

Optimal Cyclic Multiple Hoist Scheduling for Processes with Loops and Parallel Resources

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Abstract—This paper addresses the cyclic multi-hoist scheduling problem. The problem arises in automated electroplating lines where multiple hoists are operating on a shared track. The considered processing sequences may be different from the location sequence of the tanks and may contain loops, i.e. multiple usages of single tanks within a sequence. In addition the electroplating line may be equipped with identical tanks used as parallel resources. A mixed integer linear programming (MILP) formulation for optimizing the cycle time is developed considering in particular collisions of hoists, loops in the process sequence and parallel tanks. The developed model is tested and evaluated using a real world example indicating that the presented solution finds optimal cyclic schedules in reasonable time.

Keywords—Cyclic Multi-Hoist Scheduling, Parallel Resources and Process Loops, Mixed Integer Linear Programming

I. PROBLEM STATEMENT

In this paper the problem of Hoist Scheduling is addressed. In the production process of discrete semiconductors and passive components automated electroplating lines are used. The components are treated in chemical tanks, e.g. to deposit copper, tin, etc., which are passed through successively. The transport between the tanks is done by an automated hoist transport system as depicted in Fig. 1. For each tank the processing times have to be within a lower and an upper limit. The hoist moving times between the tanks are fixed depending on the position of the tanks. In addition the pickup and the lowering time of the carrier have to be considered. For the transport and the processing no preemption is allowed, there are no buffers and the transport between the tanks is done directly, i.e. with a no-wait scheme. Since the facility is used for different products the processing sequence for a single product and the tank arrangement may be different. Some of the tanks, e.g. for rinsing the components, are used more than once within the processing sequence causing loops in the processing sequence. Often the transport system becomes the bottleneck of the electroplating line if more than one carrier is processed at the same time. To overcome this problem so-called multi-hoist configuration is used, where a set of hoists operate on a shared track in both directions to perform the transport of the carriers. In such a configuration the problem of

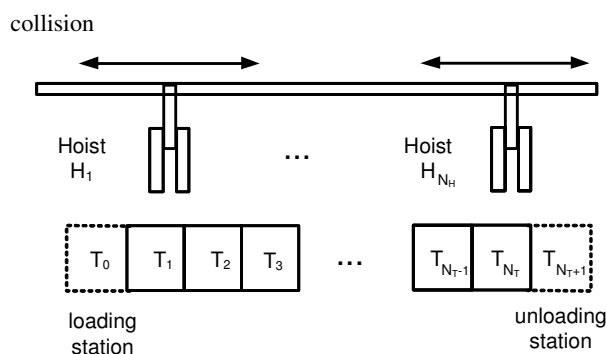


Figure 1. Galvanic process with multiple hoist

avoidance arises and needs to be inspected. In some facilities parallel process tanks may be used to improve the performance, i.e. to widen the bottleneck of intensively used tanks. These tanks can then be used alternately by successive processes. The goal is to find a transport sequence with a minimal cycle time which maximizes the throughput.

II. STATE OF THE ART

The studied problem of Hoist Scheduling using a mixed integer linear programming formulation is considered since the 1970s, when Phillips and Unger [1] gave one of the first mathematical formulations for minimizing the cycle time for a one hoist scheduling problem. Since then, the issue of optimal cyclic hoist scheduling has received a lot of attention and consistently has been examined over the past decades. The problem belongs to the class of NP-hard problems [8], although some polynomial time algorithms have been developed for special cases, e.g. for a known hoist move sequence or fixed processing times.

Since the transport system often limits the capability of an electroplating line multiple hoist configurations emerged and were studied in the research as well. For a two hoist problem a “zoned” model was taken in [9] separating the line into parts on which the hoists can operate. Constraint logic programming or hybrid approaches have also been applied on the multi-hoist scheduling problem [5], combining the advantages of both modeling methods. Investigation of a general transport

sequence and the resulting constraints for a multi-hoist problem can be found in [3]. Recently Petri net based approaches have been applied successfully for optimal cyclic multi-hoist scheduling [6], investigating process sequences with loops and parallel tank usage.

III. ASSUMPTIONS AND NOTATIONS

The electroplating facility shall be equipped with N_T processing tanks, the loading and unloading station is tank 0 resp. tank $N_T + 1$ as shown in Fig 1. N_H identical hoists operate with constant speed on a single track and cannot pass each other. The move time between two tanks i and j shall be denoted by $e_{i,j}$. The loading and unloading of the carriers consume a constant time T_i^L and T_{i+1}^U , as depicted in Fig. 2.a, which are added to the move time $e_{i,i+1}$ to obtain the duration of the transport. The process sequence consists of N_p process steps. The corresponding tank for the i^{th} step is contained in P_i with $i \in \{0, \dots, N_p + 1\}$. The upper and lower soak time limits for step i are represented by U_i and L_i . The following constants are used:

- N_T , number of tanks
- N_H , number of hoists
- N_p , number of process step
- $e_{i,j}$, empty hoist travel time
- T_i^U , unloading time
- T_i^L , loading time
- P_i , number of tank of process step i
- n_i , number of tank for process step i
- d_i , hoist travel time for transport from P_i to P_{i+1}
- L_i , lower limit for soak time of process step i
- U_i , upper limit for soak time of process step i
- M , a number larger than the cycle time

Decision variables are:

- C = cycle time
- t_i = start time of transport i
- $y_{ij} = \begin{cases} 1 & \text{if transport } j \text{ starts after move } i \\ 0 & \text{otherwise} \end{cases}$
- $z_i^k = \begin{cases} 1 & \text{if transport } i \text{ is done by hoist } k \\ 0 & \text{otherwise} \end{cases}$
- $s_i = \begin{cases} 1 & \text{if process step } i \text{ or the corresponding} \\ & \text{transport exceeds the period} \\ 0 & \text{otherwise} \end{cases}$

which define the cycle time and the transport start times t_i . The cycle time defines the period of the scheduled process. Besides that, the order and the allocation of the transports and whether a process exceeds a period and ends in the consecutive cycle or not are specified. These variables describe the optimization model in the next section to define.

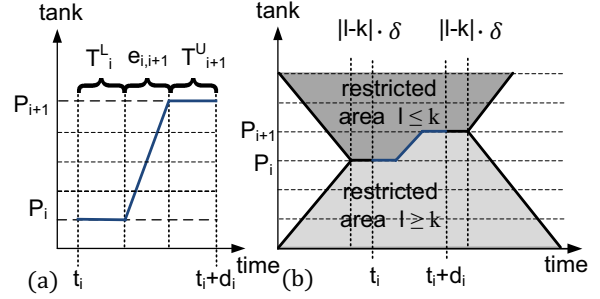


Figure 2. Transporter move model (a) and restricted area model (b)

IV. MILP MODEL FOR CYCLIC MULTI-HOIST SCHEDULING WITH LOOPS AND MULTI TANKS

For the task of allocation and sequencing all transports for the complete process while minimizing the cycle time C :

$$\min C \quad (1)$$

a mixed integer linear program shall be developed. In the next sections the necessary constraints are described in detail.

A. General Assumptions

Without loss of generality the start time of transport 0 can be set to zero. A lower bound for the cycle time can be either set by the last transport or the last transport plus the move time to the input tank, if the first transport is done by the considered hoist:

$$t_0 = 0 \quad (2)$$

$$t_i + (d_i + e_{i+1,0})z_i^1 \leq C, \forall i \in \{1..N_p\} \quad (3)$$

$$d_0 + e_{1,0} \leq C \quad (4)$$

B. Process Constraints

The soak time for each tank has to be within the specified limits. The soak time is calculated by the difference of the consecutive transport times either regarding a cycle time overstep eqns. (7)+(8) or not eqns. (5)+(6):

$$t_i - (t_{i-1} + d_{i-1}) \leq U_i, \forall i \in \{1..N_p\} \quad (5)$$

$$t_i - (t_{i-1} + d_{i-1}) + Ms_i \geq L_i, \forall i \in \{1..N_p\} \quad (6)$$

$$t_i + C - (t_{i-1} + d_{i-1}) - M \cdot (1 - s_i) \leq U_i, \forall i \in \{1..N_p\} \quad (7)$$

$$t_i + C - (t_{i-1} + d_{i-1}) \geq L_i, \forall i \in \{1..N_p\} \quad (8)$$

C. Move Ordering Constraints

For determining the ordering decision variables the constraints (9) and (10) are used, while (11) to (13) assure that all transports are allocated to one hoist only and the first resp. the last hoist perform the last transport of the process sequence.

$$t_j - t_i \leq M \cdot y_{ij}, \quad \forall i, j \in \{1..N_p\}, i \neq j \quad (9)$$

$$y_{ij} + y_{ji} = 1, \quad \forall i, j \in \{1..N_p\}, i \neq j \quad (10)$$

$$\sum_{k=1}^{N_H} z_i^k = 1, \forall i \in \{1..N_p\} \quad (11)$$

$$\text{if } P_{N_p+1} = 0: \quad z_{N_p}^1 = 1 \quad (12)$$

$$\text{if } P_{N_p+1} = N_B + 1: \quad z_{N_p}^{N_H} = 1 \quad (13)$$

D. Transport Feasibility Constraints

To assure that every hoist can perform the allocated transport the positions of the neighbored hoists have to be considered. The situation is depicted in Fig. 2b. Assuming all hoist have the same width, a constant δ is introduced which is used to define the time a hoist needs to travel the width of a hoist. Eqns. (6), (7) and (14) define the decision variable s_i , which provide the information whether the process step i exceeds the period or not. Additional eqn. (14) assures a time difference of at least δ between the removal and the lowering of the next process in the same tank:

$$t_i + T_i^L + \delta \leq t_{i-1} + T_{i-1}^L + e_{i-1,i} + M(1-s_i), \forall i \in \{1..N_p\} \quad (14)$$

$$(t_0 + d_0 + e_{1,j}) \cdot z_j^1 \leq t_j, \quad \forall j \in \{1..N_p\} \quad (15)$$

Since the first move is always performed by the first hoist, constraint (15) assures that there is enough time to reach every other transport performed by the first hoist. The next five constraints assure that the move to a required transport is possible for the allocated hoist, so that the hoists are able to perform the transports. The situation is depicted in Fig. 2.b. While a transport from P_i to P_{i+1} is performed by hoist k the correctness for all moves and transports performed by hoist l are checked. If transport i is performed by hoist k , i.e. $z_i^k = 1$; transport i starts before transport j , i.e. $y_{ij} = 1$, transport j is performed by hoist l with $l \geq k$ and $P_{i+1} \geq P_j$ eqn. (16) assures that there is enough time between the end of transport i and the beginning of transport j to perform the transport without collision. Note that the sum on the left hand side assures that there is no collision for all hoists between k and l . Eqn. (17) checks the same for the case that $k \geq l$ and $P_{i+1} \leq P_j$.

$$\forall k \in \{1..N_H\}, \forall i, j \in \{1..N_p\}, i \neq j: \\ \text{if } P_{i+1} \geq P_j: \quad (16)$$

$$t_i + d_i + e_{i+1,j} + \sum_{l=k}^{N_H} ((l-k)\delta z_j^l) \leq t_j + M(3 - y_{ij} - z_i^k - \sum_{l=k}^{N_H} z_j^l) \\ \text{if } P_{i+1} \leq P_j: \quad (17)$$

$$t_i + d_i + e_{i+1,j} + \sum_{l=1}^k ((k-l)\delta z_j^l) \leq t_j + M(3 - y_{ij} - z_i^k - \sum_{l=1}^k z_j^l) \\ \text{if } P_{i+1} \geq P_j: \quad (18)$$

$$t_i + d_i + e_{i+1,j} + \sum_{l=k}^{N_H} ((l-k)\delta z_j^l) \leq C + t_j + M(2 - z_i^k - \sum_{l=k}^{N_H} z_j^l) \\ \text{if } P_{i+1} \leq P_j: \quad (19)$$

$$t_i + d_i + e_{i+1,j} + \sum_{l=1}^k ((k-l)\delta z_j^l) \leq C + t_j + M(2 - z_i^k - \sum_{l=1}^k z_j^l) \\ \text{if } P_i \geq P_j \quad (20)$$

$$t_0 + d_0 + e_{1,j} + \sum_{l=1}^{N_H} ((l-1)\delta z_j^l) \leq t_j$$

The presented eqns. (16) and (18) resp. (17) and (19) verify the same situation but shifted by the period C . Thus the

equations check the limits shown by the lines in Fig. 2b and define a forbidden area for all hoists $l \geq k$ resp. $l \leq k$. Note that eqn. (20) assures in addition that the transport from tank 0 does not constrict any transport j .

E. Multiple Tank Usage Constraints

In Fig. 3 the effect of a loop within the processing sequence is shown in the Gantt chart for a tank used in P_i and P_j as well. The constraints must make sure that the tank is used exclusively, i.e. there is no overlap of the two process steps. The three possible constellations are formulated in eqns. (21) to (25) and are predetermined by the combination of the different values of s_i and s_j . These describe if the process step and the corresponding transport are covered completely by the period or if it exceed the period.

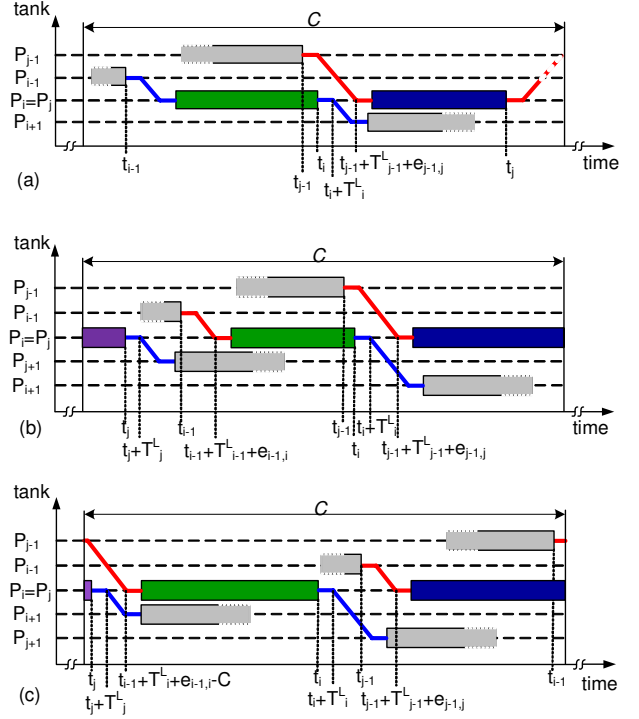


Figure 3. Resource constraint resulting from multiple tank use

Equation (21) covers the case shown in Fig. 3a. Here both of the considered processes start and end within the period. Between the removal of a process i and the arrival of a process j , at least the already known time lag δ must be kept.

$$\forall i, j \in \{1..N_p\}, i \neq j; \quad \text{if } P_i = P_j: \\ t_i + T_i^L + \delta \leq t_{j-1} + T_{j-1}^L + e_{j-1,j} + M(3 - (1-s_i) - (1-s_j) - y_{ij}) \quad (21)$$

$$t_j + T_j^L + \delta \leq t_{i-1} + T_{i-1}^L + e_{i-1,i} + M(2 - (1-s_i) - s_j) \quad (22)$$

$$t_i + T_i^L + \delta \leq t_{j-1} + T_{j-1}^L + e_{j-1,j} + M(2 - (1-s_i) - s_j) \quad (23)$$

$$t_j + T_j^L + \delta \leq t_{i-1} + T_{i-1}^L + e_{i-1,i} - C + M(3 - s_i - s_j - y_{ji}) \quad (24)$$

$$t_i + T_i^L + \delta \leq t_{j-1} + T_{j-1}^L + e_{j-1,j} + M(3 - s_i - s_j - y_{ji}) \quad (25)$$

As can be seen in Fig. 3b for this case the start and the end of process i need to be checked with process j. Note that due to the cyclic scheduling process j is shown twice, first the original process and second the one shifted by C. Again the time lag needs to be preserved, and this is done by constraint (22) and (23). The third case is shown in Fig. 3c and covered in the same way by eqns. (24) and (25).

F. Collision Avoidance for Multi Directional Transports

The assumed general process sequence requires the examination of the collisions among the hoists during the transport of the carriers. The eight scenarios displayed in Fig. 5 result from the different positions of the start and end tank for the two transports and the distinction if the shared rail interval overlaps partly (e,f,g,h) or fully (a,b,c,d). To check possible collisions it is necessary to distinguish if a transport takes place before another transport, shown by the dotted lines, or after it, shown by the solid lines. This is done with

$$x_{ij}^\lambda = \begin{cases} 1 & \text{if transport i takes place before j} \\ 0 & \text{otherwise} \end{cases} \quad \forall i, j \in \{0..N_p\}, i \neq j, \lambda \in \{1, 2, 3\}$$

which have to fulfill:

$$x_{ij}^1 + x_{ji}^1 = 1 \quad (26)$$

$$x_{ij}^2 + x_{ji}^2 = 1 \quad (27)$$

$$x_{ij}^3 + x_{ji}^3 = 1 \quad (28)$$

If both transports start within a period, x_{ij}^1 is used to determine the order, if transport i starts in the consecutive cycle after transport j x_{ij}^2 and the other way around, x_{ij}^3 is used. For a more detailed view consider Fig. 4, there scenario d is shown in more detail.

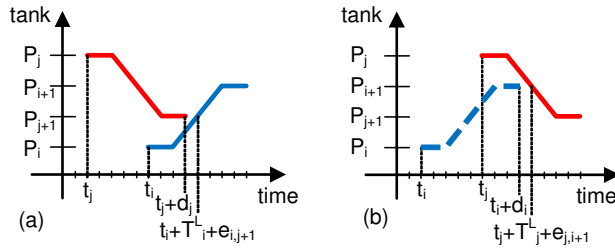


Figure 4. Collision scenario d

The collision scenario shown in Fig. 4a is checked by eqn. (29) and scenario of Fig. 4b is checked by eqn. (30):

$$\forall i, j \in \{0..N_p\}, i \neq j: \\ \text{if } P_i < P_{j+1} \leq P_{i+1} < P_j \\ t_j + d_j + \delta \leq t_i + T_i^L + e_{i,j+1} + Mx_{ij}^1 \quad (29)$$

$$t_i + d_i + \delta \leq t_j + T_j^L + e_{j,i+1} + Mx_{ji}^1 \quad (30)$$

$$t_j + d_j + \delta \leq t_i + C + T_i^L + e_{i,j+1} + Mx_{ij}^2 \quad (31)$$

$$t_i + C + d_i + \delta \leq t_j + T_j^L + e_{j,i+1} + Mx_{ji}^2 \quad (32)$$

$$t_j + C + d_j + \delta \leq t_i + T_i^L + e_{i,j+1} + Mx_{ij}^3 \quad (33)$$

$$t_i + d_i + \delta \leq t_j + C + T_j^L + e_{j,i+1} + Mx_{ji}^3 \quad (34)$$

Since there is the possibility that between t_i and t_j a new cycle begins, the collision scenario has to be checked in consecutive cycles. This is done by eqns. (31) and (32) in case that t_i

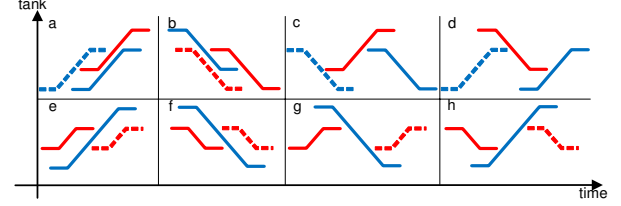


Figure 5. Collision scenarios a to h

belongs to the consecutive cycle and by eqns. (33) and (34) in case that t_j belongs to the consecutive cycle. For scenario a and b eqns. (35) to (39) are used in an analog way:

$$\forall i, j \in \{0..N_p\}, i \neq j:$$

$$\text{if } P_i < P_j < P_{i+1} < P_{j+1} \text{ or } P_{j+1} < P_{i+1} < P_j < P_i$$

$$t_j + T_j^L + \delta \leq t_i + T_i^L + e_{i,j} + Mx_{ij}^1 \quad (35)$$

$$t_i + T_i^L + e_{i,j} + \delta \leq t_j + Mx_{ji}^1 \quad (36)$$

$$t_j + T_j^L + \delta \leq t_i + C + T_i^L + e_{i,j} \quad (37)$$

$$t_j + C + T_j^L + \delta \leq t_i + T_i^L + e_{i,j} + Mx_{ij}^3 \quad (38)$$

$$t_i + T_i^L + e_{i,j} + \delta \leq t_j + C + Mx_{ji}^3 \quad (39)$$

Note that for the scenarios a and b eqn. (37) is already a simplified version of the equivalent eqns. (31) and (32). This simplification results from the fact that some constellations are not possible, and x_{ij}^2 is therefore not needed. For scenario c a similar simplification can be made:

$$\forall i, j \in \{0..N_p\}, i \neq j:$$

$$\text{if } P_{i+1} < P_j \leq P_i < P_{j+1}$$

$$t_j + T_j^L + e_{j,i} + \delta \leq t_i + Mx_{ij}^1 \quad (40)$$

$$t_i + T_i^L + e_{i,j} + \delta \leq t_j + Mx_{ji}^1 \quad (41)$$

$$t_j + T_j^L + e_{j,i} + \delta \leq t_i + C \quad (42)$$

$$t_i + T_i^L + e_{i,j} + \delta \leq t_j + C \quad (43)$$

Finally for the partly overlapped intervals, i.e. for scenarios e to h, eqns. (44) to (48) prevent the hoists from a collision during a transport:

$$\forall i, j \in \{0..N_p\}, i \neq j:$$

$$\text{if } P_i < P_j < P_{j+1} < P_{i+1}$$

$$\text{or } P_{i+1} < P_{j+1} < P_j < P_i \text{ or } P_{i+1} \leq P_j < P_{j+1} \leq P_i$$

$$\text{or } P_i \leq P_{j+1} < P_j \leq P_{i+1}$$

$$t_j + d_j + \delta \leq t_i + T_i^L + e_{i,j+1} + Mx_{ij}^1 \quad (44)$$

$$t_i + T_i^L + e_{i,j} + \delta \leq t_j + Mx_{ji}^1 \quad (45)$$

$$t_j + d_j + \delta \leq t_i + C + T_i^L + e_{i,j+1} \quad (46)$$

$$t_j + C + d_j + \delta \leq t_i + T_i^L + e_{i,j+1} + Mx_{ij}^3 \quad (47)$$

$$t_i + T_i^L + e_{i,j} + \delta \leq t_j + C + Mx_{ji}^3 \quad (48)$$

The presented eqns. (26) to (48) assure that there is no collision between the hoists during the transports.

G. Multiple Processing Tanks

If a process step takes much longer processing time than all the others, the tank in question might become the bottleneck which determines the cycle time C . A way to cope with that is to multiply the bottleneck resource. The presented model so far is not capable to handle such a process step but can be adapted to the use of multiple tanks within a process sequence. Assuming that there exist more than one tank in the electroplating line used for process step P_i and these tanks are directly neighbored and identical except for the location as depicted in Fig. 6, the tanks can be used alternately in consecutive cycles. The processing time for a single tank then exceeds the cycle time C .

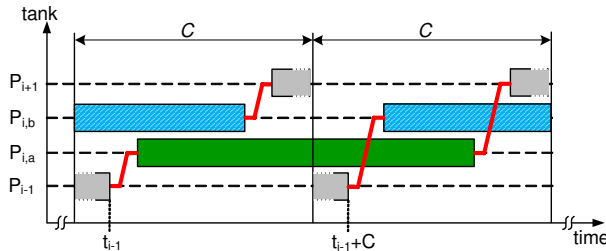


Figure 6. Parallel tanks in consecutive cycles

The idea of alternately loading the tanks is represented by introducing the multiplicity constant n_i which represents the number of tanks used for process step P_i . The soak time eqns. (5) to (8) need to be adapted. As can be seen in Fig. 6 the soak time for the two tank case is extended by C , by $2C$ for a tank multiplicity of three, and so on:

$$t_i - (t_{i-1} + d_{i-1}) \leq U_i - C(n_i - 1), \forall i \in \{1..N_p\} \quad (5)^*$$

$$t_i - (t_{i-1} + d_{i-1}) + Ms_i \geq L_i - C(n_i - 1), \forall i \in \{1..N_p\} \quad (6)^*$$

$$t_i + C - (t_{i-1} + d_{i-1}) - M \cdot (1 - s_i) \leq U_i - C(n_i - 1), \forall i \in \{1..N_p\} \quad (7)^*$$

$$t_i + C - (t_{i-1} + d_{i-1}) \geq L_i - C(n_i - 1), \forall i \in \{1..N_p\} \quad (8)^*$$

Please note that in case that all tanks are present only once in the line, the adapted equations become again the original ones. Assuming that the parallel tanks are directly neighbored, a hoist may need a traveling time Δ to travel the maximum distance between the tanks.

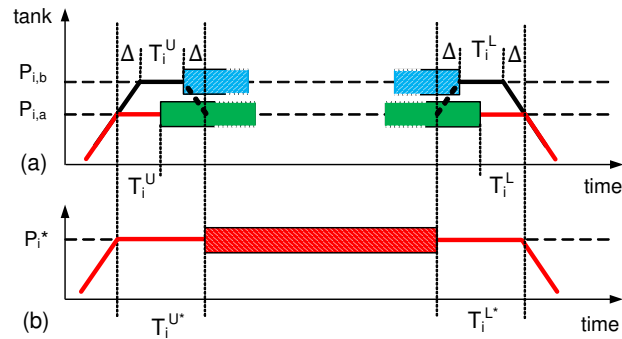


Figure 7. Parallel Tank Model (b) in consecutive Cycles for parallel tanks (a)

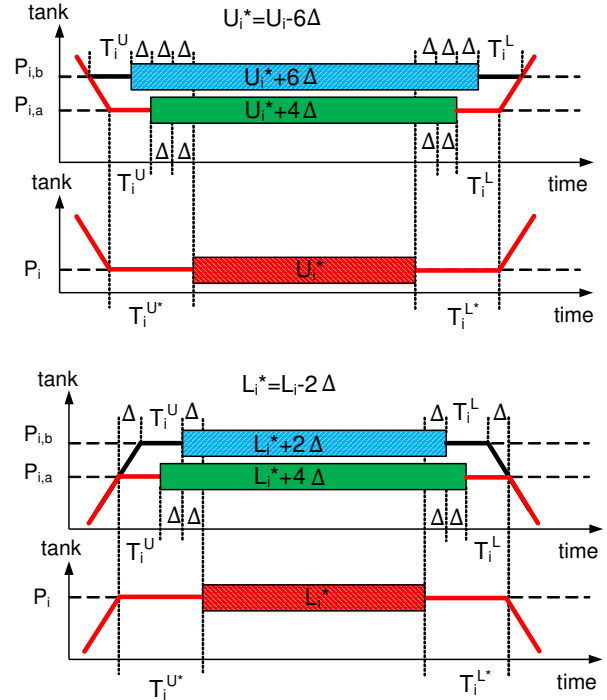


Figure 8. Adapted soak time limits for the no-wait condition

The optimization model can handle the parallel tanks using the Parallel Tank Model (PTM) as shown in Fig. 7. The main idea is that the traveling within the area in between the parallel tanks is represented by an extended loading and unloading time for the PTM tank. The collision and move feasibility constraints assume that there is only one tank for the operation. The adapted T_i^{L*} and T_i^{U*} are extended by 2Δ and used for all eqns. (1) to (48). The soak time upper and lower limits however have to be updated as well. Fig. 8 shows the reason why. Here the adapted upper limit U_i^* and the lower limit L_i^* for the PTM tank are shown. Let us assume the lower limit L_i^* is kept exactly. The resulting soak times for the real tanks are then $L_i^* + 2\Delta$ resp. $L_i^* + 4\Delta$, depending on the location and due to the no-wait condition. The value of L_i^* used for the scheduling the PTM tank is therefore $L_i^* = L_i - 2\Delta$, assuring that the soak time in any of the real tanks is at least L_i . The same observation can be made for the upper soak time limit U_i^* . In the worst case the soaking time in the upper tank is extended by 6Δ . Therefore using the upper limit $U_i^* = U_i - 6\Delta$ for the scheduling assures that the soak time for any of the real tanks will not exceed the given limit. Please note that the soak times for the process steps in consecutive cycles may differ by 2Δ , depending on the location of P_{i-1} and P_{i+1} . However, in most cases the soak time for process steps using multiple tanks will be much larger than the time a hoist needs to travel between neighbored tanks. Although all the adaptations shown and explained for the case that there is a doubled tank in the line, all equations also hold for the case $n_i > 2$. In this case the value of Δ is the traveling time between the tanks of P_i , which are furthest apart.

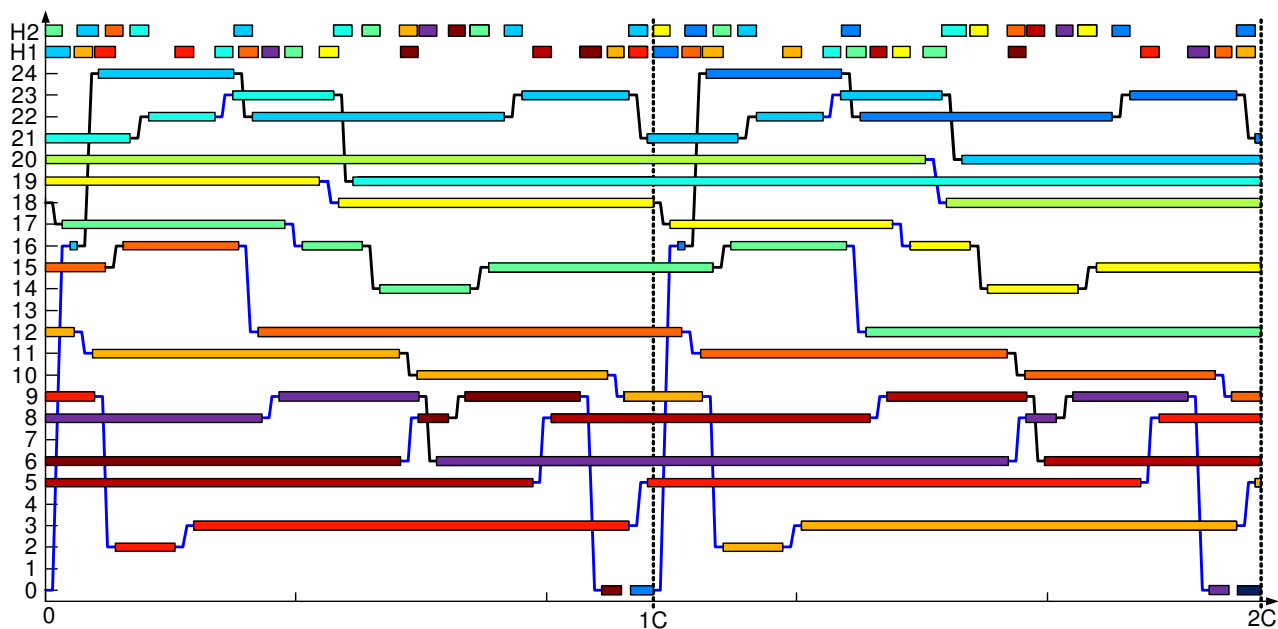


Figure 9. Gantt chart for optimal cycles scheduling an electroplating process with loops and parallel tanks

V. EVALUATION

The presented model was used to schedule a real process of semiconductor manufacturing using CPLEX [7]. The result of the cyclic scheduling is depicted in the Gantt chart of Fig. 9. The electroplating line consists of 24 chemical tanks which are operated by 2 hoists on a shared track. The tanks 19 and 20 are neighbored and identical except for their positions. The process to be scheduled consists of 26 process steps, using 21 tanks, and contains loops, since the same rinsing tanks are used repeatedly throughout the process sequence. Of course, the computation time increases as the number of hoists and process steps increases. However the complexity of the problem is highly dependent on the process data itself. If a bottleneck tank determines the minimum cycle time a schedule is found much easier. The computation can be supported by allocating transports to the hoists in advance, accepting that the solution might not be an optimal anymore. Anyhow, for the considered example the schedule was calculated and proved optimal within 13 sec. on a current PC and CPLEX 11.2. Between the transports of the carriers, empty move of the hoists may be necessary. For two hoists the moving sequence can be easily generated if the upper hoist moves maximal and the lower hoist moves minimal.

VI. CONCLUSION

In this paper a MILP model for multi-hoist scheduling is presented. The processing sequence is considered independent from the location sequence of the chemical tanks and may contain loops as in many real life applications. In addition the line may contain parallel resources in the form of duplicated

tanks which are alternately used within consecutive cycles. The presented model introduces the first compact formulation for cyclic multi-hoists with these characteristics. Although the computational expenses are highly dependent on the process data, the presented model is suitable to calculate optimal cyclic multi-hoist schedules. The model was used to schedule a real world application. We have shown that an optimal cyclic schedule can be calculated in an appropriate time. The model can also be used to analyze if multiplying tanks leads to a better overall usage of the facility or not.

REFERENCES

- [1] L. W. Phillips and P. S. Unger, "Mathematical Programming Solution of a Hoist Scheduling Program", AIIE Transactions, 1976
- [2] J. M. Y. Leung, G. Zhang, X. Yang R. Mak, K.Lam, "Optimal Cyclic Multi-Hoist Scheduling: A Mixed Integer Programming Approach", Operations Research, Vol. 52, No. 6, 2004
- [3] J. Leung, G. Zhang, "Optimal cyclic scheduling for printed circuit board production lines with multiple hoists and general processing sequence", IEEE transactions on robotics and automation, Vol. 19, No. 3, 2003
- [4] P. Brucker, Th. Kampmeyer, "A general model for cyclic machine scheduling problems", Discrete Applied Mathematics, 156, 2008
- [5] R. Rodošek, M. Wallace, "A Generic Model and Hybrid Algorithm for Hoist Scheduling Problems", Lecture Notes in Computer Science, vol.1520 Springer Verlag, 1998
- [6] W. Meyer and C. Fiedler, "On the Origin of Deadlocks in Robotic Cycle Shop Control", Proc. of IEEE International Conf. on Industrial Technology, Melbourne, Australia 2009
- [7] "ILOG OPL Development Studio 6.1 - Language User's Manual", ILOG, 2008
- [8] C. Hanen, "Study of a NP-hard cyclic scheduling problem: The recurrent job-shop", European Journal of Operational Research, 1994, vol. 72, issue 1, pages 82-101
- [9] L. Lei, T.L. Wang, "The minimum common-cycle algorithm for cyclic scheduling of two material handling hoists with time window constraints", Management Sci. 37, 1629-1639