A}$[FOU($\tilde{A}$)], i.e.,

$$\text{FOU}(\tilde{A}) = \bigcup_{z \in X} J_z, \tag{12}$$

that is,

$$\text{FOU}(\tilde{A}) = \left\{ (z, \mu(z)) : \mu(z) \in J_z = [\underline{A}(z), \overline{A}(z)] \subseteq [0,1] \right\}. \tag{13}$$

A Fuzzy Logic System described using at least one Type-2 Fuzzy Set is called a Type-2 Fuzzy Logic Systems - also called Type-2 Fuzzy Inference Systems -. Type-1 Fuzzy Inference Systems are unable to directly handle rule uncertainties, this is because they use Type-1 Fuzzy Sets that are certain. On the other hand, Type-2 Fuzzy Inference Systems are very useful in circumstances where it is difficult to determine an exact and measurement uncertainties [15].

It is known that Type-2 Fuzzy Sets let us model and minimize the effects of uncertainties in a rule-based Fuzzy Logic Systems. Unfortunately, Type-2 Fuzzy Sets are more difficult to use and understand that Type-1 Fuzzy Sets; hence, their use is not widespread yet.

Similar to a Type-1 Fuzzy Inference Systems, a Type-2 Fuzzy Inference System includes ***Type-2 fuzzifier***, ***Type-2 rule-base***, ***Type-2 inference engine*** and substitutes the ***Type-1 defuzzifier*** by the ***output processor***. The ***output processor*** includes a ***type-reducer*** [15] and a ***Type-2 defuzzifier***; it generates a Type-1 Fuzzy Set output (from the *type-reducer*), or a crisp number (from the ***Type-2 defuzzifier***). A Type-2 Fuzzy Inference System is again characterized by IF-THEN rules, but its antecedent and consequents sets are now of the Type-2, see (14). Type-2 Fuzzy Inference Systems can be used when the circumstances are too uncertain to determine exact membership grades. A model of a Type-2 Fuzzy Inference System is shown in Fig. 2.

$$R^l : \text{IF } y \text{ is } \tilde{A}_1^l \text{ AND } \dot{y} \text{ is } \tilde{A}_2^l \text{ THEN } u \text{ is } \tilde{B}_1^l, \tag{14}$$

Note that for the Type-2 Fuzzy Inference Systems in the ***rule-base*** we are describing the IF-THEN rules following the Mamdani [12]-[11] type of fuzzy rules.

In this paper we are computing a Centroid type-reducer (CTR) [15] as *Type-2 defuzzifier*, the CTR is defined as follows:

$$\tau_m = \int_{\theta_j \in J_N} \cdots \int_{\theta_N \in J_{u_N}} \left[f_{u_1}(\theta_1) \star \cdots \star f_{u_N}(\theta_N) \right] / \frac{\int u_i \theta_i d\theta}{\int \theta_i d\theta}. \tag{15}$$

A detailed definition of (15) can be found in [15].

V. Design of the Type-2 Fuzzy Logic Controller

To apply the Fuzzy Lyapunov Synthesis method, we assume that the exact equations are unknown and that we have only the following partial knowledge about the plant:

1) The system may have really two degrees of freedom referred to as x_1 and x_2, respectively. Hence by (6), $\dot{x}_1 = x_2$.
2) $\dot{x}_2$ is proportional to τ_m, that is, when τ_m increases (decreases) $\dot{x}_2$ increases (decreases).

Our objective is to design the rule-base of a Type-2 Fuzzy Controller that will carry the load to a desired position q_d, or in other words, that will carry the trajectories of x_1 to zero. Define the Lyapunov function:

$$V(x_1, x_2) = \frac{1}{2}\left(x_1^2 + x_2^2\right) \tag{16}$$

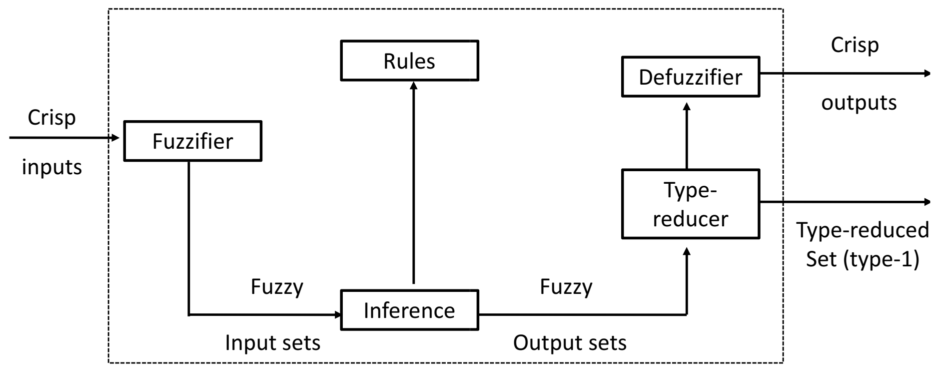

Fig. 2. Type-2 Fuzzy Inference System [14]

which is positive definite. The time derivative of $V(x_1, x_2)$ is

$$\dot{V}(x_1, x_2) = x_1\dot{x}_1 + x_2\dot{x}_2 = x_1x_2 + x_2\dot{x}_2, \quad (17)$$

hence, we require that

$$x_1x_2 + x_2\dot{x}_2 \leq 0. \quad (18)$$

We can now derive sufficient conditions so that condition (18) holds: If x_1 and x_2 have opposite signs, then $x_1x_2 < 0$ and (18) will hold if $\dot{x}_2 = 0$; if x_1 and x_2 are both positive, then (18) will hold if $\dot{x}_2 < -x_1$; if x_1 and x_2 are both negative, then (18) will hold if $\dot{x}_2 > -x_1$.

We can translate these conditions into the following fuzzy rules:

- If x_1 is *positive* and x_2 is *positive* then $\dot{x}_2$ must be *negative big*.
- If x_1 is *negative* and x_2 is *negative* then $\dot{x}_2$ must be *positive big*.
- If x_1 is *positive* and x_2 is *negative* then $\dot{x}_2$ must be *zero*.
- If x_1 is *negative* and x_2 is *positive* then $\dot{x}_2$ must be *zero*.

However, using our knowledge that $\dot{x}_2$ is proportional to τ_m, we can replace each $\dot{x}_2$ with τ_m to obtain the fuzzy rule-base for the stabilizing controller:

- If x_1 is *positive* and x_2 is *positive* then u must be *negative big*.
- If x_1 is *negative* and x_2 is *negative* then u must be *positive big*.
- If x_1 is *positive* and x_2 is *negative* then u must be *zero*.
- If x_1 is *negative* and x_2 is *positive* then u must be *zero*.

It is interesting to note that the fuzzy partitions for x_1, x_2, and u follow elegantly from expression (17). Because $\dot{V} = x_2(x_1 + \dot{x}_2)$, and since we require that $\dot{V}$ be negative, it is natural to examine the signs of x_1 and x_2; hence, the obvious fuzzy partition is *positive*, *negative*. The partition for $\dot{x}_2$, namely *negative big*, *zero*, *positive big* is obtained similarly when we plug the linguistic values *positive*, *negative* for x_1 and x_2 in (17). To ensure that $\dot{x}_2 < -x_1$ ($\dot{x}_2 > -x_1$) is satisfied even though we do not know x_1's exact magnitude, only that it is *positive* (*negative*), we must set $\dot{x}_2$ to *negative big* (*positive big*). Obviously, it is also possible to start with a given, pre-defined, partition for the variables and then plug each value in the expression for $\dot{V}$ to find the rules. Nevertheless, regardless of what comes first, we see that fuzzy Lyapunov synthesis transforms classical Lyapunov synthesis from the world of exact mathematical quantities to the world of computing with words [20], [16].

TABLE I
NOMINAL PARAMETERS.

Description	Notation	Value	Units
Motor inertia	J_i	2.8×10^{-6}	Kg-m^2
Load inertia	J_o	1.07	Kg-m^2
Motor viscous friction	f_i	7.6×10^{-7}	N-m-s/rad
Load viscous friction	f_o	1.73	N-m-s/rad

To complete the controllers design, we must model the linguistic terms in the rule-base using fuzzy membership functions and determine an inference method. Following [2], we characterize the linguistic terms *positive*, *negative*, *negative big*, *zero* and *positive big* by the type-2 membership functions shown in Fig. 3 for a Type-2 Fuzzy Logic Controller, note that the membership function *zero* is not depicted for error and change of error variables. To this end, we had systematically design a Type-2 Fuzzy Logic Controller following the Lyapunov stability criterion.

VI. Simulation Results

To perform simulations we use the dynamical model (1)-(5) of the experimental testbed installed in the Robotics & Control Laboratory of CITEDI-IPN (see Fig. 4), which involves a DC motor linked to a mechanical load through an imperfect contact gear train [1]. The parameters of the dynamical model (1) are given in Table I, while $N = 3$, $j = 0.2$ [rad], and $K = 5$ [N-m/rad]. These paremeters are taken from the experimental testbed.

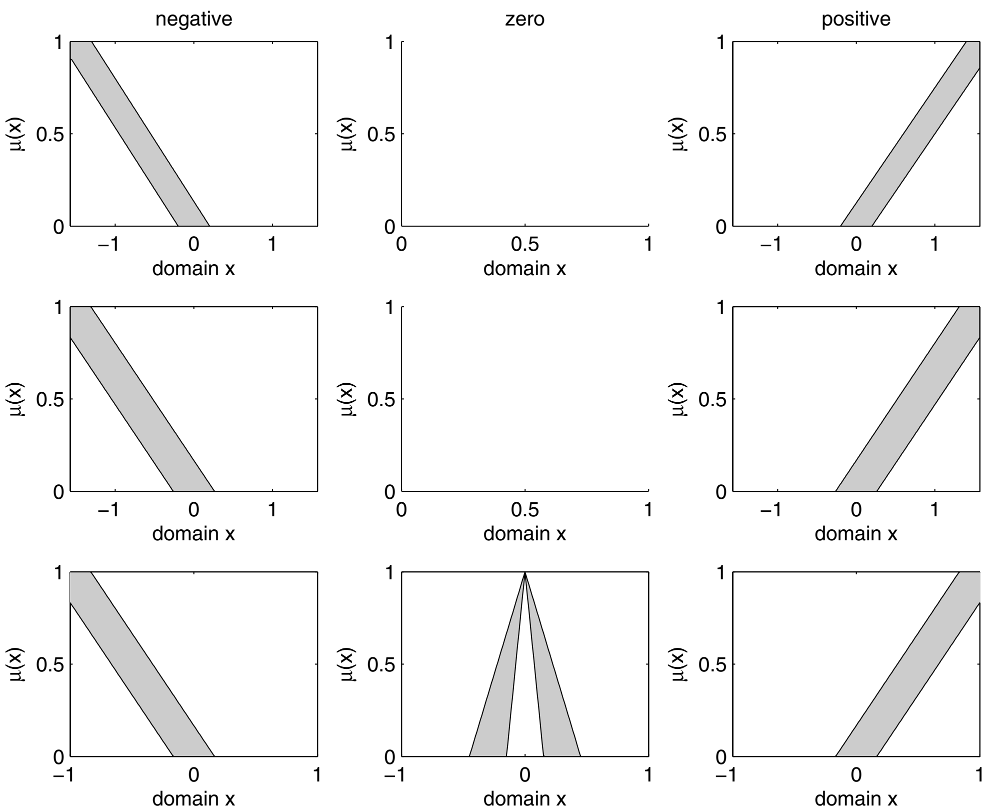

Fig. 3. Set of type-2 membership functions.

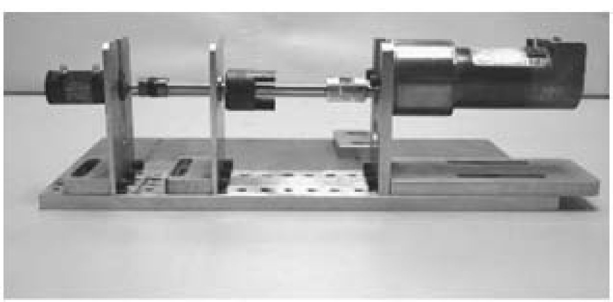

Fig. 4. Experimental testbed of Robotics & Control Laboratory of CITEDI-IPN

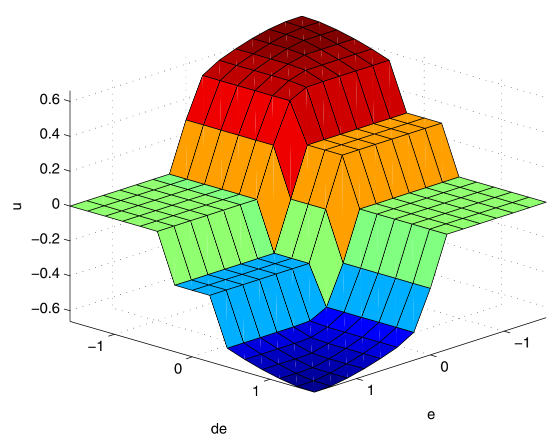

Fig. 5. Surface of control.

Performing a simulation of the closed-loop system (1)-(4) with the Type-2 Fuzzy Logic Controller designed in Section V we have the following results: the surface of control of Fig. 5 and the output of Fig. 6. It can be seen that load trajectories reach the desired position as was predicted. That is x_1 converges to zero, it can be seen in Fig. 7, where you can see that the error is stable between 10-14 seconds, at the same time that the change of the error is oscillating, this is due to the mechanism's inertia, the fuzzy system does not provide a strong control signal but the mechanism continues in motion due to its inertia moment.

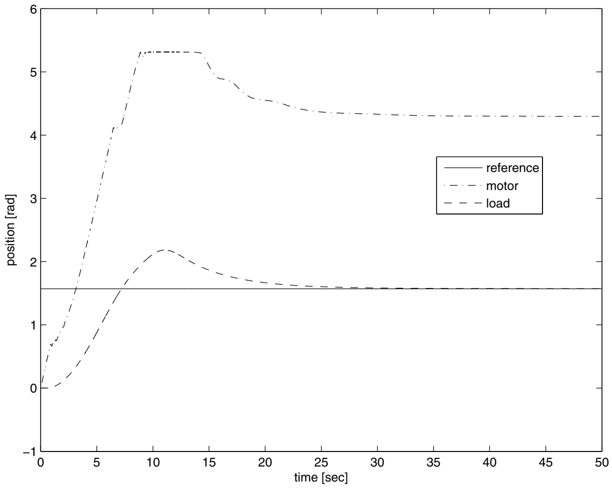

Fig. 6. Simulation result.

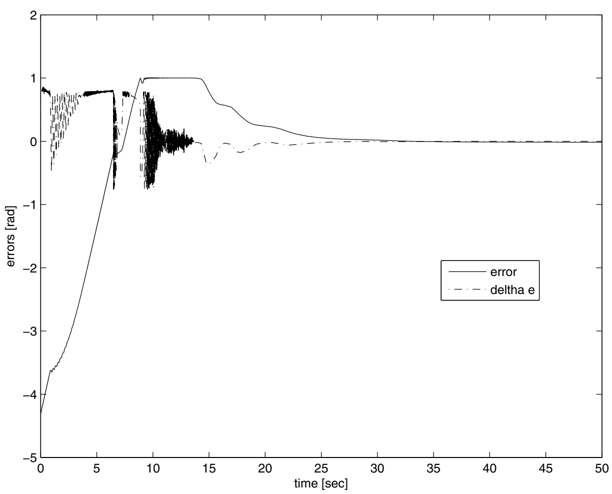

Fig. 7. Convergence of x_1 to zero.

VII. Conclusion

A Type-2 Fuzzy Logic Systems Controller was designed following the Fuzzy Lyapunov Synthesis, this controller was used to control the output regulation of a servomechanism with nonlinear backlash.

The proposed design strategy results in a controller that as was predicted, guarantees that the load reaches the desired position q_d, and the trajectories of the error x_1 converge to zero.

Type-2 Fuzzy Logic Systems were proved to be an appropriate control strategy for nonminimum phase systems, achieving regulation the proposed systems, however, the settlement time must be reduced to achieve results like those reported in [1].

Fuzzy Lyapunov Synthesis helps us to design Type-2 Fuzzy Logic Systems Controllers, but unlike the use of Genetic Algorithms reported in [3], [5], [6], [4], Fuzzy Lyapunov Synthesis is based on the theory of Lyapunov, and this fact guarantees that the designed Type-2 Fuzzy Logic System Controller is stable.

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