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Active incipient fault detection with more than two simultaneous faults

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***Abstract*—The problem of detecting small parameter variations in linear uncertain systems due to incipient faults, with the possibility of injecting an input signal to enhance detection, is considered. Most studies assume that there is only one fault developing. Recently an active approach for two simultaneous faults has been introduced. In this paper we extend this approach to allow for more than two simultaneous faults. Having more than two simultaneous incipient faults is sometimes a natural assumption. A computational method for the construction of an input signal for achieving guaranteed detection with specified precision is presented for discrete time systems. The method is an extension of a multi-model approach used for the construction of auxiliary signals for failure detection, however, new technical issues must be addressed. A case study is examined.**

***Index Terms*—Failure detection, linear system, detection systems, feedback, optimal detection**

I. Introduction

There are two main approaches to fault detection: passive and active. The passive approach monitors system outputs and then attempts to determine if system operation is normal. There is an extensive literature on passive approaches, we note only [11]. The passive approach has traditionally been the most used. Unlike the passive approach, the active approach requires that we apply an input, or test signal, to the system to detect the fault. Using the active approach, we want to do enough to the system to detect the fault, but we also want to disturb system performance as little as possible. This last requirement will be addressed using a cost function that will allow us to determine the optimal input required that will satisfy our conditions. In recent years there has been an increasing interest in active approaches, see [4], [12], [14], [15], [20], [21].

Incipient faults are those that develop over a period of time. They are often modeled as a drift or change in a parameter. The goal is to determine that they are occuring before the fault gets too severe. There has been considerable work done on detecting incipient faults [6], [7], [9], [11]. However, this work generally supposes that there is one incipient fault. The one fault assumption is common in the fault detection literature. While assuming one fault is often done with incipient faults, it is not as natural to assume that all incipient fault situations that arise behave this way. Since the faults are developing over a period of time it is reasonable to assume in many cases that there may be more than just one incipient fault. For example, with advancing age more than one electrical component may begin to undergo change in a complicated circuit. In the same way, in a mechanical system it would be reasonable to expect to have several parts experiencing wear.

One approach to detection of a single incipient fault was developed in [5], [16], [18], [19]. That approach used some of the ideas from a robust active fault detection scheme developed in [2]. Recently in [8] a way to approach the two fault problem was introduced. In this paper we will extend the ideas of [8] to more than two simultaneous faults. An example is solved. Since this paper is the first investigating the more than two incipient fault problem, our focus will be exploratory in nature and in determining what the key issues are and developing approaches for addressing them thereby laying the groundwork for future algorithms.

The key idea, following that introduced in [18], is to reformulate the incipient problem so that we we can utilize modifications of the existing algorithms from [2]. However, the presence of several faults requires addressing several technical problems. To do that we will first review just enough of the results from [2] so that we can develope the new application.

The tests are done over a short period of time so that parameter values are considered constant over the test period. While the problem with more than two incipient faults at first glance may appear to be a straight forward generalization of the one fault case, we shall see that there are additional technical issues to deal with. For our purposes here, it suffices to consider discrete time systems in the static model formulation. Once the issues are fully elaborated here, we will consider continuous time and hybrid systems in later studies.

The approach presented here is described in terms of additive uncertainty but, as explained later, it can accomodate some types of model uncertainty.

II. Classical Active Fault Detection Setup

We begin by considering the classical two model fault detection approach. We could easily do time varying coefficients but to start we take the coefficients constant. That is, the A_i etc., do not depend also on k. We will provide just enough detail to get to the multiple parameter incipient problem. We use the notation that $[i : j]$ are the integers from i to j.

We assume that we have two linear time invariant models of the form:

$$x_{k+1}^i = A_i x_k^i + B_i v_k + M_i \mu_k^i, \quad k \in [0 : K-1] \tag{1a}$$

$$y_k^i = C_i x_k^i + N_i \mu_k^i, \quad k \in [0 : K]. \tag{1b}$$

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The i indicates model 0 or model 1. Model 0 is considered to be the normal system and model 1 the faulty system.

We consider x_0^i to be uncertain or noise. In the approach of [2], it is assumed that the uncertainty in the models is bounded by a bound in the form:

$$S_i = (x_0^i)^T Q_i x_0^i + \sum^K (\mu_k^i)^T \mu_k^i < \frac{1}{2}, \tag{2}$$

where the bound of $\frac{1}{2}$ is taken in this paper for convenience. Scaling of the M_i easily accomodates other bounds. A bound of 1 is used in [2]. For a given v the output sets of each model then form a bounded convex set. A test signal v is proper if these output sets are disjoint. That is, based on the output one can always tell which model is correct. Given proper signals, then we seek the smallest one in some appropriate norm. This is done in [2], but it takes the equivalent characterization of v proper to be that if the outputs are the same for both models, then one of the noise bounds (2) are violated. Thus if v is proper, setting $y_k^1 = y_k^0$ and then taking the $\inf$ over those noises giving common outputs of the $\max_i S_i$, results in an answer that is greater then or equal to one. In the algorithms, the $\max_i S_i$ is replaced by $\max_{0 \le \beta \le 1} \beta S_0 + (1-\beta) S_1$ which, after some reformulation, allows the use of LQR control theory.

In general, the β parameter requires an iteration to solve for. However, in [18] it is seen that when the single parameter incipient problem is reformulated so that the machinery of [2] can be used, we find that $\beta = \frac{1}{2}$. Also, for the additive uncertainty case studied here, the use of a fixed β in $[0,\ 1]$ does not effect the existence of a proper test signal. Rather it may result in obtaining a suboptimal proper test signal as discussed in [3]. This is not the case with model uncertainty where fixing β can some times result in not being able to find a proper test signal. As we will see later, the addition of a second incipient parameter results in another layer of optimization. Accordingly, in this first paper we will use a noise measure that could be suboptimal.

For theoretical analysis and clarity of exposition, it is very convenient to reformulate this problem as a static problem. Thus we define the vectors

$$x^i = \begin{pmatrix} x_1^i \\ \vdots \\ x_K^i \end{pmatrix}, \quad y^i = \begin{pmatrix} y_0^i \\ \vdots \\ y_K^i \end{pmatrix},$$

$$\mu^i = \begin{pmatrix} x_0^i \\ \mu_0^i \\ \vdots \\ \mu_K^i \end{pmatrix}, \quad v = \begin{pmatrix} v_0 \\ \vdots \\ v_{K-1} \end{pmatrix}, \quad \mu = \begin{pmatrix} \mu^0 \\ \mu^1 \end{pmatrix}.$$

Then (1) can be rewritten as:

$$\mathcal{E}_i x^i = \mathcal{B}_i v + \mathcal{M}_i \mu^i \tag{3a}$$
$$y^i = \mathcal{C}_i x^i + \mathcal{N}_i \mu^i, \tag{3b}$$

where

$$\mathcal{E}_i = \begin{pmatrix} I & 0 & \dots & \dots & 0 \\ -A & I & 0 & \dots & \vdots \\ 0 & -A & I & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & -A & I \end{pmatrix}_{K \times K},$$

$$\mathcal{M}_i = \begin{pmatrix} A_i & M_i & 0 & \vdots & \vdots & 0 \\ 0 & 0 & M_i & 0 & \vdots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 0 & M_i & 0 \end{pmatrix}_{K \times K+2},$$

$$\mathcal{B}_i = \text{Diag}(B_i, \dots, B_i)_{K \times K},$$

$$\mathcal{C} = \begin{pmatrix} 0 & 0 & \dots & 0 \\ C_i & 0 & \dots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & C_i \end{pmatrix}_{K+1 \times K},$$

$$\mathcal{N}_i = \begin{pmatrix} C_i & N_i & 0 & \dots & 0 \\ 0 & 0 & N_i & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 0 & N_i \end{pmatrix}_{K+1 \times K+2}.$$

But $\mathcal{E}_i$ is invertible so we can solve (3a) for x^i and substitute into (3b) to get

$$\begin{aligned} y^i &= \mathcal{C}_i \left(\mathcal{E}_i^{-1} (\mathcal{B}_i v + \mathcal{M}_i \mu^i) \right) + \mathcal{N}_i \mu^i \\ &= \mathcal{C}_i \mathcal{E}_i^{-1} \mathcal{B}_i v + (\mathcal{C}_i \mathcal{E}_i^{-1} \mathcal{M}_i + \mathcal{N}_i) \mu^i. \end{aligned} \tag{4}$$

This shows us that in the discrete time case we may assume that we have two models of the form,

$$y^i = X_i v + H_i \mu^i, \quad i = 0, 1, \tag{5}$$

where

$$X_i = \mathcal{C}_i \mathcal{E}_i^{-1} \mathcal{B}_i, \quad H_i = \mathcal{C} \mathcal{E}_i^{-1} \mathcal{M}_i + \mathcal{N}_i.$$

The form (5) will be very convenient in writing our calculations and getting easier to program algorithms. It will not be the most efficient computationally, however, since it does not exploit the structure of the matrices involved.

Now, based on our earlier discussion that β turned out to be $\frac{1}{2}$ in the previous incipient problems, in this paper we shall use a combined noise bound of

$$\|\mu\|^2 = (\mu^0)^T \mu^0 + (\mu^1)^T \mu^1 < 1, \tag{6}$$

which is the sum of the two bounds (2). Note this is just twice what $\beta = \frac{1}{2}$ gives. Thus the effect is just a scaling of the test signals by $\frac{1}{2}$. Note also that

$$\begin{aligned} \max\{(\mu^0)^T \mu^0, (\mu^1)^T \mu^1\} &\le (\mu^0)^T \mu^0 + (\mu^1)^T \mu^1 \\ &\le 2 \max\{(\mu^0)^T \mu^0, (\mu^1)^T \mu^1\} \end{aligned} \tag{7}$$

so there is a relationship between the two ways of measuring the total noise even if β is not being taken equal to 1/2. Namely if we find a proper test signal for (6), then for the additive noise case $1/\sqrt{2}$ times it will be proper for the original problem as shown in [3].

Now we consider characterizing v being proper. That is, if the outputs are the same, then too much noise is required. So suppose that it is possible to get the same output from both models, that is, $y^0 = y^1$. Using (5) this says

$$\begin{aligned}(X_1 - X_0)v &= H_1\mu^1 - H_0\mu^0 \\ &= [-H_0,\ H_1]\begin{pmatrix}\mu^0\\ \mu^1\end{pmatrix} = \overline{H}\mu. \end{aligned} \tag{8}$$

We assume that $[-H_0, H_1]$ is full row rank. This is reasonable since it means that all the output channels are noisy. If v is proper in this case, then it will be proper if only some channels are noisy. Now v is proper if (8) implies that $\|\mu\|^2 \geq 1$ for all μ for which (8) holds. Thus it suffices to look at the minimum such μ and see if it is greater than or equal to one in norm. But for a fixed v the minimum norm μ is given by

$$\overline{H}^{\dagger}(X_1 - X_0)v, \tag{9}$$

where $\overline{H}^{\dagger}$ is the minimum norm least squares inverse of $\overline{H}$ (also called the Moore-Penrose generalized inverse of $\overline{H}$ [1]). If $\overline{H}$ is full row rank, then the singular value decomposition (SVD) of $\overline{H}$ is

$$U[\Sigma \quad 0]V$$

where U, V are orthogonal and Σ is a positive definite diagonal matrix. Then

$$\overline{H}^{\dagger} = V^T\begin{pmatrix}\Sigma^{-1}\\ 0\end{pmatrix}U^T. \tag{10}$$

One could also use

$$\overline{H}^{\dagger} = (\overline{H}\overline{H}^T)^{-1}\overline{H}, \tag{11}$$

but (10) is better for the computations that follow than (11) if $\overline{H}$ is not well conditioned.

If $\overline{H}^{\dagger}(X_1 - X_0)v$ is not greater than or equal to one in norm, then v is not proper. Now think of picking v. It needs to make (9) have norm at least one. Take the singular value decomposition of $\overline{H}^{\dagger}(X_1 - X_0)$. Let σ be the largest singular value of $\overline{H}^{\dagger}(X_1 - X_0)$ and let γ be a normalized ($\|\gamma\| = 1$) right singular vector that goes with the singular value σ. Then if we take $v^* = \sigma^{-1}\gamma$ we will have that $\overline{H}^{\dagger}(X_1 - X_0)v^*$ has norm one and v^* is proper. In fact, v^* is the smallest test signal that works with the given noise bounds (6). Having summarized the one fault case we turn to the incipient case where there is more than one fault.

III. Several Incipient Faults

For the incipient problem, we start with the normal problem

$$\begin{aligned} x_{k+1} &= A(\theta)x_k + B(\theta)v_k + M\mu_k, \\ & \qquad k \in [0 : K-1] \end{aligned} \tag{12a}$$

$$y_k = C(\theta)x_k + N\mu_k,\ k \in [0 : K]. \tag{12b}$$

where $\theta = \left(\theta_1, \ldots \theta_r\right)^T$ is a vector of r parameters. Thus we allow r simultaneous faults. Let $\delta\theta = \left(\delta\theta_1, \ldots, \delta\theta_r\right)^T$ be the vector of parameter variations. Model (12) with parameter drift $\delta\theta$ is

$$\begin{aligned} \hat{x}_{k+1} &= A(\theta + \delta\theta)\hat{x}_k + B(\theta + \delta\theta)v_k + M\hat{\mu}_k, \\ & \qquad k \in [0 : K-1] \end{aligned} \tag{13a}$$

$$\hat{y}_k = C(\theta + \delta\theta)\hat{x}_k + N\hat{\mu}_k, k \in [0 : K]. \tag{13b}$$

Again using the notation of Section II we can rewrite (12) and (13) in the static formulation.

$$y^0 = X(\theta)v + H(\theta)\mu^0 \tag{14}$$

$$y^1 = X(\theta + \delta\theta)v + H(\theta + \delta\theta)\mu^1. \tag{15}$$

Our goal is to determine the shape of a good test signal independent of the value of $\delta\theta$ and then be able to scale the test signal depending on the threshold with which we wished to guarantee detection of the fault. We again assume (2) and (6). Setting $y^1 = y^0$ we get

$$0 = X(\theta + \delta\theta)v + H(\theta + \delta\theta)\mu^1 - X(\theta)v - H(\theta)\mu^0$$

or

$$0 = \sum_{j=1}^{r} X_{\theta_j}(\theta)\delta\theta_j v + H(\theta)\mu^1 - H(\theta)\mu^0 + O(\delta\theta),$$

where $X_{\theta_j} = dX/d\theta_j$.

Dropping the O terms except for those with v, we get after a little calculation that

$$\sum_{j=1}^{r} X_{\theta_j}(\theta)\delta\theta_j v = H(\theta)\mu^1 - H(\theta)\mu^0. \tag{16}$$

We could let $w_j = \delta\theta_j v$ and we would get a problem with r vector valued controls in it. But this is deceptive since all the w_j are multiples of each other and that fact is getting lost.

We address this by using a formulation that lets us use the results from the one parameter incipient case as a subproblem. Since we know that the w_j are multiples of the same test signal, we suppose that

$$\delta\theta_j = \alpha_j\delta\kappa, \quad j = 1, \ldots, r. \tag{17}$$

Next, we assume that $\alpha = (\alpha_1, \ldots, \alpha_r)$ lies in some closed bounded subset Ω of R^r for a fixed $\delta\kappa$. For detection to be possible, we need that $0 \notin \Omega$. Then the equation (16) for the smallest μ that results in equal outputs for a given α becomes

$$\overline{H}^{\dagger}\left(\sum_{j=1}^{r}\alpha_j X_{\theta_j}(\theta)\right)w = \mu, \tag{18}$$

where $w = \delta\kappa v$ and $\overline{H} = [H(\theta),\ -H(\theta)]$.

We want to find a w that works for all $\alpha \in \Omega$. One way to proceed is this. Fix $\|w\|$=1. Then we minimize the norm of $\overline{H}^{\dagger}(\sum_{j=1}^{r}\alpha_j X_{\theta_j}(\theta))w$ over $\alpha \in \Omega$. That is, we find that fault condtion that w is worst for. We then maximize the worst case over the w to do the best possible. We may then need to adjust the size. This leads to the following expression.

Let α^*, $\bar{w}^*$ be the solutions of

$$\zeta = \max_{\|w\|=1} \min_{\alpha \in \Omega} \|\overline{H}^\dagger(\sum_{j=1}^r \alpha_j X_{\theta_j}(\theta))w\|. \quad (19)$$

Then the test signal shape would be $\frac{1}{\zeta}\bar{w}^*$

There are a variety of different choices to make for Ω depending on the particular application.

One option is to suppose that $\sum_{j=1}^r \delta\theta_j = \delta\kappa$. This says that we are taking a certain level of perturbation but not worrying about what variable causes it. But then

$$\sum_{j=1}^r \alpha_j = 1. \quad (20)$$

Another natural possibility is when

$$\underline{\alpha}_j \leq \alpha_j \leq \overline{\alpha}_j \quad (21)$$

and at least one interval $[\underline{\alpha}_j,\ \overline{\alpha}_j]$ does not contain zero. This formulation is attractive for several reasons. For one, it leads to box constraints which many optimizer packages are build to consider. Secondly it has a natural interpretation in terms of each parameter value falling into a certain interval.

A. *Solving* (19)

At first glance the optimization problem (19) can appear difficult to solve. In fact, it can be rapidly solved for small to medium sized problems as we shall now explain.

We consider the inner minimization problem first. Suppose that we have a given w such that $\|w\| = 1$. Let $b_j = \overline{H}^\dagger X_{\theta_j}(\theta)w$. Then the square of the norm inside (19) is

$$\|\sum_{j=1}^r \alpha_j b_j\|^2 = \sum_{j=1}^r \sum_{i=1}^r c_{i,j}\alpha_i\alpha_j, \quad (22)$$

where $c_{i,i} = b_i^T b_j$. This is a positive semi-definite quadratic form over Ω. There are a number of fast routines for minimizing quadratic functions. If the region Ω is given by (21) or by (20) with $r = 2$, then we get the minimization of a quadratic form over box constraints.

The inner optimization is thus of a smooth function. For the outer optimization in (19) the cost may not be smooth and there is also the constraint $\|w\| = 1$. One could just use this constraint but many optimization packages prefer simpler box type constraints. Suppose that w is n-dimensional. Using the usual parameterization of the n-sphere we can replace $\|w\| = 1$ by box constraints. The parameterization is

$$\begin{aligned}
w_1 &= \cos(\gamma_1) && (23a)\\
w_2 &= \sin(\gamma_1)\cos(\gamma_2) && (23b)\\
w_3 &= \sin(\gamma_1)\sin(\gamma_2)\cos(\gamma_3) && (23c)\\
&\vdots && (23d)\\
w_{n-1} &= \sin(\gamma_1)\cdots\sin(\gamma_{n-2})\cos(\gamma_{n-1}) && (23e)\\
w_n &= \sin(\gamma_1)\cdots\sin(\gamma_{n-1}), && (23f)
\end{aligned}$$

where $0 \leq \gamma_i \leq \pi$ for $i \in [1, n-2]$ and $0 \leq \gamma_{n-1} \leq 2\pi$. Note that there is symmetry to the optimization problem so that if w is optimal, then so is $-w$. Thus we can reduce the size of the optimization domain by taking

$$0 \leq \gamma_i \leq \pi \text{ for } i \in [1 : n-1] \quad (24)$$

as parameterizing w. This is true in a mathematical sense but note the comments on Example 2.

The inner optimization problem is thus over an $r-1$ dimensional box where r is the number of incipient faults. Typically r is not a large integer. The outer optimization is over K variables. The static formulation is most practical for small to moderate sized values of K and the state dimension.

IV. Using the Test Signal

Suppose that we have our test signal v and an observed output $\tilde{y}$. To use the test signal we compute the norm squared of the smallest noise that makes $\tilde{y}$ consistent with model zero. That is

$$\rho = \|H_0^\dagger(\theta)(\tilde{y} - X_0(\theta)v)\|^2. \quad (25)$$

If $\rho < \frac{1}{2}$, then we conclude there is no fault. If $\rho > \frac{1}{2}$, then we conclude there is an incipient fault. We can do this since the noise in both models was assumed to be bounded by (2) and v was designed to produce disjoint output sets.

If the model equations are affine in $\delta\theta$, then there is guaranteed fault detection. The only conservatism is in perhaps using a slightly larger test signal then is minimally necessary due to our use of the one norm for the combined norm.

V. Two Examples

If the $\delta\theta$ enter in a nonlinear manner then there is an approximation whose error could possibly increase with the size of the parameter change. This is similar to the one incipient fault case. Note that the we are seeking to detect incipient faults early when they are still small. We solve both the inner and outer optimization problems using the MATLAB command fmincon.

1) Example 1: As a simple academic example we consider

$$y = \begin{pmatrix} 2+\theta_1 & -1+3\theta_2 \\ 2 & 1+\theta_3 \end{pmatrix} v + H.\mu \quad (26)$$

We take the nominal value to be $\theta = 0$ and $H = I$. Using the notation of (21) we assume Ω is of the form

$$\begin{aligned}
0.5 &\leq \alpha_1 \leq 1.0 && (27a)\\
0.1 &\leq \alpha_2 \leq 0.5 && (27b)\\
0 &\leq \alpha_3 \leq 0.1. && (27c)
\end{aligned}$$

We know that if w is an optimum, then so is $-w$ so it suffices to paramterize half the w sphere. We have then, using the sphere parameterization (23) that

$$q(w,\alpha) = \begin{pmatrix} \frac{1}{\sqrt{2}}I \\ -\frac{1}{\sqrt{2}}I \end{pmatrix} \begin{pmatrix} \alpha_1 & 3\alpha_2 \\ 0 & \alpha_3 \end{pmatrix} \begin{pmatrix} \cos(\gamma) \\ \sin(\gamma) \end{pmatrix}, \quad (28)$$

where $0 \leq \gamma \leq \pi$ and we solve

$$-\min_{0\leq\psi\leq\pi} -\min_{\alpha} \|q(w,\alpha)\|^2. \quad (29)$$

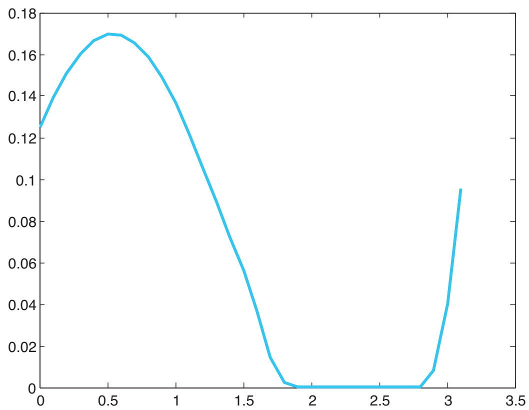

Fig. 1. Inner min as a function of γ.

Figure 1 graphs the inner min as a function of γ. Note that it is highly asymmetrical unlike the examples in [8].

The outer optimization gives the maximum of γ as 0.17 at $x = 0.5407$. Figure 2 gives the incipient test signal in red. The black vectors are the two model test signals for various values of $\alpha \in \Omega$. That is, they are the optimal test signal for the nominal model and one specific parameter variation.

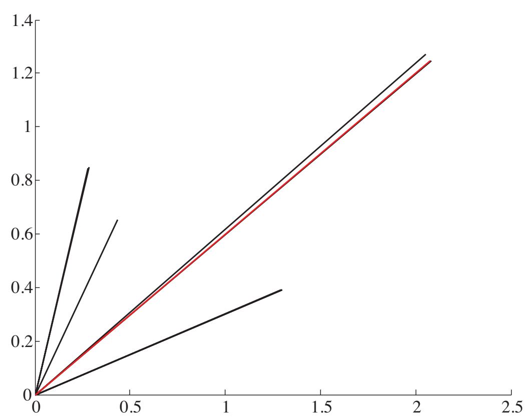

Fig. 2. Incipient test signal (red) and two model signals for several values of α (black).

2) Example 2: To illustrate the use of this approach on a problem which is in the form of (1) and has the parameters entering nonlinearly, we consider the discrete system

$$x_{k+1} = \begin{pmatrix} 1 & h \\ -hsm & -hmd \end{pmatrix} x_k + \begin{pmatrix} 0 \\ hm \end{pmatrix} u + M\mu_k, \quad k = 0\ldots, K-1 \tag{30a}$$

$$y_k = \begin{pmatrix} 0 & 1 \end{pmatrix} x_k + N\mu_k, \quad k = 0\ldots, K. \tag{30b}$$

This problem is the Euler approximation of a mechanical system with damping coefficient d, spring constant s, and mass $\frac{1}{m}$. h is the step size. The output is the position of the mass. Note that the parameters enter nonlinearly.

We assume that the normal values are $\theta_1 = m = 2$, $\theta_2 = s = 12$, $\theta_3 = d = 0.2$. This is a lightly damped mechanical system with eigenvalues $0.4600 \pm 0.8176i$ with modulus 0.9381. It is lightly damped since $0.9381 < 1$. We assume that we want a test signal that is guaranteed to determine a fault has occured where the fault condition is $\theta_1 + \delta\theta_1 \in [0.9,\ 1.1]$, $\theta_2 + \delta\theta_2 \in [8, 10]$ and $\theta_3 + \delta\theta_3 \in [0.4, 0.6]$. Thus there may have been change or uncertainty in the mass, but the spring has weakened and friction has increased due to say a break down in lubricant. Then $\delta\theta_1 \in [-1.1,\ -0.9]$, $\delta\theta_2 \in [-4, -2]$ and $\delta\theta_3 \in [0.2, 0.4]$. We can take $\delta\kappa = 1$ and then $\alpha_i = \delta\theta_i$. For this example we take $h = 0.2$ and $K = 5$.

Our set Ω is now a rectangular box that does not include the origin.

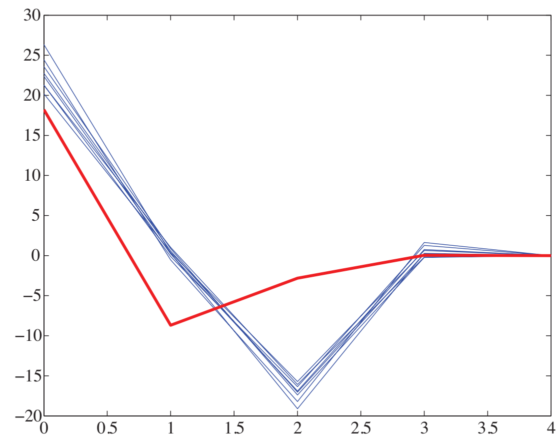

Fig. 3. Incipient test signal (red) and two model signals for several values of α (black).

With these parameter values we get

$$v_{\text{inc}} = \begin{pmatrix} 18.2136 \\ -8.6901 \\ -2.8236 \\ 0.0485 \\ 0 \end{pmatrix}.$$

Figure 3 shows the incipient test signal in red and the two model test signals for the values of α at the corners of Ω. Since the noise is symmetric about the origin we have that $-v$ is proper if v is. To better compare the incipient and two model test signals we have plotted the negative of the incipient test signal in Figure 3. It should be noted that the parameterization (23) can create artificial local minimum when using a numerical optimization code. For example, the parameterization has $0 \leq \gamma_1 \leq \pi$. However, if the actual optimum appears at say $-\frac{1}{6}\pi$ and $\frac{5}{6}\pi$, and we use $0 \leq \gamma_1 \leq \pi$, then depending on the initial value, the optimizer might go to $\gamma_1 = 0$. In fact, this happens with this particular example. This was easily overcome by giving wider bounds on the γ_i when the optimizer is called.

VI. Conclusion

When considering incipient faults the usual assumption that there is only one fault happening at a given time is no longer always appropriate. In this paper we have begun the examination of an active approach for incipient fault detection when there are more than two faults in the discrete time case. As with some of the earlier results we specify a threshold and guarantee detection if some combination of the uncertainties exceeds that threshold. The problem to be solved is a $\max\min$ problem. In principle this can be solved by software. However, more efficient algorithms are needed for higher state dimensions and longer time horizons. Investigations into developing these more efficient algorithms and also into extending the algorithms to the continuous time case are under investigation.

Section II of this paper used additive uncertainty. In the previous work based on [2], finding a test signal in the presence of model uncertainty was carried out with a reformulation that lead to a new optimization problem which added the uncertainty to the noise. The approach of this paper has provided an alternative approach to including some types of model uncertainty when the faults are incipient. In this use of the ideas of this paper, some of the parameters are treated as faults and some of them as model uncertainties. The difference is that if the jth parameter is uncertain, then we have $0 \in [\underline{\alpha}_j,\ \overline{\alpha}_j]$ and if the jth parameter is incipient, then $0 \notin [\underline{\alpha}_j,\ \overline{\alpha}_j]$. This approach to test signal design in the presence of model uncertainty is a topic for further study.

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