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Representations of Status Quo Analysis in the Graph Model for Strength of Preference

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***Abstract*—In this paper, status quo analysis is addressed by using both logical and matrix representations in the graph model with strength of preference. The graph model for conflict resolution (GMCR) provides a convenient and effective means to model and analyze a strategic conflict. The graph model has entertained diverse preference structures to characterize decision-makers' (DMs) preference over feasible states, including simple preference, preference uncertainty, and strength of preference. The "simple preference" structure consists of a strict preference relation and an indifference relation to represent a DM's preference for one state relative to another. Another "strength of preference" framework allows a DM to express its strong or mild preference for one state over another, as well as the indifference relation. When a graph model is established for a strategic conflict, the standard practice is to carry out a stability analysis first, and then, followed by a post-stability analysis such as coalition analysis and status quo analysis. Status quo analysis complements stability analysis and aims at assessing whether predicted equilibria are attainable from the status quo. So far, status quo analysis has been examined for the graph model with simple preference and both logical and matrix representations of status quo analysis have been developed. This article extends these results to handle status quo analysis for the graph model with strength of preference. The algebraic method is illustrated using a conflict over proposed bulk water exports from Lake Gisborne in Newfoundland.**

I. Introduction

A strategic conflict involves two or more interactive DMs with different objectives and preferences, and their collective choice together determine the outcome of the conflict. Many models have been proposed to tackle strategic conflicts, and the Graph Model for Conflict Resolution **(GMCR)** [1], [7] is probably the most simple but flexible framework. A graph model for a strategic conflict is comprised of a finite set of decision makers (DMs), N, a set of feasible states, S, a preference structure on S for each DM i to express its preference over feasible states, and a directed graph $G_i = \{S, A_i\}$ to characterize possible moves controlled by DM i. In each directed graph, S is the common vertex set, and each oriented arc in $A_i \subseteq S \times S$ indicates that DM i can make a one-step unilateral move from the initial state to the terminal state of the arc.

Preference information plays an essential role in graph models. In the original graph model, the simple preference structure [1] is employed, which allows DMs to express its preference with a pair of binary relations, a strict preference relation $\succ$ and an indifference (or equal) relation $\sim$. Subsequently, a triplet of binary relations $\{\gg_i, >_i, \sim_i\}$ on S that takes strength of preference (strong or mild preference) into account was developed by Hamouda *et al.* [5], [6]. There binary relations are described as follows. For $s, q \in S$ and $i \in N$,

- $s \succ_i q$ denotes that DM i prefers state s to q;
- $s \gg_i q$ means that DM i strongly prefers state s to q;
- $s >_i q$ denotes that DM i mildly prefers state s to q;
- $s \sim_i q$ means that DM i is indifferent between states s and q; and
- $s \geq_i q$ indicates either $s >_i q$ or $s \sim_i q$.

Note that $\{\gg_i, >_i, \sim_i\}$ is assumed to be complete [5], [6], i.e. if $s, q \in S$, then exactly one of the following relations holds: $s \gg_i q$, $q \gg_i s$, $s >_i q$, $q >_i s$, and $s \sim_i q$.

If for any states k, s, and q, $k \succ s$ and $s \succ q$ imply $k \succ q$, then the preference relation $\succ$ is transitive. Otherwise, it is intransitive. In this paper, transitivity of the strict, strong, and mild preference relations is not required and, hence, our results apply to both transitive and intransitive preferences.

When a conflict is modeled within the graph model paradigm, a point in time has to be selected first and the current (or initial) state of the conflict is referred to as the status quo. When the stability analysis is carried out to determine equilibria or potential resolutions, it is not a concern whether the predicted equilibria are achievable or not from the status quo state. As a follow-up analysis, status quo analysis, on the other hand, is designed to trace the evolution of a conflict from the status quo state to any desirable outcome. Thus, status quo analysis provides useful dynamic and forward-looking insights into a strategic conflict [9], [10].

In the original graph model, the stability analysis is carried out within a well-designed logic structure [1]. When status quo analysis algorithms were developed by Li et al. [9]–[11] in the graph model with simple preference and preference uncertainty, this line of thinking was retained and pseudo-codes were furnished following a similar logic structure. A drawback of this structure is that it is difficult to integrate these new developments into the existing DSS GMCR II [3], [4]. To overcome this challenge, Xu et al. [12] developed a matrix system to represent status quo analysis in the graph model with simple preference.

In this paper, the algorithms [9], [10] of status quo analysis for simple preference are extended to the graph model with strength of preference. In addition, a matrix representation of status quo analysis is developed to effectively convert the logic structure to an algebraic system. The algebraic structure is flexible and can be easily adapted to new techniques.

The rest of the paper is organized as follows. In Section 2, an algorithm in a logical structure is proposed for conducting status quo analysis in the graph model with strength of preference. A matrix representation of the logical structure is then presented in Section 3, followed by a case study that is modeled with strength of preference, and then, the newly proposed status quo analysis is conducted to provide more insights regarding this conflict. The paper concludes with some comments in Section 5.

II. A Logical representation of status quo analysis in the graph model with strength of preference

A. Reachable lists from a status quo

Based on different preference frameworks, DM i can categorize the feasible state set S into different subsets. Under strength of preference, a sensible way to partition the state set S is provided by Hamouda *et al.* [5], [6] and described as follows. For $s, q \in S$ and $i \in N$,

- $\Phi_i^{++}(s) = \{q : q \gg_i s\}$ denotes all states that DM i strongly prefers to s;
- $\Phi_i^{+}(s) = \{q : q >_i s\}$ indicates all states that DM i mildly prefers to s;
- $\Phi_i^{=}(s) = \{q : q \sim_i s\}$ denotes all states that are indifferent to s for DM i;
- $\Phi_i^{-}(s) = \{q : s >_i q\}$ means all states that DM i mildly less prefers than s;
- $\Phi_i^{--}(s) = \{q : s \gg_i q\}$ indicates all states that DM i strongly less prefers than s.

Let $\cup$ and $\cap$ denote the union and intersection operations, respectively. Since the preference structure with strength is complete, the state set S can be expressed as: $S = \Phi_i^{++}(s) \cup \Phi_i^{+}(s) \cup \Phi_i^{=}(s) \cup \Phi_i^{-}(s) \cup \Phi_i^{--}(s)$.

Let $R_i(s)$ denote DM i's reachable list from a state s, containing all states to which DM i can move from state s in one step and representing DM i's unilateral moves (UMs). Similar to the partition of feasible state set S, DM i's reachable list $R_i(s)$ can also be categorized into the following five subsets by considering appropriate preference information.

- $R_i^{++}(s) = R_i(s) \cap \Phi_i^{++}(s)$ denotes strong unilateral improvements from state s for DM i;
- $R_i^{+}(s) = R_i(s) \cap \Phi_i^{+}(s)$ indicates all mild unilateral improvements from state s for DM i;
- $R_i^{=}(s) = R_i(s) \cap \Phi_i^{=}(s)$ includes equally preferred states reachable from state s by DM i;
- $R_i^{-}(s) = R_i(s) \cap \Phi_i^{-}(s)$ denotes all mild unilateral disimprovements from state s for DM i;
- $R_i^{--}(s) = R_i(s) \cap \Phi_i^{--}(s)$ includes all strong unilateral disimprovements from state s for DM i.

For convenience, let $R_i^{+,++}(s) = R_i^{+}(s) \cup R_i^{++}(s)$ depict unilateral mild improvements or strong improvements, hereafter, referred to as weak improvements (W-Is) from state s for DM i.

B. Legal sequences of UMs and W-Is

Any subset H of DMs in the set N is called a coalition. Generally, a coalition $H \subseteq N$ and $|H| > 1$ is non-trivial. For a two-DM model, DM i's opponent is a single DM, denoted by j, so DM j's reachable list from state s by one step move is $R_j(s)$. In an n-DM model $(n > 2)$, the opponents of a DM constitute a group of two or more DMs. A legal sequence of UMs by a coalition H is a series of states linked by UMs by the members of H, in which any DM may move more than once, but not twice consecutively. Similarly, a legal sequence of *W-Is* by a coalition H denotes a series of states linked by strong unilateral improvements or mild unilateral improvements (*W-Is*) by its members, in which a DM may make two or more *W-Is*, but not twice consecutively.

Let $i \in N$ and $H \subseteq N$ and let $k \geq 1$ be an integer. Some notation for status quo analysis is furnished below to facilitate the description of the algorithm for status quo analysis.

- SQ is the status quo state;
- $S_i^{(k)}(s)$ and $S_i^{(k,+,++)}(s)$ stand for states reachable from $SQ = s$ in exactly k legal UMs and legal *W-Is*, respectively, with last mover DM i;
- $V_H^{(k)}(s)$ and $V_H^{(k,+,++)}(s)$ denote states reachable from $SQ = s$ in at most k legal UMs and legal *W-Is* by H, respectively. $V_i^{(k)}(s)$ and $V_i^{(k,+,++)}(s)$ consist of all states reachable from $SQ = s$ in at most k legal UMs and *W-Is*, respectively, with last mover DM i. i.e., $V_i^{(k)}(s) = S_i^{(k)}(s) \cup V_i^{(k-1)}(s)$ and $V_i^{(k,+,++)}(s) = S_i^{(k,+,++)}(s) \cup V_i^{(k-1,+,++)}(s)$;
- $A_i^{(k)}(s)$ and $A_i^{(k,+,++)}(s)$ indicate arcs with last mover DM i in sequences of at most k $(k > 1)$ legal UMs and legal *W-Is*, respectively;
- A_i and $A_i^{+,++} \subseteq S \times S$ are two oriented arc sets, denoting DM i's one-step UMs and *W-Is*, respectively. Let $A_i(s)$ and $A_i^{+,++}(s)$ denote, respectively, the set of arcs associated with DM i in one-step UMs and *W-Is* from state s. Therefore, $A_i = \bigcup_{s \in S} A_i(s)$, and $A_i^{+,++} = \bigcup_{s \in S} A_i^{+,++}(s)$.

C. An Algorithm for status quo analysis in the graph model with strength of preference

In the status quo analysis, if a DM moves twice in succession, the DM is deemed to make illegal moves. The following Theorem 2.1 asserts that if there does not exist any new appropriate arc in the graph model, then corresponding moves will stop. Let δ_1 and δ_2, respectively, stand for the number of iteration steps required to construct $V_H(s) = V_H^{(\delta_1)}(s)$ and $V_H^{+,++}(s) = V_H^{(\delta_2,+,++)}(s)$.

Theorem 2.1: For $SQ = s$ and $H \subseteq N$, the following results hold:

(1) If $\bigcup_{i \in H} A_i^{(k_1+1)}(s) = \bigcup_{i \in H} A_i^{(k_1)}(s)$, then $V_H^{(k_1+1)}(s) = V_H^{(k_1)}(s)$ and $\delta_1 = k_1$;

(2) If $\bigcup_{i\in H} A_i^{(k_2+1,+,++)}(s) = \bigcup_{i\in H} A_i^{(k_2,+,++)}(s)$, then $V_H^{(k_2+1,+,++)}(s) = V_H^{(k_2,+,++)}(s)$ and $\delta_2 = k_2$.

The proofs of the two statements (1) and (2) are similar to those presented in [12].

The algorithms for status quo analysis in the graph model for simple preference developed by Li et al. [9], [10] can be extended to models with strength of preference. Here, the algorithm for status quo analysis allowing W-Is only in the graph model for strength of preference is presented in Table I.

III. A Matrix representation of status quo analysis in the graph model with strength of preference

A. Components of the matrix representation of status quo analysis

Next, we introduce basic components of the matrix representation for status quo analysis in the graph model with strength of preference. Let $m = |S|$. $Q = M \vee G$ is defined as the $m \times m$ matrix with its (s, q) entry

$$Q(s,q) = \begin{cases} 1 & if\ M(s,q) + G(s,q) \neq 0, \\ 0 & otherwise. \end{cases}$$

A sign function, $sign(\cdot)$, maps an $m \times m$ matrix with its (s, q) entry $M(s, q)$ to another $m \times m$ matrix

$$sign[M(s,q)] = \begin{cases} 1 & M(s,q) > 0, \\ 0 & M(s,q) = 0, \\ -1 & M(s,q) < 0. \end{cases}$$

For $i \in N$, $s \in S$, an $m \times m$ 0-1 matrix J_i is defined by

$$J_i(s,q) = \begin{cases} 1 & if\ (s,q) \in A_i, \\ 0 & otherwise. \end{cases}$$

It is clear that $R_i(s) = \{q : J_i(s, q) = 1\}$, so J_i is called DM $i's$ UM matrix (or adjacency matrix). For DM i, a *W-I* matrix $J_i^{+,++}$ is defined by

$$J_i^{+,++}(s,q) = \begin{cases} 1 & if\ J_i(s,q) = 1\ and\ q \gg_i s, or q >_i s \\ 0 & otherwise. \end{cases}$$

Obviously, $R_i^{+,++}(s) = \{q : J_i^{+,++}(s, q) = 1\}$. UM and *W-I* matrices, J_i and $J_i^{+,++}$, depict DM i's one-step UMs and *W-Is*.

B. Status quo matrices by legal UMs and legal W-Is

We now demonstrate how to find matrices to trace conflict evolution by legal UMs or legal W-Is from a status quo for DM i. First, define two $m \times m$ matrices $M_i^{(t)}$ and $M_i^{(t,+,++)}$ with their (s, q) entries as follows:

Definition 3.1: For $i \in N$ and $t = 1, 2, 3, \cdots$,

$$M_i^{(t)}(s,q) = \begin{cases} 1 & \text{if } q \in S \text{ is reachable from } s \in S \text{ in} \\ & \text{exactly } t \text{ legal UMs with last mover} \\ & \text{DM } i, \\ 0 & \text{otherwise,} \end{cases}$$

and

$$M_i^{(t,+,++)}(s,q) = \begin{cases} 1 & \text{if } q \in S \text{ is reachable from } s \in S \\ & \text{in exactly } t \text{ legal \textit{W-Is} with last} \\ & \text{mover DM } i, \\ 0 & \text{otherwise.} \end{cases}$$

Based on Definition 3.1, we have

Theorem 3.1: The two $m \times m$ matrices $M_i^{(t)}$ and $M_i^{(t,+,++)}$ can be expressed inductively by

$$M_i^{(t)} = sign[(\bigvee_{j\in H-\{i\}} M_j^{(t-1)}) \cdot J_i],\ with\ M_i^{(1)} = J_i, \quad (1)$$

and

$$M_i^{(t,+,++)} = sign[(\bigvee_{j\in H-\{i\}} M_j^{(t-1,+,++)}) \cdot J_i^{+,++}], \quad (2)$$

$$with\ M_i^{(1,+,++)} = J_i^{+,++}.$$

The proofs of equations (1) and (2) are similar to those shown in [12].

Next we define two status quo analysis matrices $M_i^{SQ^{(t)}}$ and $M_i^{SQ^{(t,+,++)}}$ to trace conflict evolution from a status quo state to any desirable outcome as follows:

Definition 3.2: For $i \in N$ and $t = 1, 2, 3, \cdots$,

$$M_i^{SQ^{(t)}}(s,q) = \begin{cases} 1 & if\ q \in V_i^{(t)}(s), \\ 0 & otherwise, \end{cases}$$

$$M_i^{SQ^{(t,+,++)}}(s,q) = \begin{cases} 1 & if\ q \in V_i^{(t,+,++)}(s), \\ 0 & otherwise. \end{cases}$$

Specifically, $M_i^{SQ^{(t)}}(s,q) = 1$ and $M_i^{SQ^{(t,+,++)}}(s,q) = 1$ denote that state q is reachable from status quo state s in at most t UMs and *W-Is*, respectively, with last mover i. Based on Definitions 3.1 and 3.2, Theorem 3.2 can be easily derived.

Theorem 3.2: Let $s \in S$, $H \subseteq N$, and $H \neq \emptyset$. Then status quo analysis matrices $M_i^{SQ^{(k)}}$ and $M_i^{SQ^{(k,+,++)}}$ can be respectively expressed as

$$M_i^{SQ^{(k)}} = \bigvee_{t=1}^{k} M_i^{(t)}, \quad (3)$$

$$M_i^{SQ^{(k,+,++)}} = \bigvee_{t=1}^{k} M_i^{(t,+,++)}. \quad (4)$$

Proof: The proof of equation (3) is presented in [12]. We prove equation (4). Let $M_i^{SQ^{(k,+,++)}}(s,q)$ denote the (s, q) entry of the matrix $M_i^{SQ^{(k,+,++)}}$. Based on Definition 3.2, $M_i^{SQ^{(k,+,++)}}(s,q) = 1$ iff $q \in V_i^{(k,+,++)}(s)$. i.e., q is reachable from $SQ = s$ in at most k legal *W-Is*, with last mover DM i.

Let $\bigvee_{t=1}^{k} M_i^{(t,+,++)}(s,q)$ denote the (s, q) entry of the matrix $\bigvee_{t=1}^{k} M_i^{(t,+,++)}$. By Definition 3.1, $\bigvee_{t=1}^{k} M_i^{(t,+,++)}(s,q) = 1$ iff there exists $1 \leq t \leq k$, such that $M_i^{(t,+,++)}(s,q) = 1$. i.e., q is reachable from $SQ = s$ in exactly t legal W-Is, with last

TABLE I
ALGORITHM FOR STATUS QUO ANALYSIS IN THE GRAPH MODEL ALLOWING *W-Is* ONLY

Initialize //initialize the necessary parameters

H: any subset of DMs;

n: the number of DMs;

m: the number of states;

s: the status quo state;

δ_2: the max step we want to calculate;

$R_i^{+,++}(s)$: *W-Is* from state s by DM i, $i = 1, \cdots, n$;

$k = 1$

$S_i^{(k,+,++)}(s) = R_i^{+,++}(s), i = 1, \cdots, n$

$V_i^{(k,+,++)}(s) = S_i^{(k,+,++)}(s), i = 1, \cdots, n$

$A_i^{(k,+,++)}(s) = \bigcup_{q \in R_i^{+,++}(s)} (s, q)$, (s, q) is a *W-I* arc from state s to state q, $i = 1, \cdots, n$

loop 1

$k = k + 1$

loop 2 i from 1 to n // the last mover is DM i

$S' = \bigcup_{j \in H \setminus \{i\}} S_j^{(k-1,+,++)}(s)$

$S_i^{(k,+,++)}(s) = \bigcup_{s' \in S'} R_i^{+,++}(s')$

$V_i^{(k,+,++)}(s) = V_i^{(k-1,+,++)}(s) \bigcup S_i^{(k,+,++)}(s)$

$A_i^{(k,+,++)}(s) = A_i^{(k-1,+,++)}(s) \bigcup \{(s_1, s_2) : s_1 \in \bigcup_{j \in H \setminus \{i\}} S_j^{(k-1,+,++)}(s)$, and $s_2 \in R_i^{+,++}(s_1)\}$

return to loop 2

$V_H^{(k,+,++)}(s) = \bigcup_{i \in H} V_i^{(k,+,++)}(s)$

return to loop 1 if $\bigcup_{i \in H} A_i^{(k,+,++)}(s) \neq \bigcup_{i \in H} A_i^{(k-1,+,++)}(s)$

$\delta_2 = k$

$V_H(s) = V_H^{(\delta_2)}(s)$.

mover DM i. It implies that q is reachable from $SQ = s$ in at most k legal *W-Is*, with last mover DM i. Consequently, $\bigvee_{t=1}^{k} M_i^{(t,+,++)}(s,q) = 1$ iff $M_i^{SQ^{(k,+,++)}}(s,q) = 1$. Since $M_i^{SQ^{(k,+,++)}}$ and $\bigvee_{t=1}^{k} M_i^{(t,+,++)}$ are $m \times m$ 0-1 matrices, it follows that $M_i^{SQ^{(k,+,++)}} = \bigvee_{t=1}^{k} M_i^{(t,+,++)}$.

Any nonzero entry (s, q) of the two status quo analysis matrices $M_i^{SQ^{(t)}}$ and $M_i^{SQ^{(t,+,++)}}$ reveal that the desired outcome state q is reachable from the status quo state s in at most t legal UMs and t legal *W-Is*, respectively.

IV. AN APPLICATION

In this section, the matrix approach to status quo analysis is applied to a practical problem—the Lake Gisborne conflict. Lake Gisborne is located near the south coast of Newfoundland, an Atlantic province in Canada. In 1995, a company called Canada Wet Incorporated in Newfoundland proposed a project to export bulk water from Lake Gisborne. Owing to possible economic benefits, the **Provincial Government** of Newfoundland and Labrador registered this project. However, many environmental lobby groups opposed to it and were concerned with potentially devastating consequences to the environment in the lake basin. The **Federal Government of Canada** supported the opposition groups and prohibited water exports. In view of the impoverished state, however, several groups supported the project, arguing for implementing the project. Thus, the contentious Lake Gisborne water export project creates a conflict among the Federal Government of Canada, Provincial Government of Newfoundland and Labrador, and Support Groups [2].

This conflict is modeled with three DMs: DM 1, the **Federal**

TABLE II
FEASIBLE STATES FOR THE LAKE GISBORNE MODEL

Federal								
1. Continue	N	Y	N	Y	N	Y	N	Y
Provincial								
2. Lift	N	N	Y	Y	N	N	Y	Y
Support								
3. Appeal	N	N	N	N	Y	Y	Y	Y
State number	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8

Government (Federal); DM 2, the **Provincial Government (Provincial)**; and DM 3, the **Support Groups (Support)**; and a total of three options. Specifically,

- Federal continues to prohibit bulk water export;
- Provincial lifts the ban on bulk water export; and
- Support appeals to continue this project.

Because each option can either be selected (Y for yes) or not taken (N for no), there is a total of 2^3 possible states, s_1, s_2, $\cdots$, s_8 in the Lake Gisborne conflict. The results are presented in Table II. Based on the 8 feasible states, the graph model of this conflict is shown in Fig. 1, in which a label on an arc denotes which DM controls the move between the two states connected by the arc.

Li et al. [8] expanded the graph model of the Lake Gisborne conflict introduced in [2] by allowing preference uncertainty. Here, the graph model for the Gisborne conflict is considered with strength of preference. The preference information for this conflict over the feasible states is given in Table III. We assume that state s_7 is strongly less preferred to all other states by the Federal Government, the Provincial Government is strongly less preferred s_6 to other states, and the Support Groups consider state s_2 to be strongly less preferred relative to all other states. Additionally, this representation of preference information presented in Table III implies that the preferred relations, $>$ and $\gg$, are transitive. For instance, since $s_5 > s_3$ and $s_3 \gg s_7$, then $s_5 \gg s_7$. However, in general, the preference structure presented in this paper does not require the transitivity of preference relations and, hence, the proposed approach can be used to handle intransitive preferences as well.

TABLE III
PREFERENCE WITH STRENGTH FOR THE GISBORNE MODEL

DMs	Preference with strength
Federal	$s_2 > s_6 > s_4 > s_8 > s_1 > s_5 > s_3 \gg s_7$
Provincial	$s_3 > s_7 > s_4 > s_8 > s_1 > s_5 > s_2 \gg s_6$,
Support	$s_3 > s_4 > s_7 > s_8 > s_5 > s_6 > s_1 \gg s_2$

Using the strong stability definitions proposed by Hamouda et al. [6] for the graph model with strength of preference, states s_4 and s_6 are identified as strong equilibria for the Gisborne conflict. Therefore, it is natural to ask whether states s_4 and s_6 are attainable from the status quo state. Status quo analysis is designed to address this attainability issue and examines the dynamics of a conflict model. As such, by taking status quo

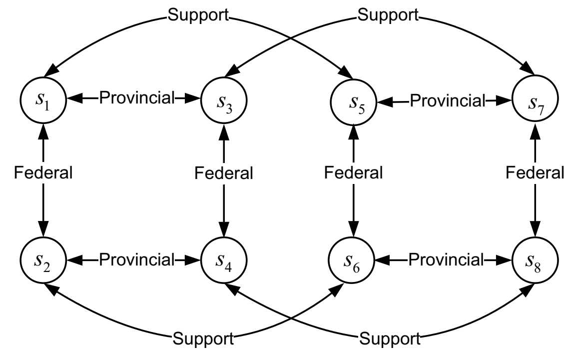

Fig. 1. Graph model for the Gisborne conflict.

analysis into account, additional insights can be garnered about conflict under study. The conflict evolution of the graph model for the Gisborne conflict by allowing *W-Is* only is portrayed in Fig. 2. Based on Fig. 2, an evolution path by allowing *W-Is* only from statu quo s_1 to the desirable equilibrium s_4 is illustrated in Fig. 3.

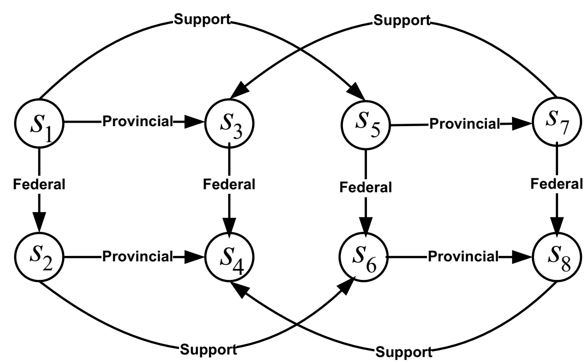

Fig. 2. Graph model for the Gisborne conflict allowing W-Is only.

However, the shortest paths from s_1 to s_4 can be achieved by two-step *W-Is* as shown below:

$$s_1 \underrightarrow{Federal} s_2 \underrightarrow{Provincial} s_4;$$

and

$$s_1 \underrightarrow{Provincial} s_3 \underrightarrow{Federal} s_4.$$

Similarly, there exist two paths from status quo s_1 to another equilibrium s_6 by two-step *W-Is* as well.

$$s_1 \underrightarrow{Federal} s_2 \underrightarrow{Support} s_6;$$

and

$$s_1 \underrightarrow{Support} s_5 \underrightarrow{Federal} s_6.$$

V. CONCLUSIONS AND FUTURE WORK

A. Conclusions

Status quo analysis aims at looking forward from the status quo by identifying attainable states and assessing how readily

The evolution of the Gisborne conflict by W - Is from status quo s_1

Options	Status quo	Transitional states			Equilibrium
Federal	N	N	Y	Y	Y
Provincial	N	N	N	Y	Y
Support	N	Y	Y	Y	N
State number	s_1	s_5	s_6	s_8	s_4

Fig. 3. **The Gisborne conflict evolution from the status quo s_1 to state s_4.**

they can be reached. Although pseudo-codes for status quo analysis were introduced by Li et al. [9]–[11] in the context of simple preference and preference uncertainty, strength of preference has not been considered, nor have the algorithms been implemented into a decision support system.

In this paper, the pseudo-codes [9]–[11] and the matrix approach [12] of status quo analysis for simple preference are extended to include strength of preference. Both logical and matrix representations of status quo analysis are developed in the graph model for conflict resolution with strength of preference. Due to the nature of the explicit algebraic expressions, the matrix method is more convenient for computer implementation and easier for adapting to new analysis techniques compared with the logical structure.

B. Future work

The proposed method is based on the adjacency matrix to search state-by-state paths. If a graph model contains common moves [1] characterized by multiple arcs between the same two states controlled by different DMs, the adjacency matrix would be unable to track all aspects of conflict evolution from the status quo state. It is well known that the incidence matrix can represent multidigraphs if all edges are appropriately labeled. An algebraic approach based on the incidence matrix is expected to be able to demonstrate more aspects of conflict evolution in the graph model for strength of preference.

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