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A Fast Region-based Image Segmentation Based on Least Square Method

Gang Chen[1,2], Tai Hu[1], Xiaoyong Guo[1], Xin Meng[1]
1 Lab of information simulation, Center for Space Science and Applied Research,
Chinese Academy of Sciences
2 Graduate University of Chinese Academy of Science
Beijing, China
chengangvip@tom.com, {hutai, xyguo, mengxin}@cssar.ac.cn

Abstract—**Image segmentation is always very important for computer vision and pattern recognition. Moreover, how to fast extract objects from a given image is still a problem for real time image processing. Most of the traditional region-based models depend on global information to converge to minimum error segmentation, but they are always time-consuming, and result in no effective segmentation. In this paper, we propose a region-based model with weight matrix to detect objects fast based on Least Square Method. The basic ideal of our model is to build up a minimum error functional by approximating objects and background of original image with two constants respectively. At the same time, we introduce a weight matrix into the region-based model, which can enhance the weight of objects while reducing the influence from background. Our method can fast converge through alternating iterations under Least Square Method. We also compare it with other region-based methods to show the improvements that can be achieved. Experimental results show the advantages of our method in terms of efficiency in image segmentation without losing accuracy.**

Keywords—**Active contour model, region-based model, image segmentation, weight matrix, threshold detection method, least square method**

I. INTRODUCTION

Image segmentation is to partition a given image into a set of regions that are meaningful and easier to analyze and recognize. There have been many methods to deal with this problem.

Active contour models have become one of the widely used models in image segmentation since M. Kass, A. Witkin, and D. Terzopoulos first put out snake model in 1987. Then, S. Osher and J. A. Sethian put out the level set method [7], which could tackle with curve topological change in image processing. With the development of active contour models [1-4, 6-8] in image segmentation, they are gradually formed into two classes: edge-based [1,3-5,11,13] models and region-based [2,8,10,12,16,18] models. These two types of models both have their pros and cons, and the choice of them in applications depends on different characteristics of images. Edge-based models utilize image gradients to stop the evolving contours on the object boundaries, and therefore can detect image boundaries. This type of highly localized image information is adequate in some situations, but has been found to be very sensitive to image noise and highly dependent on initial curve placement. In addition, this type of method is likely to pass through weak object boundary.

Compared with edge-based methods, the region-based model has the following advantages. First, region based methods are significantly less sensitive to the location of initial contours, such as the effects of noises. Another advantage is that the region-based methods do not utilize the gradient and have better performance for the image with weak boundary. Many works have been done in active contours since Mumford and Shah first put forward a region-based functional framework [8] in 1989. One of the most well-known and widely used region-based active contour models, such as Chan-Vese model [2] and A. Tai et al [18], assume the image is piece-wise constant or smooth to fit the original image to find an energy optimum. Recently, many models [12, 15-17] have been proposed for image segmentation. N. Paragios and R. Deriche[16] proposed a model based both region and edge information for supervised image segmentation. Li et al [15] put out a fitting functional to approximate original images to deal with intensity inhomogeneity. Shawn Lankton and Allen Tannenbaum in 2008 introduced the local mask functional to improve the accuracy of image segmentation. However, all these methods cannot be used for real time image processing. In addition, although these active contour models based on level set methods can topologically change, the process of the curve converging to the minimum functional is time-consuming and need re-initialization if we want the curve to evolve stably.

Another kind of methods that are widely used is the thresholding methods. Essentially there are two types of thresholding available to date: global thresholding and locally adaptive thresholding. Often, due to the variability in the gray level intensities and because of noise, the local thresholding method does not work satisfactorily. On the

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contrary, the global method does not consider local information in a given image. The Otsu's method [14] is based on histogram to find optimal threshold globally and adaptively that minimize the weighted within-cluster point scatter to segment image into a set of regions. This turns out to be the same as the maximizing the between-class scatter. However, the key weakness of the Otsu's method is also time-consuming because it needs to construct histogram and search all gray-level to find an optimal solution.

In this paper, we propose a region-based model with weight matrix to fast detect objects based on Least Square Method. The basic ideal of our model is to build up a minimum error functional by approximating objects and background of a given image with two constants respectively (Only gray-level image consider in this paper). At the same time, we introduce weight matrixes into our model by considering local image information to extract the objects of interests accurately and effectively. Unlike the Chan-Vese model under the level set methods, we implement the proposed model based on Least Square Method. Moreover, our model can converge faster than traditional methods by alternating iterations. Experimental results show our method advantages over traditional methods.

The rest of the article is organized as follows. In section 2, previous work on region-based models is reviewed. Details of the proposed method are described in section 3. Section 4 discusses implementation issues, and experimental results are given and analyzed. A summary is provided in section 5.

II. BACKGROUND

A. Mumford-Shah energy functional

Let us briefly describe the classical models based on Mumford-Shah model. Let Ω be the image domain, a bounded open subset of R^2, with $\partial\Omega$ its boundary, and $I:\Omega->\mathrm{R}$ be a given image in which we want to detect its objects boundaries. Mumford and Shah [8] proposed the following energy functional:

$$E(u,\overline{\mathrm{C}})=\beta\iint_{\Omega}(u-I)^2\,dA$$
$$+\alpha\iint_{\Omega\setminus C}|\nabla u|^2 dA+\gamma\oint_C ds, \qquad (1)$$

where α, β and γ control the competition between the various terms above and determine the "scale" of the segmentation and smoothing. u is the fidelity of the original image I in the first term, the second term means u does not varies too much in each sub-regions. The Mumford-Shah problem is to minimize $E(u,\vec{C})$ over admissible u and $\vec{C}$. However, $E(u,\vec{C})$ contains the partial differential term of u, which makes the calculation is very complicated in the practical usage.

B. Chan-Vese model

For practical applications, much works have been down to modify or simplify the above Mumford-Shah functional. One of well known approaches is the piecewise constant model proposed by Chan and Vese (C-V model) [2], which can be written as:

$$E(\overline{\mathrm{C}},c1,c2)=\lambda_1\iint_{in(C)}|I-c_1|^2\,dA$$
$$+\lambda_2\iint_{out(C)}|I-c_2|^2\,dA+\nu\left|\overline{C}\right|, \qquad (2)$$

where $in(\vec{C})$ and $out(\vec{C})$ represent the region inside and outside of the contour C respectively, and c_1 and c_2 are two constants that approximate the image intensity in $in(\vec{C})$ and $out(\vec{C})$. $|\vec{C}|$ is the arc-length of the curve or object boundaries. λ_1, λ_2 and ν are positive constant coefficients. However, such method can only be effective for homogeneity image composed of many piece constant regions. The global energies find segmentations that are consistent with their underlying assumptions about image content but are ultimately incorrect. For example, the building image is displayed in the Fig.5 (a). The experimental result in the Fig. 5(b) makes using of C-V model, which can only detect brightest areas and lead to incorrect segmentation. Although, Chan and Vese later put out piece-smooth model [10], this method needs to solve second order derivates and always have complex calculations.

C. Otsu's method

Image thresholding or histogram threshold detection algorithm is very useful for keeping the significant part of an image and getting rid of the unimportant part or noise. This holds true under the assumption that a reasonable threshold value is chosen.

In computer vision and image processing, Otsu's method [14] is used to automatically perform histogram shape-based image thresholding, or, the reduction of a gray level image to a binary image. The algorithm assumes that the image to be thresholded contains two classes of pixels (e.g. foreground and background) then calculates the optimum threshold separating those two classes so that their combined spread (within-class variance) is minimal.

By using the Otsu's method we exhaustively search for the threshold that minimizes the within-class variance, defined as a weighted sum of variances of the two classes:

$$\sigma_w{}^2(T)=\min_t\{\varpi_1(t)\sigma_1{}^2(t)+\varpi_2(t)\sigma_2{}^2(t)\} \quad (3)$$

where weights ϖ_i ($i=1,2$) are the probabilities of the two classes separated by a threshold t and σ_i^2 variances of these two classes.

III. DESCRIPTION OF THE MODEL

In this section, we propose a minimum error functional which introduces a weight matrix into region-based model. Moreover, our model can converge fast under the Least Square Method. The disadvantage of Chan-Vese is that it converges slowly and needs to reinitialize in order to keep stable evolution. Although many thresholding methods can adaptively find the optimal threshold, it is time-consuming for them to converge to minimum or maximum. The proposed model is inspired by the Chan-Vese model and thresholding methods, and wants to overcome the limitations of these two methods. The following parts will give details description to our model.

A. Region-based model with weights

For a given image I_0 with size $m \times n$ in image domain Ω, assume that the image I_0 is formed by two regions of approximately piecewise-constant intensities, of distinct values c_1 and c_2. Assume further that the object to be detected is represented by the region with the value c_1, which will approximate the object by minimizing the error functional. Considering the region-based models, we define the following energy functional as:

$$E(w_1, w_2, c_1, c_2) = \sum_{j=1}^{n}\sum_{i=1}^{m} w_1 \,| I_0(i,j) - c_1 |^2 + \sum_{j=1}^{n}\sum_{i=1}^{m} w_2 \,| I_0(i,j) - c_2 |^2 \tag{4}$$

where $I_0 : \overline{\Omega} -> R$ is a given image(only grey valued image considered in this paper), c_1 and c_2 are the constants to approximate the object and background respectively. w_1 and w_2 are weight matrixes in the two terms respectively. The purpose of w_1 is to enhance the weight of object and ignore the contribution from the background, while w_2 will enhance the background and restrict the object's weight. Note that we want to minimize the error functional in order to find the best c_1 and c_2 to fit the original image.

From the definition of $E(w_1, w_2, c_1, c_2)$, we can see that the energy functional is the accumulated error which satisfies the principle of the least square method. The weight matrixes w_1 and w_2 are crucial for the successful image segmentation, which will be defined in the next part. Keeping w_1 and w_2 fixed and minimizing the energy functional $E(w_1, w_2, c_1, c_2)$ with respect to the constants c_1 and c_2, it is easy to express these constants by the weights w_1 and w_2 as

$$c_1 = \frac{\sum_{j=1}^{n}\sum_{i=1}^{m} w_1(i,j) I_0(i,j)}{\sum_{j=1}^{n}\sum_{i=1}^{m} w_1(i,j)} \tag{5},$$

if $\sum_{j=1}^{n}\sum_{i=1}^{m} w_1(i,j) > 0$ (i.e. if the all $w_1(i,j)$ has positive value in image domain Ω), and

$$c_2 = \frac{\sum_{j=1}^{n}\sum_{i=1}^{m} w_2(i,j) I_0(i,j)}{\sum_{j=1}^{n}\sum_{i=1}^{m} w_2(i,j)} \tag{6},$$

if $\sum_{j=1}^{n}\sum_{i=1}^{m} w_2(i,j)$ (i.e. if the all $w_2(i,j)$ >0 in the image domain Ω).

According to the Least Square Method, c_1 and c_2 are the best fitting values, given w_1 and w_2 respectively. If the matrixes w_1 and w_2 are fixed, we can find the optimal c_1 and c_2. However, if w_1 and w_2 are changing in the iterations, we should update c_1 and c_2, then update in turn w_1 and w_2 in an alternating way.

As a natural application of our model, the finial fitting matrixes w_1 and w_2, as well as c_1 and c_2 can be used for image denosing, we define the following formula:

$$u = c_1 w_1 + c_2 w_2 \tag{7}$$

The above computer image u can be used to approximate original image while reducing image noise.

B. Weight matrix definition

Different usages can have their own definition of weight matrix. In this part, we present a way to design a weight matrix. The purpose of our method is to extract the objects of interest. In order to achieve this goal, we can design a matrix that can enhance the weight of objects at the same time reduce the influence from the background.

For a given gray level image with size $m \times n$, we want to normalize the two weighted matrixes w_1 and w_2. In this paper, we define the following weighted matrix:

$$\begin{cases} w_1 = | I_0 - c_2 | / 255 \\ w_2 = 1 - w_1 \ \ or \ \ | I_0 - c_1 | / 255 \end{cases} \tag{8}$$

where I_0 is a given image, c_1 and c_2 are the optimal intensities. w_1 and w_2 are the weights of the object and background respectively. Just as the Otsu's method, we introduce the weighted matrixes in our model. We can see that w_1 approximates to zero in the background of image, while enhance the weight of the object which is represented by c_1. By the same analyzing, the w_2 also satisfy our requirement.

From the definition of Eq. (8), we can see the weighted matrixes are constructed based on the original image I_0, so that our model consider the every pixel's contribution by introducing the weighted matrix in our global energy functional (4), while means our model consider both the local and global information in the image segmentation. Thus, our model can segment image more effectively and accurately.

C. Relationship with other region-based models

The Chan-Vese model makes use of the Heaviside function H and level set function ϕ to represent the two weighted matrixes of the object and background with $H(\phi)$ and $1-H(\phi)$ respectively, and then keep the contour evolving under the level set framework. Besides, the energy functional in the Chan-Vese model is also an accumulated error, which needs to be minimized to find the best fitting constants to approximate the original image. But the Chan-Vese model does not take local information into account. Our model constructs the weights w_1 and w_2 from a different perspective become we consider the every pixel's contribution in the energy function. What's more, we employ alternating iteration to update both the weights (w_1 and w_2) and constants (c_1 and c_2) rather than utilize the level set function to evolve the curve.

Just like the Otsu's method, our model leverages two weighted matrixes, and also needs to find the optimal thresholds c_1 and c_2 by iterations. But our model can converge fast without operating on the gray level histogram like the Otsu's method. Moreover, our method is based on least square method, which can help us find a best candidate in each iteration.

In fact, the basic principle behind the region-based models, including both the Chan-Vese model and Otsu's method is to minimize the accumulated error functional, which is the same with our model. The main advantage of our model can converge fast through alternating iteration under the least square method. Moreover, our model can segment image more accurately with weighted matrix.

IV. IMPLEMENTATIONS AND EXPERIMENTAL RESULTS

In this section, we address the algorithm steps in implementing the presented model. Moreover, our method has been applied to images of different modalities.

A. Algorithm implementation

*step*1: initialize the weights w_1 and w_2 respectively, the iterative times *n*.
*step*2: *while* $i < n$ *do*
*step*3: *update* c_1 and c_2 according to Eq. (5) and (6) respectively. *If* $|c_{1,i+1} - c_{1,i}| < \varepsilon$ and $|c_{2,i+1} - c_{2,i}| < \varepsilon$ *then* break out.
*step*4: calculate w_1 and w_2 according to Eq. (8) respectively. *Update* $i = i + 1$;
step5: *end while*;
setp6: output the optimal segmentation results.

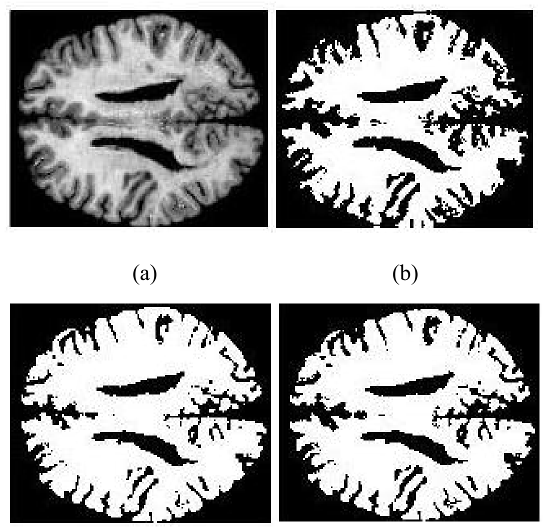
where ε is a small positive constant. Note that weights w_i and constants c_i ($i = 1, 2$) are updated in an alternating way. That means w_1 and w_2 will update the energy functional, which in turn will update c_1 and c_2, then they will update weights w_1 and w_2. When c_1 and c_2 do not change, it means we have find an optimal or approximate solution. Generally, our algorithm can converge from 10 to 30 iterations, which is very fast compared with Otsu's method and Chan-Vese model.

B. Experimental results

In this part, we present the experimental results using our model described in this work. Here, we demonstrate the improvements that are offered by introducing the weight matrix into region based model. The proposed method has been tested with real images of different modalities.

We initialize ε =10^{-5}, w_1 = 0.5*ones(m, n), which indicates that every element equals to 1 in the $m \times n$ matrix, $w_2 = 1 - w_1$. We realized our model with matlab code run on a Dell GX270 PC, with Pentium 4 processor, 3.0 GHz, 512M RAM.

Fig. 1(a) is the MR image of head. Fig.1 (b) is the result using level set method in [2]. Our result is shown in Fig.1 (d). It is hard to say which method is better because their results are almost identical. But, our method is very fast because our model only uses 0.0938 second in the Tab. (1) to extract object.



(a) (b) (c) (d)

Fig. 1. (a) The original MRI brain image. (b) The result using level set. (c) The result of Otsu's method. (d) The result of our model

(a) (b)

Fig. 2. (a) The original Lena image. (b) The result using level set. (c) The result of Otsu's method. (d) The result of our model

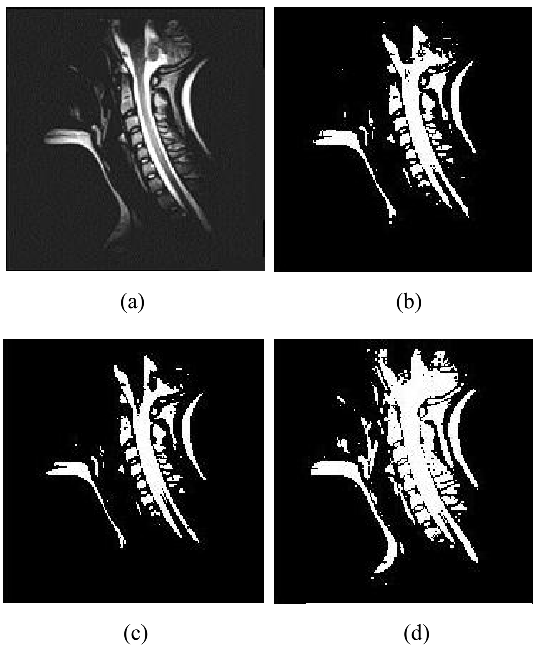


Fig. 3. (a) The original MRI Cervical image. (b) The result using level set. (c) The result of Otsu's method. (d) The result of our model

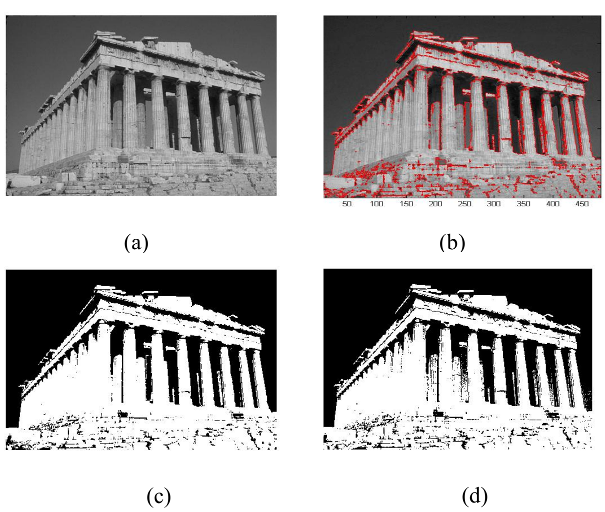


Fig. 4. (a) The original image of monkey. (b) The result using level set. (c) The thresholding result of Chan-Vese model of (b). (d) The result of our model

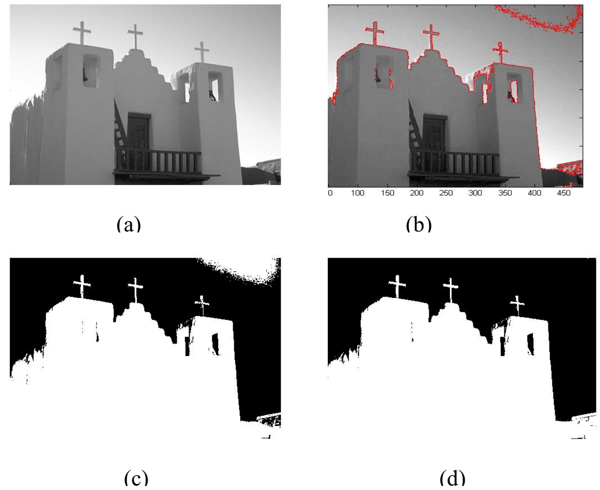


Fig. 5. The building detection with Chan-Vese model and our method. (a) The original image. (b) The result of Chan-Vese model with contour. (c) The thresholding result of Chan-Vese model of (b). (d) The result of our method.

TABLE 1. The time consumed for both the Chan-Vese model and our method. Our method is very faster compared with Chan-Vese model.

Time / Experiment	Image size (m*n)	C-V model	Our model(sec)
Fig. 1(Brain)	157*122	2.4375	0.0938
Fig. 2(Lena)	131*131	3.0313	0.0469
Fig. 3(Cervical)	182*182	17.2031	0.1250
Fig. 4(Building)	481*321	244.8125	1.0313
Fig. 5(Building)	481*321	102.7500	1.0625

Fig. 2(a) is the Lena image. The Fig. 2(b) is the result of level set method. The Fig. 2(c) is result using Otsu's method, which is almost the same as the Chan-Vese model. The Fig. 2(d) is the result of our model. Our method extracts more details. In a sense, the results of these methods are almost same.

Fig. 3 is a cervical MR image. The results of Fig. 3(b) and Fig. 3(c) are almost the same. The level set methods can only extract the brightest area, while ignore weak intensity area as shown in Fig. 3(b). From the Fig. 3(c), we can understand Otsu's method also cannot extract weak intensity region. Fig. 3(d) is our result, which extract weak boundary information, and successfully partition the cervical MR image.

Fig. 4 and Fig. 5 show the results of both Chan-Vese model and our method. Sometimes, our model can extract the building boundary more accurately than Chan-Vese model in Fig. 5. Moreover, our model converge very fast even if the image size increase greatly.

Table. 1 is the time consumed by both Chan-Vese model and our method. It shows that our method is very fast for image segmentation by alternating iterations. Moreover, when the intensity difference of background and foreground is small, the Chan-Vese model converges slowly, which could be found the in Fig. 4 and Table. 1. In our experiment, it cost the Chan-Vese model almost about 244 seconds to segment the building image in the Fig. 4. In addition, when image size increase, the time for the curve evolving to object boundaries also increase faster.

Thus, we can get that Otsu's method and Chan-Vese model are not suitable for real time image processing. Moreover, the global energy using level set methods or Otsu's method finds only the brightest parts of the image, which might lead to incorrect segmentation, while our model consider local information by introducing the weighted matrix. Although both Otsu's method and Chan-Vese model rely on minimum segmentation error requirements, it ignores the local image information in the image segmentation. As a result, our model can segment images more effectively and accurately. Besides, our model can converge fast in less than 30 iterations, which is efficient in practical application.

C. *Discussion about our model*

The weight matrix in our model is very important for the successful image segmentation because it takes the local image information into consideration. We also test our model with different initialization of matrix w_1 and w_2, and we get the same results. So our model is insensitive to the initialization of w_1 and w_2.

Although all the methods above, including our method, Chan-Vese and Otsu's method are based on the minimum segmentation error requirement, they might work differently because they have different energy functional. From the experiments above, our method can segment image faster than Chan-Vese model or Otsu's method through alternating iteration under least square methods. Moreover, our model can always segment image accurately by introducing a weight matrix into region-based models.

As for the weight matrix, we only consider a very simple method to construct it. Thus, it may result unsatisfied image segmentation. For example, both our model and Chan-Vese model cannot extract the correct building boundaries in the Fig. 4. In practice, different kind of images can design their specific weight matrix according to different requirement in order to improve the accuracy of image segmentation.

V. CONCLUSIONS

In this paper, we have presented a region-based model utilizing both global and local information for real time image segmentation. Considering many region-based methods always result in image details losing, we introduce a weight matrix into region-based model, which takes the local information into consideration. Moreover, our model can be used for real time image processing because our model find optimal solution fast through alternating iteration under least square method. Compared with other region-based models, this proposed model can detect the objects of interests more accurately and effectively. Thus, our model has practical application in image segmentation.

In the further work, we will design more sophisticated weight matrix in our model to improve the segmentation accuracy for more complex images.

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