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Restarting Multi-type Particle Swarm Optimization Using an Adaptive Selection of Particle Type

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***Abstract*—The particle swarm optimization method (PSO) is one of popular metaheuristic methods for global optimization problems. Although the PSO is simple and shows a good performance of finding a good solution, it is reported that almost all particles sometimes converge to an undesirable local minimum for some problems. Thus, many kinds of improved methods have been proposed to keep the diversity of the search process. In this paper, we propose a novel multi-type swarm PSO which uses two kinds of particles and multiple swarms including either kind of particles. All particles in each swarm search for solutions independently where the exchange of information between different swarms is restricted for the extensive exploration. In addition, the proposed model has the restarting system of inactive particles which initializes a trapped particle by resetting its velocity and position, and adaptively selects the kind of the particle according to which kind of particles contribute to improvement of the objective function. Furthermore, through some numerical experiments, we verify the abilities of the proposed model.**

***Index Terms*—particle swarm optimization, global optimization, multi-type swarms, restarting method**

I. Introduction

The particle swarm optimization method (PSO) is one of metaheuristic methods for global optimization inspired by swarms of birds or fish [6]. This method is very simple, and has a high performance to obtain a desirable solution. The exploration ability of this method is critical to find a high-quality solution. Thus, in order to improve the ability, various kinds of improved methods have been investigated [4].

The multi-swarm PSO is one of improved models in which all particles are divided into multiple swarms and particles in each swarm search for a solution independently to other swarms, where exchange of information between particles in different swarms is restricted to prevent all particles from gathering around a undesirable local minimum ([1], [2], [3], [7]). In Tribe-PSO proposed by Chen, Li and Cao [3], the information between different swarms are exchanged through the elite particles which find the best solution in each swarm. It is reported that this method enables particles in multiple swarms to search for solutions individually.

On the other hand, the method which restarts particles trapped at a local minimum has been also proposed [8], [9]. In these methods, if the element or the norm of the velocity of a particle is sufficiently small, its velocity is initialized by randomized numbers to avoid the stagnation. Then, since the risk of stagnation is reduced, particles are accelerated to converge. However, it remains possible that the restarted particle might be trapped at the same local minimum again.

In this paper, first, we propose a multi-swarm PSO with restarting method of inactive particles by considering the advantages and drawbacks of existing improved methods. The proposed model uses the elite and standard particles which are updated by the same dynamical systems in Tribe-PSO, and restarts a trapped particle by initializing not only its velocity but also its position. Furthermore, we improve the above multi-swam PSO by modifying the structure of multiple swarms, where two kinds of particles are used, and each swarm includes only one kind of particles. In addition, when a particle is restarted, the type of the particle and its assigned swarm are adaptively determined according to contribution of two kinds of particles to improving the global best solution. Thus, the proposed multi-type PSO can be expected to adjust the numbers of two kinds of particles to each objective function. Finally, through some numerical experiments, we compare the abilities of searching for solutions of the proposed models.

II. Particle Swarm Optimization

A. *Original Particle Swarm Optimization*

In this paper, we focus on the following global optimization problems having many local minima and the rectangular constraint.

$$\min \quad f(x) \quad \text{s.t.} \quad x \in \prod_{i=1}^{n} [d_{\min}, d_{\max}].$$

In order to solve this problem, in the PSO system a number of candidate solutions called *particles* are simultaneously updated by exchanging the information each other. At each iteration, particles move toward a linear combination of two best solutions called *the local best* $l^i(t)$ and *the global best* $g(t)$, where the former is the best solution obtained by each particle i until iteration t and the latter is the best one obtained by all particles until iteration t. Then, the update formula of particle $i \in \{1, \ldots, L\}$ is given by

$$\begin{aligned} v^i(t+1) &:= wv^i(t) + c_1 r^1 \otimes (g(t) - x^i(t)) \\ &\quad + c_2 r^2 \otimes (l^i(t) - x^i(t)), \qquad (1) \\ x^i(t+1) &:= x^i(t) + v^i(t), \end{aligned}$$

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where $x^i(t) \in \Re^n$ is a current point of particle i at iteration t, and w, c_1, $c_2 > 0$ are constant weights, while r_1, r_2 are randomized numbers uniformly selected from $(0,1)^n$. The operation $\otimes : \Re^n \times \Re^n \to \Re^n$ is defined by $(s \otimes t)_i := s_i t_i,\ i = 1, \ldots, n$. This extremely simple approach has been surprisingly effective across a variety of problem domains [6]. However, if parameter values are not appropriate, particles sometimes tend to converge quickly to a local minimum, and it is difficult to find a desirable solution. In this method, the exploration ability is critical to find desirable solutions. Hence, Eberhart and Shi proposed that a suitable selection of the inertia parameter w will provide a balance between global and local explorations called the inertia weight approach (IWA) [5], which linearly decreases w by

$$w(t) := w_{\max} - t\,\frac{w_{\max} - w_{\min}}{T}, \tag{2}$$

where $w_{\max}$ and $w_{\min}$ are the initial and final values of w. It is reported that PSO-IWA is better than OPSO, but that its performance is not so efficient for high-dimensional problems.

Besides, in order to improve the ability, various kinds of improved methods have been investigated [4]. The multi-swarm PSO is one of improved models in which all particles are divided into multiple swarms and particles in each swarm search for a solution independently to other swarms. Moreover, exchange of information between particles in different swarms is restricted to prevent all particles gathering around a local minimum ([1], [2], [3], [7]). In Tribe-PSO proposed in [3], the information of the best solutions found by each swarm are exchanged through a part of particles.

In this paper, first, we propose a multi-swarm PSO with a restarting system of inactive particles, which uses the update formulas of particles and the exchange rule of the information obtained by particles similar to Tribe-PSO [3]. Thus, in the next subsection, we briefly introduce Tribe-PSO.

B. Tribe-PSO

The Tribe-PSO proposed by Chen, Li and Cao uses multiple swarms, where all particles are divided into some groups. Although they call each group a *tribe*, in this paper we call the group a *swarm* for convenience. This method has three phases in which different rules are used to exchange the information of obtained solutions between swarms in order to take a balance of intensification and diversification. In the first phase, particles in each swarm search for solutions individually without exchange of any kind of information between different swarms. Particle i in swarm k is updated by the following equation:

$$\begin{aligned} v^i(t+1) := {} & wv^i(t) + c_1 r_1^i \otimes (l^i(t) - x^i(t)) \\ & + c_2 r_2^i \otimes (s^k(t) - x^i(t)), \end{aligned} \tag{3}$$

where $s^k(t)$ denotes the *swarm best* which is defined as the best solution found so far by all particles in swarm k. We call a particle updated by (3) a *standard* particle. Next, in the second phase, the particle that found $s^k(t)$ in swarm k is called the *elite particle* at iteration $t+1$, and elite particles are updated by the following equation:

$$\begin{aligned} v^i(t+1) := {} & wv^i(t) + c_1 r_1^i \otimes (s^k(t) - x^i(t)) \\ & + c_2 r_2^i \otimes (g(t) - x^i(t)), \end{aligned} \tag{4}$$

where $g(t)$ is the global best solution found so far by all particles which is the same one used in OPSO. All particles in each swarm except the elite particle search for solutions independently to other swarms by (3). In this phase, the information of swarm bests obtained in multiple swarms is exchanged gradually. Finally, in the third phase, all particles behave similarly to OPSO irrelevantly to swarms. The rates of the first, second and third phases to the maximal number of iterations are selected as 20%, 30% and 50%, respectively. It is reported that this model can keep the intensification and diversification at the same time, and thus, it achieves a good performance.

However, we can easily expect that there also exist particles trapped at an undesirable solution in the Tribe-PSO. In order to verify it, we executed an experiment where Tribe-PSO was applied to a 40-dimensional Rosenbrock function. If the velocity $v^i(t)$ of a particle i satisfies the condition:

$$||v^i(t)|| < (d_{\max} - d_{\min})\ \sqrt{n}\ v_{\mathrm{th}}, \tag{5}$$

where v_{th} is a threshold value and set by $v_{\mathrm{th}} = 0.001$, then particle i is regarded as being trapped at an undesirable solution and called *inactive*. Otherwise, it is called *active*. Fig. 1 displays the particle numbers which are inactive at each iteration $t \in [0, 500]$, that is, early stages. Fig. 1 shows that

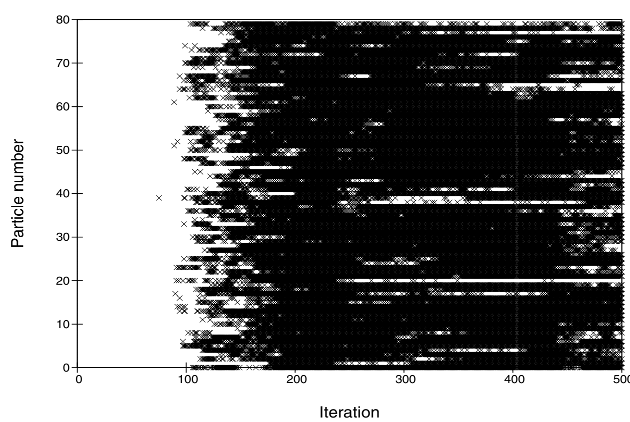


Fig. 1. Stagnation of particles in Tribe-PSO

when the global best is improved steeply at $t = 250$ or 450, some of inactive particles become active again, and however, almost all particles still remain inactive. Even though in the final stages the local search is important to find a high-quality solution around the global best, in early stages such searches executed by a large number of particles are not so effective because there still exist a lot of unexplored regions. Therefore, in this paper we propose a multi-swarm PSO which restarts inactive particles in order to avoid the above drawbacks.

III. Restarting Multi-Swarm Particle Swarm Optimization

In this section, we propose a multi-swarm PSO which uses elite and standard particles in the second phase of Tribe-PSO. We consider that since the proposed model with a restarting method can keep a balance of the diversification and intensification of the search, the first and third phases of Tribe-PSO are not necessarily required. Thus, the proposed model uses update rules (3) and (4) for elite and standard particles, respectively, throughout all stages of the search.

A. The proposed restarting method

In addition, the proposed model has the following restarting method. If the velocity $v^i(t)$ of particle i satisfies the following condition at iteration t:

$$||v^i(t)|| < (d_{\max} - d_{\min}) \sqrt{n}\; v_{\rm th}(t), \qquad (6)$$

then the particle is regarded as inactive. Then, its velocity $v^i(t+1)$ and position $x^i(t+1)$ at the next iteration $t+1$ are initialized by randomized numbers and its local best is also initialized as $l^i(t+1) := x^i(t+1)$. In this method, a selection of threshold value $v_{\rm th}(t)$ in (6) is important for the extensive search. However, since it is difficult to determine an appropriate constant value $v_{\rm th}$, it is varied on the basis of the number of particles restarted in the last 20 iterations. First, $v_{\rm th}(0)$ is initialized as 10^{-3}, and at each iteration t such that $t \bmod 20 = 0$, $v_{\rm th}(t)$ is adjusted as follows:

$$v_{\rm th}(t+1) := 1.07\, v_{\rm th}(t), \quad \text{if} \quad p_r(t) < 0.1\, N^p, \qquad (7)$$

$$v_{\rm th}(t+1) := 0.8\, v_{\rm th}(t), \quad \text{if} \quad p_r(t) > 0.2\, N^p, \qquad (8)$$

where N^p and $p_r(t)$ denote the number of all particles and the number of particles restarted within an interval $[t-19, t], t > 20$. Here, in order not to restart so many particles, the increasing rate of threshold in (7) is less than the decreasing rate in (8). Thus, this method can prevent the premature convergence without interrupting the local search of each particle. We call this model the restarting multi-swarm PSO (RMS-PSO).

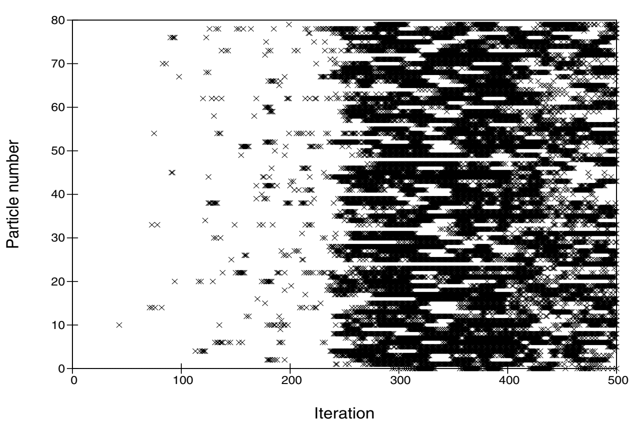


Fig. 2. Stagnation of particles in RMS-PSO

B. Comparison of other methods

Improved PSOs which restarts inactive particles have been investigated so far [8], [9], which restart a particle by initializing its velocity or an element of the velocity by randomized numbers. On the other hand, the proposed method initializes not only its velocity but also its position and its local best, which might seem to discard the important information of promising regions. However, the best solution obtained at each swarm is kept as the swarm best. In addition, we observed that when inactive particles are trapped at an undesirable solution, several active particles move around there in some experiments. Therefore, the proposed method can be expected to offer lower risk of losing the important information. Moreover, the method prevents restarted particles from being trapped again at the same local minimum.

C. Numerical experiments

Now, we executed the same experiment in the previous section which checks the particle numbers of inactive ones at each iteration. Then, we used the condition (5) for this experiment. Hence, notice that particles which do not satisfy (6) may be plotted. The result is shown in Fig. 2, which indicates that there are less inactive particles in RMS-PSO than in Tribe-PSO due to the restarting method.

Next, we compare the proposed RMS-PSO with other models. Thus, we applied RMS-PSO, PSO-IWA and Tribe-PSO, as mentioned in Section II, to six kinds of 20- or 40-dimensional benchmark problems, that is, Rastrigin, Rosenbrock, Griewank, 2n-minima, Schwefel and Ackley functions, where we used scaled 2n-minima and Schwefel functions whose optimal function value are 0. We executed preparatory experiments for Tribe-PSO and RMS-PSO, and selected a set of parameter values $(w, c_1, c_2) = (0.729, 1.494, 1.494)$ for Tribe-PSO, and two sets $(w, c_1, c_2) = (0.5, 1.85, 1.85)$ and $(0.6, 1.85, 1.0)$ for RMS-PSO. We call RMS-PSOs with the former and latter parameter sets RMS-PSO with type A and RMS-PSO with type B, respectively. The number of particles was 80 for all models, and the number of swarms was eight and each swarm includes ten particles in Tribe-PSO and RMS-PSO. The maximal number of iterations was set to be 10000. In order to compare three methods, we used the following two indices: One is the average function values over 100 trials which are obtained by three methods for each problem, and the other is the rate of trials in which the global optimal solution is found to 100 trials, which are shown in Table I at Section IV. We can observe that RMS-PSOs obtain considerably better function values than tribe PSO and PSO-IWA on average, and that RMS-PSOs find the global best solution in higher probability than two methods. These results imply that the proposed restarting method effectively enables inactive particles to search for solutions again without losing the important information obtained before restarting.

At the same time, we can see that RMS-PSO with type A is superior to Type B in Rastrigin, 2n-minima and Schwefel functions, while type B is to type A in other three functions. The result means that it is difficult to find an appropriate set of parameter values for all problems. Therefore, we furthermore propose a method which selects adaptively the parameters of restarted particles in the next section.

IV. Adaptive multi-swarm Particle Swarm Optimization

In this section, we improve RMS-PSO by introducing a method which adaptively selects the type of restarted particles. We call this method the adapting multi-type PSO (AMT-PSO).

This model uses multiple swarms and a restarting method similar to RMS-PSO, where elite and standard particles are updated by (3) and (4). Moreover, each particle uses either of two sets of parameter values $(w, c_1, c_2) = (0.5, 1.85, 1.85)$ and $(0.6, 1.85, 1.0)$, which are called A-type and B-type particles, respectively, and each swarm consists of either type of particles. The numbers of A-type and B-type particles at iteration t are represented by $N^p_{\mathrm{A}}(t)$ and $N^p_{\mathrm{B}}(t)$, and the numbers of A-type and B-type swarms at t are represented by $N^S_{\mathrm{A}}(t)$ and $N^S_{\mathrm{B(t)}}$, respectively. In addition, the number of particles in a swarm is called *swarm size* and the swarm size of swarm k is represented by N^S_k.

A. Adaptive restarting method

In this model, if a particle satisfies the condition (6) in which $v_{\mathrm{th}}(t)$ is updated by (7) and (8), the particle is restarted similarly to RMS-PSO. Then, the type of the particle is selected according to the contribution of two kinds of particles to improve the global best. Thus, we define the degree of contribution of particles with type X by

$$f^X_{sc}(t) = \frac{\sum_{i \in P_X} \sum_{\tau \in [0.8t, t]} \max\left[0,\ f(g(\tau-1)) - f(x^i(\tau))\right]}{|P_X|}, \quad X \in \{\mathrm{A}, \mathrm{B}\}, \quad (9)$$

which denotes the amount of function value reduced by X-type particles within an interval $[0.8t, t]$. When the function value of the global best $g(t)$ is steeply reduced by X-type particles, the degree of contribution $f^X_{sc}(t)$ becomes high. Otherwise, it is low. Then, the probability of selecting type X for a restarted particle at iteration t is given by

$$p^X(t) = \frac{f^X_{sc}(t)}{f^{\mathrm{A}}_{sc}(t) + f^{\mathrm{B}}_{sc}(t)},\ X \in \{\mathrm{A}, \mathrm{B}\}. \quad (10)$$

If the type of a particle is changed into type X, the particle is assigned to the X-type swarm with the smallest size. Hence, this method changes the rate of two kinds of particles which is suitable for each problem.

However, by the adaptive type selection of restarted particles, the number of particles in a swarm may become too large or too small. In most of multi-swarm PSO, it is reported that the method shows a good performance, when each swarm size is approximately ten [3], [7]. Thus, in the next subsection, we propose a method of adjusting each particle size and keeping it ten roughly.

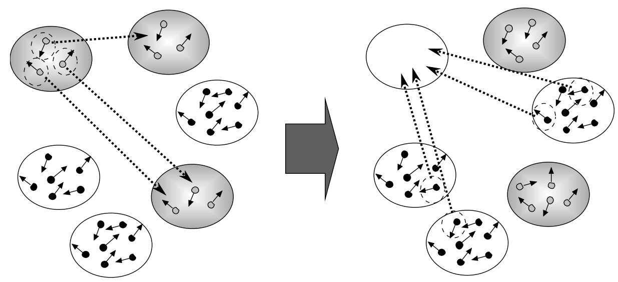


Fig. 3. Deletion of a small size swarm and formation of a new swarm

B. Deletion and formation of a swarm

This model adjusts the swarm sizes to avoid a large difference between swarm sizes, where the target average size of the swarm is set to be ten. At first $t = 0$, the initial numbers of both types of particles are equal, and thus, the initial numbers of both types of swarms are also equal. If the number of particles with type X increases by the adaptive type selection in the proposed restarting method, and in addition, if

$$\frac{N^p_X(t)}{10} - N^S_X(t) > 1 \quad (11)$$

is satisfied, a new X-type swarm is formed and $N^p_X(t+1) := N^p_X(t) + 1$, and for any X-type swarm k such that $N^S_k > 10$, $(N^S_k - 10)$ particles in swarm k are randomly selected and moved into the new swarm (Fig. 3). Next, the other type of swarm (Y-type) with the smallest size is deleted and $N^p_Y(t+1) := N^p_Y(t) - 1$, and its members are moved into the same type of swarms in decreasing order of swarm size. Here, in order not to delete all particles with either type, if the number of either type of swarms becomes one, the change of the particle type from the type to the other is not executed.

C. Numerical experiments

In this subsection, we applied the improved model, AMT-PSO, to some problems and verify its performance.

First, we compared the average transition of the global bests obtained by OPSO, Tribe-PSO and AMT-PSO for 40-dimensional Rosenbrock function over 100 trials. Fig. 4 shows that Tribe-PSO and AMT-PSO reduce the global best more rapidly than OPSO at early stages, which can be explained as follows: If once the search is stagnant in OPSO, all particles tend to gather together around the global minimum, and thus, the global best is not improved for the time being. On the other hand, since Tribe-PSO and AMT-PSO have multiple swarms, they can prevent all particles from converging to a solution at early stages. Moreover, when comparing Tribe-PSO and AMT-PSO, we can observe that AMT-PSO improves the global best more steeply than Tribe-PSO, which exhibits the effectiveness of the restarting system of particles and the adaptive type selection of restarted particles in AMT-PSO.

Next, we applied AMT-PSO to the same six benchmark problems in III-C, and compared with the results of PSO-IWA, Tribe-PSO and RMS-PSOs with type A and B. AMT-

TABLE I
THE AVERAGE FUNCTION VALUES FOR FOUR METHODS OVER 100 TRIALS

Function	n	PSO-IWA	Tribe-PSO	RMS-PSO (Type A)	RMS-PSO (Type B)	AMT-PSO
Rastrigin	20	8.04922 (0%)	8.29795 (0%)	**0.00000 (94%)**	0.17944 (62%)	0.13931 (90%)
	40	57.0195 (0%)	58.7422 (0%)	**5.3012** (0%)	19.4850 (0%)	12.8101 (**1%**)
Rosenbrock	20	7.83730 (0%)	7.03208 (0%)	0.08087 (0%)	0.11966 (21%)	**0.03989 (23%)**
	40	29.0570 (0%)	27.7698 (0%)	18.7695 (0%)	**0.2809** (0%)	0.3240 (0%)
Griewank	20	0.032814 (15%)	0.022638 (21%)	0.003325 (70%)	**0.000394 (95%)**	0.001118 (86%)
	40	0.012842 (30%)	0.021509 (16%)	0.021509 (36%)	**0.002413 (80%)**	0.002735 (74%)
2n-minima	20	182.95 (0%)	164.05 (0%)	**0.00 (100%)**	**0.00 (100%)**	**0.00 (100%)**
	40	401.80 (0%)	425.83 (0%)	**26.06 (43%)**	104.39 (4%)	39.33 (30%)
Schwefel ($\times 10^{-3}$)	20	1.06264 (0%)	1.08708 (0%)	**0.80692** (0%)	0.83071 (0%)	0.83130 (0%)
	40	2.33500 (0%)	2.25453 (0%)	**1.85516** (0%)	2.63486 (0%)	1.88580 (0%)
Ackley	20	0.39980 (98%)	**0.00000 (100%)**	**0.00000** (99%)	**0.00000 (100%)**	**0.00000 (100%)**
	40	2.79663 (86%)	1.86818 (72%)	1.59826 (48%)	**0.00000 (100%)**	**0.00000 (100%)**

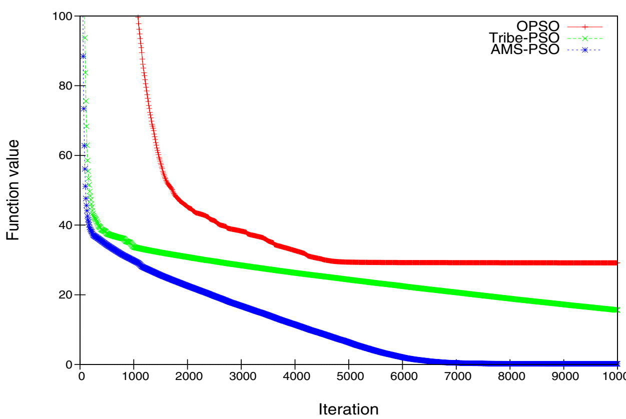


Fig. 4. The global bests obtained by OPSO, Tribe-PSO and AMT-PSO at each iteration t for 40-dimensional Rosenbrock function

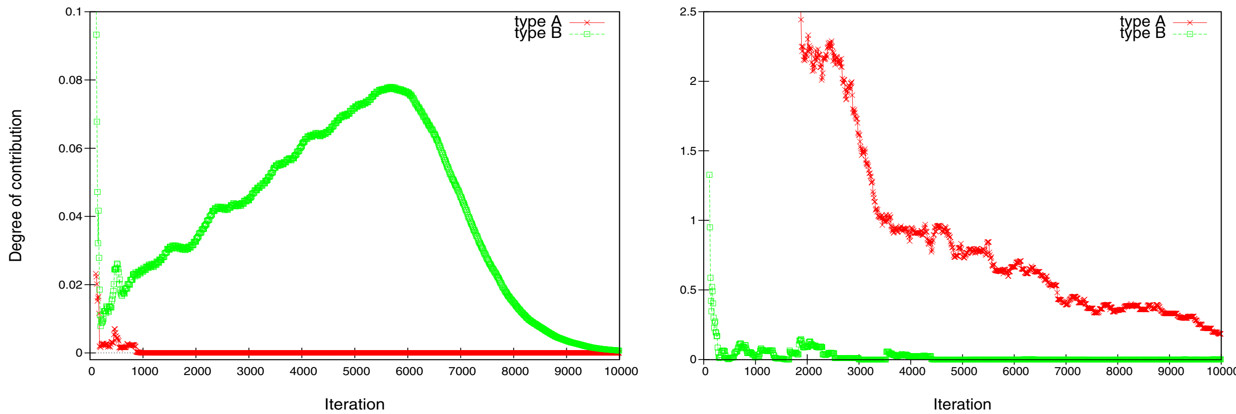
PSO uses 80 particles and eight swarms, and each swarm includes ten particles at the first iteration $t = 0$. The maximal number of iterations was set to be 10000. We evaluated the same two indices in III-C, the average obtained function value and the rate of trials in which the global optimal solution are found, which is shown in Table I. The bold and underlined numbers indicate the first and second best function values among four methods, respectively. This table shows that the proposed RMS-PSOs with type A and B and AMT-PSO obtained considerablely lower function values and found the global best solutions in higher probability than other two methods. Moreover, the results of AMT-PSO are almost the same as or second only to the best results of RMS-PSOs for each problem, which implies that the proposed adaptive type selection of restarted particles worked appropriately.

Finally, we analyze the process of adaptive type selection in detail. We focus on Rosenbrock and Schwefel functions. Fig. 5 shows the average degree of contribution $f_{sc}^{\mathrm{A}}(t)$ and $f_{sc}^{\mathrm{B}}(t)$ at each iteration t over 100 trials for 40-dimensional problems, while Fig. 6 shows the average numbers of the A-type particles at each t for 20- and 40-dimensional problems, . These results are not widely different from the result of each trial for the same functions, and rough shapes of the graphs are similar to each other. Figs. 5(a) and 6(a) imply that the degree of contribution $f_{sc}^{\mathrm{B}}(t)$ of B-type particles becomes large, and thus, the number of B-type particles becomes larger than that of A-type ones. Conversely, in Figs. 5(b) and 6(b) the opposite results can be observed. These results mean that the particle type which achieved the best results in RMS-PSO for each problem was finally selected in AMT-PSO. Therefore, we can conclude that the proposed adaptive type selection of of restarted particles works efficiently in the process of the search.

V. CONCLUSIONS

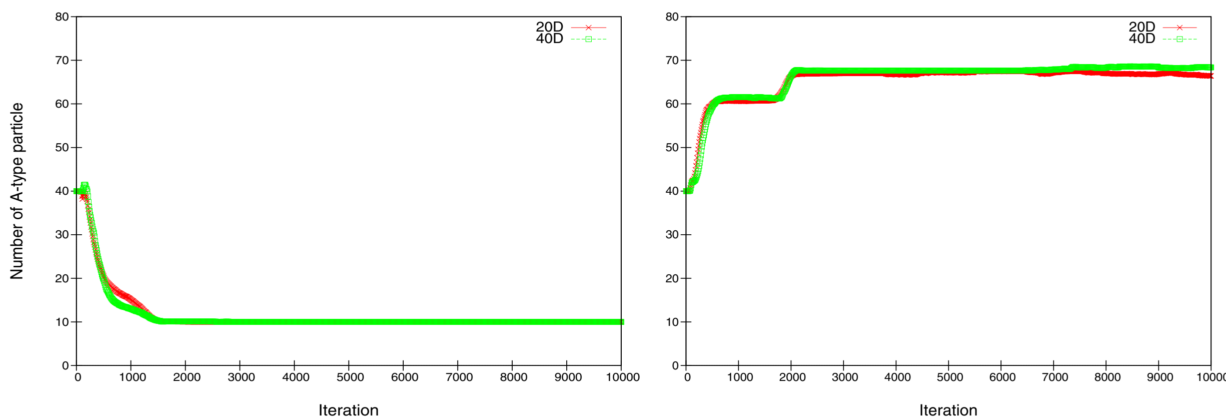
In this paper, we have focused on the PSO having multiple swarms, especially, Tribe-PSO proposed by Chen, Li and Cao. In order to overcome the drawbacks of the model, we have proposed a new model called RMS-PSO which restarts inactive particles. Moreover, we have improved it by introducing particles with two kinds of sets of parameter values, and proposing an adaptive type selection of restarted particles, which is called AMT-PSO. Then, we have applied the proposed models to six benchmark problems and observed that the proposed models are superior to Tribe-PSO and PSO-IWA in the sense of the average of obtained function values and the number of trials in which the global optimal solutions were found in 100 trials. In addition, by comparing RMS-PSOs using appropriate two sets of parameter values and AMT-PSO, we can see that the results of AMT-PSO are the almost same as or second only to the best results of two RMS-PSOs for almost all problems. In addition, the analysis of the numbers of two kinds of particles in AMT-PSO shows that the numbers are adaptively changed according to each problem. These results indicate that the proposed adaptive type selection works effectively.

Furthermore, the proposed model can not only resolve the drawbacks of the existing models, but also achieve the further improvements of the searching ability. Although we have used two kinds of particles in this paper, the proposed method of adaptively selecting the particle type can be applied to the multiple-swarm PSO using multiple kinds of particles updated by different dynamical systems. Therefore, as the future study, we should investigate such improvement of the proposed



(a) Rosenbrock (b) Schwefel

Fig. 5. The average degree of contribution of each type of particles over 100 trials at each iteration t



(a) Rosenbrock (b) Schwefel

Fig. 6. The average number of A-type particles over 100 trials at each iteration t

method which can adaptively select one among several particle types for restarted particles which is suitable for each problem.

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