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Obtaining an FOU for a Word from a Single Subject by an *Individual Interval Approach*

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***Abstract*— Recently a simple and practical type-2-fuzzistics methodology called an *Interval Approach* (IA) was presented for obtaining interval type-2 fuzzy set (IT2 FS) models for words using data collected from a group of subjects. There may be times, however, when a group of subjects is not available. This paper proposes a way to obtain IT2 FS models from words collected from a single subject using an IA, and is called an *Individual IA* (IIA). Two methods are presented for doing this. Both use end-point and uncertainty data that are collected from an individual, assume a probability distribution on each interval, map them into pre-specified T1 membership functions (MF), interpret the MFs as nine embedded T1 FSs of an IT2 FS, and then aggregate the FSs using union to obtain the footprint of uncertainty (FOU) for the word. This approach not only captures the strong points of the previously developed IA but simplifies it. Experiments show that the IIA is easy to implement and the resulting FOUs match our intuition.**

Keywords*— Computing With Words, Interval Approach, IT2 FS, Perceptual Computer

I. Introduction

Zadeh coined the phrase "*computing with words*" (CWW) [9, 10]. According to [10], CWW is "*a methodology in which the objects of computation are words and propositions drawn from a natural language.*" There are at least two types of uncertainties associated with a word [8]: intra-personal uncertainty and inter-personal uncertainty. Wallsten and Budescu [8] suggest to model intra-personal uncertainty by type-1 fuzzy sets (T1 FSs). Mendel [3] points out inter-personal uncertainty as "*words mean different things to different people.*" Because an IT2 FS can be viewed as a group of T1 FSs, it can model both types of uncertainty; hence, we suggest IT2 FSs be used in CWW [2, 5, 8].

A specific architecture is proposed in [5] for making subjective judgments by CWW, as shown in Fig. 1. It is called a *Perceptual Computer*—Per-C for short. In Fig. 1, the *encoder* transforms linguistic perceptions into IT2 FSs that activate a *CWW engine*. The CWW engine performs operations on the IT2 FSs. The *decoder* maps the output of the CWW engine into a recommendation, which can be a word, rank, or class.

There are three problems one needs to solve in order to operate the Per-C: 1) *How to transform words into IT2 FSs, i.e., the encoding problem*, 2) *How to construct the CWW engine, which maps IT2 FSs into IT2 FSs*, and 3) *How to map the output of the CWW engine into a recommendation, i.e., the decoding problem.* Only the first problem is considered in this paper since the last two problems are outside of its scope.

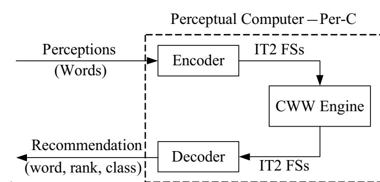


Fig. 1. Conceptual structure of the Perceptual Computer.

When a group of subjects are available so that data can be collected from them about a word, the *encoding problem* can be solved using Liu and Mendel's *Interval Approach* [1]. The basic idea of the IA is to collect interval end-point data for a word from them (the subjects are asked: *On a scale of 0-10, what are the end-points of an interval that you associate with the word ______?*), map each subject's data interval into a pre-specified type-1 (T1) person membership function (MF), interpret the latter as an embedded T1 fuzzy set (FS) of an IT2 FS, and obtain a mathematical model for the FOU for the word from these T1 FSs.

The strong points of this approach are that it: only requires interval endpoint data from subjects who don't need to be knowledgeable about FSs; maps data into an FOU straightforwardly; doesn't require an assumption about whether or not an FOU is symmetric or non-symmetric; and, leads to an IT2 FS word model that reduces to a T1 FS word model if the uncertainty disappears. A weak point of the IA is that the interval endpoint data have to be collected from a group of subjects. Sometimes this is not possible, due, e.g., to budget limitations, time-constraints, or unavailability of a subject pool.

In this paper, a method is proposed for obtaining an FOU from a single subject by an IA and is called an *Individual IA* (IIA). It collects interval end-point and uncertainty band data from a single subject and maps them into an IT2 FS. The IIA is not meant to replace the IA when a group of subjects is available. It is meant to be used only when a group of subjects is not available.

The rest of the paper is organized as follows. Section II provides an overview of the IIA, in which it is explained that the IIA has two parts, the *data part* and the *FS part*. Section III demonstrates the FOU classification diagram, which is the basis for the IIA. Section IV describes the data part, in which two methods are explained for collecting the data. Section V describes the FS part. Section VI illustrates the IIA by means of some examples. Section VII provides conclusions.

II. Overview of the IIA

In this section, the IIA to obtain an FOU from a single subject is overviewed. The IIA consists of two parts, the *data part* and the *FS part*. In the data part, data are collected from an individual on a scale of 0–10, after which nine data intervals are computed. There are two methods for collecting the data and they are summarized in the top portion of Fig. 2.

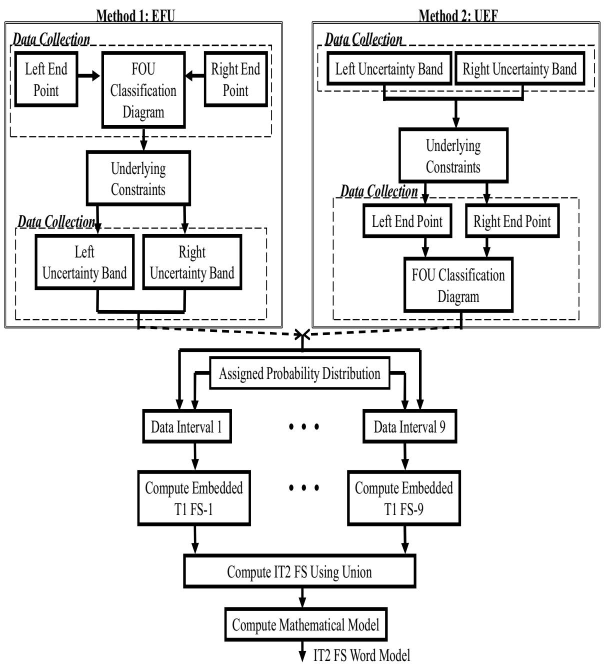


Fig. 2. The data part (top portion) and FS part (bottom portion) of the IIA. Two methods for the data part are shown within the two big boxes: Method 1: EFU (End-points → FOU classification diagram → Uncertainty-bands), and Method 2: UEF (Uncertainty- bands → End-points → FOU classification diagram).

One can choose either method to provide the data since the rest of the steps in this figure are the same. The individual's data are then located on the *FOU classification diagram* that was developed in [1]. In the FS part, the parameters of nine T1 MFs are determined using nine data intervals that derive from the data part. These T1 MFs are then aggregated using union, leading to an FOU for a word, and finally to a mathematical model for the FOU. The FS part is shown in the bottom portion of Fig. 2.

To begin, the FOU classification diagram is described in Section III. Details of each step in the data part and the FS part, are described in Sections IV and V, respectively.

III. FOU Classification Diagram

In the IIA, the FOU classification diagram, that was developed in [1] and is depicted in Fig. 3, is used in the data part, unlike in the IA where it is used in the fuzzy set part. The FOU classification diagram shows the admissible regions for Interior, left-shoulder (LS) and right-shoulder (RS) FOUs. The three lines that define the boundaries for the three kinds of FOUs are obtained from three simple requirements [1]: (1) the right-end of a data-interval for a word has to be larger than the left-end of that interval; (2) the left-end of a symmetric triangle T1 MF has to be greater than or equal to zero; and, (3) the right-end of that triangle has to less than or equal to 10.

Whenever an individual chooses two interval end points for a word, α and β, they can be located on this diagram and she can see what kind of an FOU the word will map into. If she doesn't want an Interior FOU for a word, she can relocate α and β so that the FOU for the word is a LS or RS FOU. In this way, the individual can interact in an adaptive manner with the FOU classification diagram.

Fig. 3 is similar to Fig. 9 in [1], with the following changes: the x and y axes in the IA are labeled m_l and m_r where m_l and m_r are the mean of the left (right) end-point of the surviving intervals (after pre-processing) that were collected from a group of subjects; whereas in the IIA α and β are used in place of m_l and m_r, respectively.

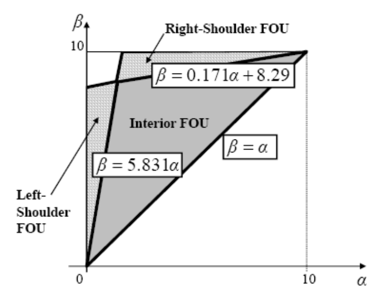


Fig. 3. Classification diagram with its FOU decision regions [1].

IV. Data Part of the IIA

In this section, we explain two methods for collecting data from an individual and how that data can be used in conjunction with the FOU classification diagram that was described in Section III. Because data are collected from a single subject in the IIA, more data are needed from that subject than are needed in the IA. Instead of asking a subject to provide only the two end-points of an interval that they associate with a word, as is done in the IA, a subject is now asked to provide four data, namely two interval end-points [α, β] and two *end-point uncertainty bands* P_α and P_β. Uncertainty bands are in percentage and indicate how uncertain one is about the end points, e.g., a 20% left uncertainty-band indicates that one is 20% uncertain, both to the left and to the right of the left-end point. We note that an uncertainty band should be specified as a percentage because this is easier to do by an individual. We assume that the uncertainty bands are symmetric about the end points. For later computation, the % uncertainty bands are converted into numbers, $\delta\alpha$ and $\delta\beta$, i.e.

$$\left.\begin{aligned} \delta\alpha &= \alpha \times \frac{P_\alpha}{100} \geq 0 \\ \delta\beta &= \beta \times \frac{P_\beta}{100} \geq 0 \end{aligned}\right\} . \tag{1}$$

The two interval end points α and β and the two uncertainty bands that are situated symmetrically about α and β are depicted in Fig. 4.

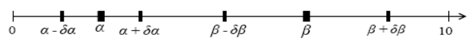


Fig. 4. Six numbers are needed in the IIA.

The two methods for collecting the four data from an individual are explained next.

A. EFU (End-points → FOU classification diagram → Uncertainty-bands) Method to Collect Data

1) End-points: First, an individual is asked to provide left- and right end points, α and β, respectively, for a specific word, i.e. the following question is answered: On a scale of 0-10, what are the end-points of an interval that you associate with the word *W*? She chooses numbers between zero and ten, subject to the following obvious constraints:

$$\left.\begin{aligned} 0 \leq \alpha \leq 10 \\ 0 \leq \beta \leq 10 \\ \alpha \leq \beta \end{aligned}\right\} . \tag{2}$$

2) FOU classification diagram: α and β are located on the FOU classification diagram in Fig. 3, so that she can see whether the word is going to be mapped into an Interior, LS, or RS FOU.

3) Uncertainty-bands: End-point uncertainty bands are limited by some constraints, which are derived next, and are deduced from Fig. 4.

The left-most end-point, $\alpha - \delta\alpha$, has to be greater than or equal to zero, and the right-most end-point, $\beta + \delta\beta$, cannot be greater than ten, i.e.,

$$\left.\begin{aligned} \alpha - \delta\alpha \geq 0 \\ \beta + \delta\beta \leq 10 \end{aligned}\right\} . \tag{3}$$

Additionally, the right end of the uncertainty band of the left end-point, $\alpha + \delta\alpha$, has to be smaller than the left end of the uncertainty band of the right end-point, $\beta - \delta\beta$ (or else the left end-point can become the right end-point, and vice-versa), i.e.,

$$\alpha + \delta\alpha \leq \beta - \delta\beta . \tag{4}$$

Note, that for the first inequality in (3), if $\alpha = 0$ then $\delta\alpha$ must be zero, which means that, if $\alpha = 0$ then $P_{\alpha=0} = 0$. Similarly, for the second inequality in (3), if $\beta = 10$ then $\delta\beta$ also has to be zero, which means that if $\beta = 10$ then $P_{\beta=10} = 0$. These constraints seem very reasonable and mean that in the IIA, no one can have uncertainties about the two boundary values of 0 and 10.

From (1), (3) and (4), the following constraints are obtained for the two uncertainty bands $\delta\alpha$ and $\delta\beta$:

$$\left.\begin{aligned} 0 \leq \delta\alpha \leq \alpha \\ 0 \leq \delta\beta \leq 10 - \beta \\ \delta\beta \leq -\delta\alpha + (\beta - \alpha) \end{aligned}\right\} . \tag{5}$$

Because we have already argued that it is easier for an individual to express the uncertainty bands as percentages, (5) is re-expressed in terms of P_α and P_β, by substituting (1) into (5), i.e.:

$$\left.\begin{aligned} 0 \leq P_\alpha \leq 100 \\ 0 \leq P_\beta \leq \frac{100(10-\beta)}{\beta} \\ P_\beta \leq -\frac{\alpha}{\beta} P_\alpha + 100(1 - \frac{\alpha}{\beta}) \end{aligned}\right\} . \tag{6}$$

Even though it has not been stated explicitly, it is also true that $0 \leq P_\beta \leq 100$. Below, we show that this constraint is not needed because it is superceded by the constraints on P_β in (6).

Given values for α and β, (6) demonstrates that one is not free to arbitrarily choose P_α and P_β. The three inequalities in (6) are shown in four diagrams in Fig. 5.

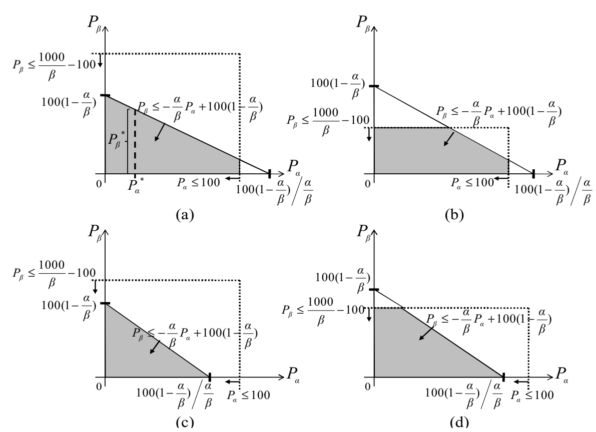


Fig.5. Four cases of the uncertainty-band constraints. The shaded region in each figure corresponds to the simultaneous satisfaction of the three inequalities in (6). P_α^* and P_β^* in (a) are possible choices of P_α and P_β that an individual can make.

Observe from Fig. 5 that $100(1-\alpha/\beta)/\alpha/\beta$ and $100(1-\alpha/\beta)$ are the P_α and P_β intercepts of the line $P_\beta = -\frac{\alpha}{\beta} P_\alpha + 100(1 - \frac{\alpha}{\beta})$, respectively. The slope of this line is always negative and greater than or equal to -1, since $\alpha \leq \beta$. Observe also that $0 \leq P_\beta \leq 100$ does not appear in all four cases in Fig. 5 because P_β is always bounded by $100(1-\alpha/\beta)$ and it is always true that $100(1-\alpha/\beta)\le 100$.

There are two limits for both P_α and P_β in Fig. 5, and these limits are determined according to the values of the two interval end points α and β, as the following *uncertainty-band constraints*:

$$\left.\begin{aligned} P_\alpha &\le \min\left[100, 100\left(\frac{\beta}{\alpha}-1\right)\right] \\ P_\beta &\le \min\left[-\frac{\alpha}{\beta}P_\alpha + 100\left(1-\frac{\alpha}{\beta}\right), \frac{1000}{\beta}-100\right] \end{aligned}\right\}. \quad (7)$$

In order to choose values for P_α and P_β, once an individual chooses α and β, she chooses $P_\alpha = P_\alpha^*$ that satisfies the first inequality in (7) and then chooses P_β^* with the help of the second inequality in (7), as shown in Fig. 5 (a).

The uncertainty bands are converted from percentages to numbers, using (1), in order to perform the calculations of the FS Part of the IIA.

B. UEF (Uncertainty-bands → End-points → FOU classification diagram) Method to Collect Data

1) Uncertainty-bands: In this second method for collecting data, an individual is asked to first provide two end-point uncertainty bands for a specific word, i.e. the following question is answered: What are the percentage end-point uncertainties of an interval that you associate with the word *W*? She provides a percentage end-point uncertainty for both the left and right end-points, P_α and P_β, respectively, and, they are usually not the same. Clearly, $0\le P_\alpha \le 100$ and $0\le P_\beta\le 100$.

2) End-points: Given values for P_α and P_β, the interval end points α and β are limited by some constraints that are derived next. By re-expressing the last two inequalities in (6), one obtains:

$$\left.\begin{aligned} \beta &\le \frac{1000}{100+P_\beta} \\ \beta &\ge \frac{100+P_\alpha}{100-P_\beta}\alpha \end{aligned}\right\}. \quad (8)$$

Note from the second inequality in (8) that, because $(100+P_\alpha)\ge(100-P_\beta)$, the slope of the line $\beta=[(100+P_\alpha)/(100-P_\beta)]\alpha$ is always greater than or equal to 1. Note also that $P_\beta \ne 100$, because $P_\beta = 100$ means an individual is 100% uncertain about the end point, and if this is so, she will not choose that end point.

The two inequalities in (8) are shown in Fig. 6. Observe that $\alpha = 1000(100-P_\beta)/[(100+P_\alpha)(100+P_\beta)]$ is the intercept of the lines $\beta=[(100+P_\alpha)/(100-P_\beta)]\alpha$ and $\beta = 1000/(100+P_\beta)$.

In this second method for collecting data, only one diagram for end points constraints is needed because it is always true that $1000/(100+P_\beta)\le 10$ for $\forall\alpha$ and $1000(100-P_\beta)/\{(100+P_\alpha)(100+P_\beta)\}\le 10$ for $\forall\beta$.

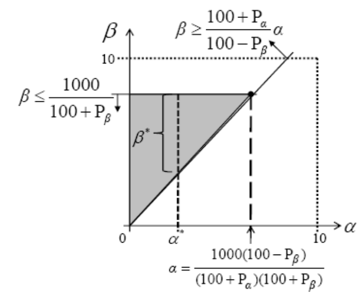


Fig. 6. The end points constraints. α^* and β^* are the choices made by an individual when P_α and P_β are given.

In order to assist an individual in choosing values for α and β from Fig. 6, (8) can be re-expressed as the following *end points constraints*:

$$\left.\begin{aligned} 0\le\alpha &\le \frac{1000(100-P_\beta)}{(100+P_\alpha)(100+P_\beta)} \\ \frac{100+P_\alpha}{100-P_\beta}\alpha \le \beta &\le \frac{1000}{100+P_\beta} \end{aligned}\right\}. \quad (9)$$

Given P_α and P_β, an individual first chooses $\alpha=\alpha^*$ with the help of the first inequality in (9), and then she chooses β^* with the help of the second inequality in (9), as shown in Fig. 6.

3) FOU classification diagram: With α and β chosen at this point, one can observe whether the word is going to be mapped into an Interior, LS or RS FOU by locating α and β on the FOU classification diagram in Fig. 3. If the resulting FOU seems plausible (e.g., an Interior FOU for a word such as *very small* is not plausible, nor is a LS FOU for the word *moderate amount*) then it is accepted for the word; otherwise P_α and P_β are re-specified until a plausible FOU is obtained.

The uncertainty bands are converted from percentages to numbers, using (1), in order to perform the calculations of the FS Part of the IIA.

V. FS Part of the IIA

In this section, we explain the FS part of the IIA that is depicted in the lower portion of Fig. 2. Note that the FS part of the IIA is quite similar to the FS part of the IA [1].

A. Assigned Probability Distribution

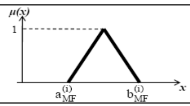
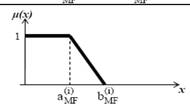
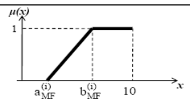
Regardless of the method in which data are collected, the following nine intervals are determined from the data: $[\alpha-\delta\alpha, \beta-\delta\beta]$, $[\alpha-\delta\alpha,\beta]$, $[\alpha-\delta\alpha,\beta+\delta\beta]$, $[\alpha,\beta-\delta\beta]$, $[\alpha,\beta]$, $[\alpha,\beta+\delta\beta]$, $[\alpha+\delta\alpha,\beta-\delta\beta]$, $[\alpha+\delta\alpha,\beta]$ and $[\alpha+\delta\alpha,\beta+\delta\beta]$. These intervals are denoted $[a^{(i)},b^{(i)}]$ (i=1,…,9). Note that these intervals overlap with one another. As in the IA, we assume that a uniform probability distribution is assigned to these two intervals.

B. Compute Embedded T1 FSs

Once a decision has been made in the data part of the IIA about the kind of FOU to use for a specific word, the nine intervals $[a^{(i)}, b^{(i)}]$ (i=1,…,9) are mapped into their respective embedded T1 FSs using the equations given in Table I (as in Table III of [1]), i.e.

$$[a^{(i)}, b^{(i)}] \Rightarrow (a_{MF}^{(i)}, b_{MF}^{(i)}), \quad i = 1,...,9. \tag{10}$$

TABLE I. TRANSFORMATIONS OF THE NINE UNIFORMLY DISTRIBUTED INTERVALS INTO THE PARAMETERS OF A T1 FS.

Name	MF	Transformations
Interior		$a_{MF}^{(i)} = \frac{1}{2}[(a^{(i)}+b^{(i)}) - \sqrt{2}(b^{(i)}-a^{(i)})]$ $b_{MF}^{(i)} = \frac{1}{2}[(a^{(i)}+b^{(i)}) + \sqrt{2}(b^{(i)}-a^{(i)})]$
Left-shoulder		$a_{MF}^{(i)} = \frac{(a^{(i)}+b^{(i)})}{2} - \frac{(b^{(i)}-a^{(i)})}{\sqrt{6}}$ $b_{MF}^{(i)} = \frac{(a^{(i)}+b^{(i)})}{2} + \frac{\sqrt{6}(b^{(i)}-a^{(i)})}{3}$
Right-shoulder		$a_{MF}^{(i)} = \frac{(a^{(i)}+b^{(i)})}{2} + \frac{\sqrt{6}(b^{(i)}-a^{(i)})}{3}$ $b_{MF}^{(i)} = \frac{(a^{(i)}+b^{(i)})}{2} - \frac{(a^{(i)}-b^{(i)})}{\sqrt{6}}$

C. Compute an IT2 FS Using the Union

Using the RT for an IT2FS [1], we compute the word's IT2 FS $\tilde{A}$ as

$$\tilde{A} = \bigcup_{i=1}^{9} A^{(i)}. \tag{11}$$

where $A^{(i)}$ denotes the just-computed T1 FSs for a word.

D. Compute Mathematical Model for FOU($\tilde{A}$)

In order to compute a mathematical model for FOU ($\tilde{A}$), both LMF and UMF of $\tilde{A}$ must be approximated. To do this we use the same approach as in [1].

Regardless of the type of FOU, the following four numbers must be computed first:

$$\left.\begin{aligned} \underline{a}_{MF} &\equiv \min_{i=1,...,9}\{a_{MF}^{(i)}\} \\ \overline{a}_{MF} &\equiv \max_{i=1,...,9}\{a_{MF}^{(i)}\} \end{aligned}\right\} \tag{12}$$

$$\left.\begin{aligned} \underline{b}_{MF} &\equiv \min_{i=1,...,9}\{b_{MF}^{(i)}\} \\ \overline{b}_{MF} &\equiv \max_{i=1,...,9}\{b_{MF}^{(i)}\} \end{aligned}\right\}. \tag{13}$$

For an Interior FOU, the following four numbers must be computed as well:

$$\left.\begin{aligned} \underline{c}_{MF} &\equiv \min_{i=1,...,9}\{(a_{MF}^{(i)}+b_{MF}^{(i)})/2\} \\ \overline{c}_{MF} &\equiv \max_{i=1,...,9}\{(a_{MF}^{(i)}+b_{MF}^{(i)})/2\} \end{aligned}\right\} \tag{14}$$

$$\left.\begin{aligned} p &= \frac{\underline{b}_{MF}(\overline{c}_{MF}-\overline{a}_{MF}) + \overline{a}_{MF}(\underline{b}_{MF}-\underline{c}_{MF})}{(\overline{c}_{MF}-\overline{a}_{MF}) + (\underline{b}_{MF}-\underline{c}_{MF})} \\ \mu_p &= \frac{\underline{b}_{MF}-p}{\underline{b}_{MF}-\underline{c}_{MF}} \end{aligned}\right\}. \tag{15}$$

These numbers are depicted in Fig. 7. As can be seen from the three parts of Fig. 7, the LMF and UMF of $\tilde{A}$ are bounded using piecewise-linear functions.

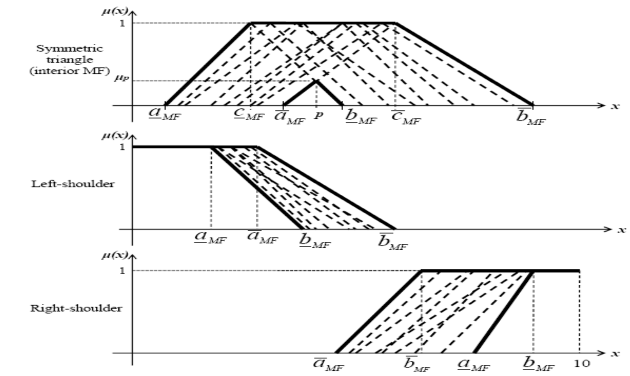


Fig. 7. IIA construction of Interior, LS, and RS FOUs. The heavy lines are the UMFs and LMFs for the FOUs.

The following parameters completely define the three FOUs:

$$\text{Interior FOU} \Leftrightarrow \{LMF(\overline{a}_{MF}, p, \mu_p, \underline{b}_{MF}), UMF(\underline{a}_{MF}, \underline{c}_{MF}, \overline{c}_{MF}, \overline{b}_{MF})\}. \tag{16}$$

$$\text{LS FOU} \Leftrightarrow \{LMF(\underline{a}_{MF}, \underline{b}_{MF}), UMF(\overline{a}_{MF}, \overline{b}_{MF})\}. \tag{17}$$

$$\text{RS FOU} \Leftrightarrow \{LMF(\underline{a}_{MF}, \underline{b}_{MF}), UMF(\overline{a}_{MF}, \overline{b}_{MF})\}. \tag{18}$$

VI. EXAMPLES

This section demonstrates the IIA for six words: *None to Very Little*, *Tiny*, *Small*, *Medium, Large*, and *Very High Amount*. The two different methods for collecting the data and their resulting FOUs are compared for these six words.

A. The Data Part of the IIA

Table II summarizes the data collection for our two methods for collecting data. The words are stated in the first column.

The next six columns show the data collection for the EFU method for collecting the data. The second and third columns show the two end points for each word that are provided by a typical individual. Those end points are depicted in the first diagram in Fig. 8 for each word to show where they are located on the FOU classification diagram. Using (7) one obtains the constraints on the left- and right-end uncertainty bands that are given in the fourth and sixth columns, respectively. Specific choices for those uncertainty bands, made by a typical individual, are in the fifth and seventh columns of the table.

The next six columns show the data collection for the UEF method for collecting data. The eighth and ninth columns show the uncertainty bands for each word that are provided by a typical individual. Using (9) one obtains the constraints on the left- and right-end points that are given in the tenth and twelfth columns, respectively. Specific choices for those two end points, made by a typical individual, are in the eleventh and thirteenth columns, respectively. Those end points are depicted in the second diagram in Fig. 8 for each word to show where they are located on the FOU classification diagram.

B. Results

The mathematical models for each word are given as codebooks in Tables III for the EFU and the UEF methods for collecting data. Observe that the numbers for the LMF and UMF are presented in the same order as in (16)-(18). Observe also that the end-points of the centroid of each FOU and the mean of each centroid are computed in the last two columns of the table as in [1]. The rank-ordering of the mean of the centroids provides a good way to rank order the words.

Regardless of which method the data are collected, *None to Very Little* and *Tiny* are mapped into LS FOUs, *Small*, *Medium,* and *Large* are mapped into Interior FOUs, and *Very High Amount* is mapped into a RS FOU. Their respective FOUs are depicted in Fig. 9. Observe that the results match our intuitions, i.e., the FOUs of small-sounding words are located to the left of large-sounding words. Observe, also, that the FOUs are not symmetric for Interior FOUs. Similar results were obtained in [1].

TABLE II. DATA COLLECTION AND CONSTRAINTS FOR TWO METHODS FOR COLLECTING DATA.

Word	The EFU Method for Collecting Data						The UEF Method for Collecting Data					
	α	β	Constraint on P_α	P_α (%)	Constraint on P_β	P_β (%)	P_α (%)	P_β (%)	Constraint on $\alpha \geq 0$	α	Constraint on β	β
None to Very Little	0	0.7	$P_{\alpha=0} = 0$	0	$P_\beta \leq 100$	25	0	40	$\alpha \leq 4.29$	0	$0 \leq \beta \leq 7.14$	1
Tiny	0.09	1.13	$P_\alpha \leq 100$	2	$P_\beta \leq 91.88$	13	1	7	$\alpha \leq 8.61$	0.2	$0.22 \leq \beta \leq 9.35$	2
Small	1.10	2.79	$P_\alpha \leq 100$	18	$P_\beta \leq 53.48$	21	22	27	$\alpha \leq 4.71$	1.5	$2.51 \leq \beta \leq 7.87$	3.5
Medium	4	6	$P_\alpha \leq 50$	10	$P_\beta \leq 26.67$	16	18	9	$\alpha \leq 7.08$	3.26	$4.23 \leq \beta \leq 9.17$	7
Large	7.5	9.21	$P_\alpha \leq 22.8$	5	$P_\beta \leq 8.58$	2	10	7	$\alpha \leq 7.9$	7.23	$8.55 \leq \beta \leq 9.35$	9.05
Very High Amount	8.5	9.8	$P_\alpha \leq 15.29$	7	$P_\beta \leq 2.04$	2	4	1	$\alpha \leq 9.43$	8.73	$9.17 \leq \beta \leq 9.90$	9.89

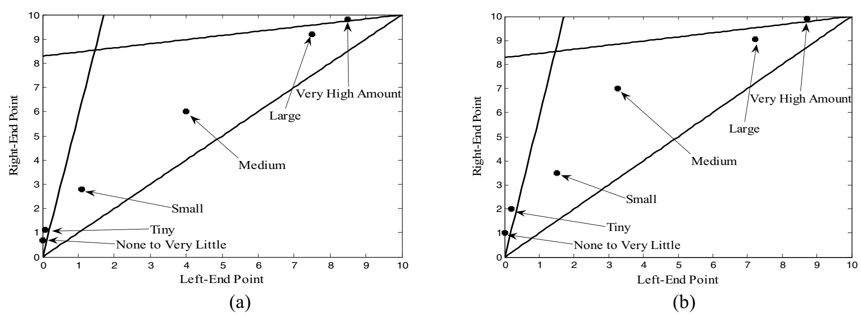


Fig. 8: FOU Classification diagram for six words using two methods for collecting data: (a) EFU and (b) UEF.

TABLE III. CODEBOOK – FOU DATA AND REGION FOR SIX WORDS OBTAINED BY USING TWO METHODS FOR COLLECTING DATA.

Word	The EFU Method for Collecting Data				The UEF Method for Collecting Data			
	LMF	UMF	Centroid	Mean	LMF	UMF	Centroid	Mean
None to Very Little	(0.05, 0.69)	(0.08, 1.15)	[0.23, 0.40]	0.31	(0.06, 0.79)	(0.13, 1.84)	[0.26, 0.65]	0.46
Tiny	(0.17, 1.27)	(0.20, 1.65)	[0.43, 0.56]	0.50	(0.35, 2.39)	(0.38, 2.75)	[0.81, 0.94]	0.87
Small	(1.11, 1.87, 0.62, 2.39)	(0.39, 1.55, 2.34, 3.89)	[1.41, 2.51]	1.96	(1.68, 2.33, 0.45, 2.71)	(0.49, 1.86, 3.14, 5.12)	[1.57, 3.58]	2.58
Medium	(4.27, 4.83, 0.40, 5.17)	(2.90, 4.32, 5.68, 7.66)	[3.97, 6.20]	5.08	(3.32, 5.12, 0.75, 6.89)	(1.65, 4.52, 5.74, 8.66)	[4.33, 5.93]	5.13
Large	(7.64, 8.38, 0.74, 9.26)	(6.66, 8.08, 8.63, 9.86)	[7.98, 8.72]	8.35	(7.86, 8.17, 0.33, 8.51)	(5.85, 7.46, 8.82, 10)	[6.94, 9.31]	8.12
Very High Amount	(7.24, 9.45)	(8.94, 9.91)	[8.97, 9.65]	9.30	(7.87, 9.66)	(8.85, 9.91)	[9.25, 9.62]	9.43

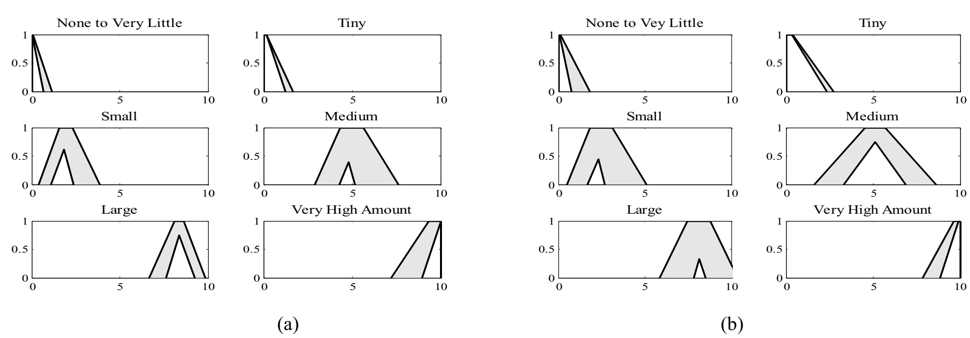


Fig. 9: Derived FOUs for six words using two methods for collecting data: (a) EFU and (b) UEF.

VII. Conclusions

This paper has presented a simple approach for obtaining an FOU of a word from a *single subject* by an IA called an *Individual IA* (IIA). In the IIA two interval end-points and their respective uncertainty bands are collected from an individual, and this can be done by two different methods. After the data collection, which also includes establishing whether the word will be modeled as a LS, Interior or RS FOU, by using the FOU classification diagram developed in [1], nine intervals are established, a uniform probability distribution is assumed for each interval, and those intervals are then mapped into their corresponding T1 MFs which are interpreted as embedded T1 FSs of an IT2 FS. These nine T1 FSs are then aggregated using union and their lower and upper bounds are used to obtain the mathematical model for the FOU of the word. Our examples have illustrated the IIA for the two methods for collecting the data. Although two methods have been described for collecting the data, it seems to us that the EFU method is preferable, because it is more natural to provide two interval end-points first than their respective uncertainty bands.

This IIA inherits all of the strong points of the IA and it simplifies it; however, this does not mean that we advocate using it in place of the IA. The IA is used when a group of subjects is available, whereas the IIA is used when only one subject is available.

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