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# Interval type-2 fuzzy logic system to simulate the environment resources stochasticity inducing the population growth shape

Cecilia Leal-Ramirez*, Oscar Castillo§, Antonio Rodriguez-Diaz*and Patricia Melin§

*Faculty of Chemical Sciences and Engineering
Autonomous University of Baja California
Tijuana Baja California, México
cleal@cicese.mx, ardiaz@uabc.mx

§Research and Graduate Studies
Tijuana Institute of Technology
Tijuana Baja California, México
ocastillo@hafsamx.org, pmelin@hafsamx.org

***Abstract*—An interval type-2 fuzzy logic system (IT2-FLS) is designed to evaluate the parameters of a population growth model based on the environment resources stochasticity, in time and space. Interval type-2 fuzzy sets are used to measure the uncertainties of the environment resources. The main goal of this work is to demonstrate how the IT2-FLS integrated into a population growth model can make a suitable evaluation of the parameters required to make that the population size reach a stable equilibrium level on which it fluctuates into a time interval and after that, population size goes down as consequence of insufficient resources. The simulation results are compared against the results of others mathematical models made through the years in Ecology to study the population dynamics.**

***Keywords*—*interval* type-2 fuzzy logic system, population growth model, environment resources.**

## I. INTRODUCTION

Population ecology is concerned with fluctuations in population size and the parameters (natality, mortality, emigration and immigration) that regulate these fluctuations. The majority of the mathematical models made through the years in Ecology describe the population growth by reaching a stable equilibrium level on which the population size fluctuates in a time interval [1]. The parameter's values that determine this growth shape are uncertain because the values depend of the presence of many environment resources [2]. It is difficult to measure the exact amount of the environment resources present within the place wherein the population is developed. Fuzzy logic (FL) originally introduced by [3] to model vagueness and uncertainty in the real world, plays an important role in the development of ecological control problems [4]. FL allows evaluating the population growth parameters using fuzzy rules. In this paper an IT2-FLS is designed and integrated into a population growth model, whose structure is defined on cellular automata (CA), which was introduced by [4] to model complex dynamic systems in time and space. The combination between FL and CA has had a great variety of applications, for example: the propagation of epidemics [5], patterns recognition [6], [7] and as a model to generate interesting images mimicking nature [8], [9].

In a previous research work made by [10], a type-1 fuzzy logic system (T1-FLS) was designed and integrated into a population growth model. T*he validation of this model was made within a comparative frame with others two different population models, in one of them variability in the mortality and reproduction rates were not considered [11], and the other one was the Verhulst's model [12], wherein these rates are constants and there is no emigration. This comparison* demonstrated that T1-FLS allows a better evaluation of the growth parameters than the models proposed by [11] and [12].

Interval type-2 fuzzy logic systems have been successfully applied in different investigation fields such as automatic control, data classification, decision analysis, expert systems, and computer vision [13]. We want to show that this also can be applied in ecology to study the population dynamics.

In the second section of this paper the architecture of the population growth model is presented in simplified form. In the third section a description on the IT2-FLS is detailed. A simulation based on the model proposed in the present work is made in forth section. The trajectories described by our model are compared against the trajectories described by Verhulst'*s model, and* are shown as results. In a last section conclusions about the representativeness and applicability of this kind of model are presented.

## II. POPULATIONN GROWTH MODEL

Our population growth model is based on the model structure proposed by [10]

$$\begin{aligned} N(C(i,j),t+1) = \beta(\cdot), N(C(i,j),t) - \lambda(\cdot)N(C(i,j),t) \\ -\sigma(\cdot)N(C(i,j),t) + \prod_{r}(k,l,t) \end{aligned}, \quad (1)$$

wherein the change in the population size is mainly given by the difference from four related terms: birth, death, emigration (individuals that leave one place to settle in another) and immigration (arrival of new individuals into population) of the individuals, which are proportional to the population size [11]:

Births: $$\beta(\cdot)N(C(i,j),t), \quad (2)$$

Deaths: $$\lambda(\cdot)N(C(i,j),t), \quad (3)$$

Emigrations: $$\sigma(\cdot)N(C(i,j),t), \quad (4)$$

Immigrations: $$\prod_r (k,l,t) \cdot \quad (5)$$

The model structure (1) is constituted by a collection of $M \times N$ cells arranged in a bi-dimensional array. Each cell $C(i,j)$, where $1 \le i \le M$ and $1 \le j \le N$, has a set of states with different values, which change at successive discrete steps by the iteration of a local transition function that depends on the states of the “neighboring" cells. The global evolution of the model (1) is defined by the synchronous update of all states according to the local function applied to each cell. The neighbourhood of the cell $C(i,j)$ in terms of a radius $r$ is given by

$$\Pi_r(i,j) = \left\{ C(k,l) \middle| \begin{array}{l} \max\{|k-i|,|l-j|\} \le r \\ 1 \le k \le M; 1 \le l \le N \end{array} \right., \quad (6)$$

which is a positive integer number, and $(k,l)$ is the coordinate of another cell where the magnitude of the difference between $(i,k)$ and $(l,j)$ does not exceed the value of $r$. In model (1) the population emigrates towards the neighbouring cells from a cell $C(i,j)$ according to a local rule.

The stochasticity of the environmental resources affects the population growth in two important ways, both positive and negative. The population always looks for environment resources under which can be developed. Areas rich in resources (e.g. food, available space, etc) tend to have higher population density, whereas areas with few resources tend to have lower population density.

In the model (1) the reproduction rate $\beta$, mortality rate $\lambda$ and emigration rate $\sigma$ are defined as variables that depend only on the state of the environment resources available ($R$) in the cell $C(i,j)$ and carrying capacity ($K$) that can afford the cell $C(i,j)$, in the time $t$.

$$\beta(\cdot) = \beta(R(C(i,j),t), K(C(i,j),t)), \quad (7)$$

$$\lambda(\cdot) = \lambda(R(C(i,j),t), K(C(i,j),t)), \quad (8)$$

$$\sigma(\cdot) = \sigma(R(C(i,j),t), K(C(i,j),t)). \quad (9)$$

The carrying capacity is the number of individuals that the environment can support without significant negative impacts to the population growth [14].

## III. Interval Type-2 Fuzzy System

Type-2 fuzzy sets (T2-FSs) are fuzzy-fuzzy sets wherein membership grades are type-1 fuzzy sets [3]. This kind of sets can be used when the circumstances are too uncertain to determine exact membership grades [15]. Unfortunately, T2-FSs are more difficult to use and to understand than type-1 fuzzy sets. T2-FSs are three-dimensional and the amplitude of their secondary membership functions (called secondary grade) can be in [0,1]. When the domain of a secondary membership function (called the primary membership) bounds a region (called the footprint of uncertainty, FOU) whose secondary grades all equal one, the resulting T2-FS is called interval type-2 fuzzy set (IT2-FS) [13], which can easily be depicted in two dimensions instead of three (Fig. 1).

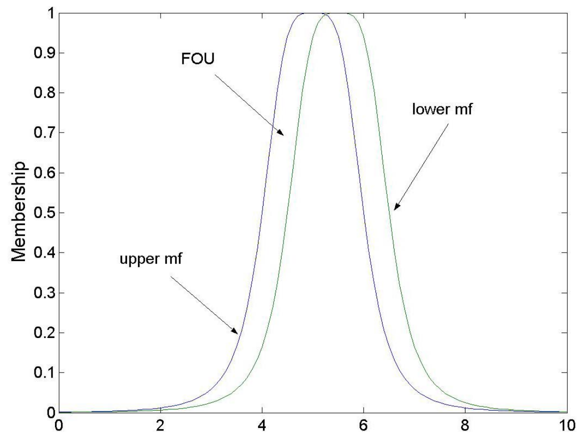


Figure 1. FOU for Gaussian primary upper and lower membership functions.

The interval type-2 fuzzy sets are not computationally as complicated as the type-2 fuzzy sets; however they can handle noisy data by making use of an interval of uncertainty (FOU in Fig. 1). We briefly introduce the notation of an IT2-FS and their associated membership functions (lower and upper). For details, please see [13].

$$\widetilde{A} = \{(x,u,1) \mid \forall x \in X, \forall u \in [\underline{\mu}_{\lambda}(x), \overline{\mu}_{\lambda}(x)] \subseteq [0,1]\} \quad (10)$$

An upper membership function (upper mf) and a lower membership function (lower mf) of $\widetilde{A}$ are two type-1 membership functions that bound for the FOU of an IT2-FS $\widetilde{A}$.

### A. Definition of the input and output variables

Systems using IT2-FSs are called IT2-FLS. The terms used to describe the parts of an IT2-FLS is far from standard [13], which include the interval type-2 fuzzyfier, rule-base, inference engine, type-reducer and a defuzzyfier. In this work, an IT2-FLS was designed using the Interval Type-2 Fuzzy Logic

Toolbox [15] under the Mamdani-type interval type-2 fuzzy inference.

Rates change values ($\beta$, $\lambda$ and $\sigma$) are considered as uncertain in our model, therefore $R$ and $K$ are elucidated as input variables into the IT2-FLS, and are defined in terms of interval type-2 fuzzy sets according to the definition expressed in (10)

$$\beta(\cdot) = \beta\left(\widetilde{R}(C(i,j),t), \widetilde{K}(C(i,j),t)\right), \tag{11}$$

$$\lambda(\cdot) = \lambda\left(\widetilde{R}(C(i,j),t), \widetilde{K}(C(i,j),t)\right), \tag{12}$$

$$\sigma(\cdot) = \sigma\left(\widetilde{R}(C(i,j),t), \widetilde{K}(C(i,j),t)\right). \tag{13}$$

There is no defined pattern to determine the interval given by the upper and lower membership functions that characterize the FOU for the fuzzy sets $F$ and $K$. However, it is necessary to define a suitable interval to bring stability in the model and enhance surviving of the species in a longer time interval [16].

$K$ is characterized by three interval type-2 fuzzy sets (Few, Moderate, Much) over a values range from 0 to 100 (Fig. 2), which represent the number of individuals in the cell $C(i,j)$ at time $t$, wherein 100 is the local carrying capacity of the cell $C(i,j)$ at time $t$.

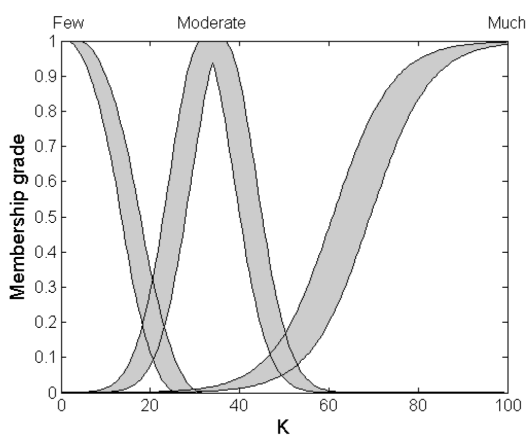


Figure 2. Upper and Lower membership functions for the antecedent corresponding to local population present in a cell at the time $t$

$R$ is characterized also by three different interval type-2 fuzzy sets (Few, Moderate, Much) over a values range from 0 to 100 (Fig. 3), which represent the level of available resource in the cell $C(i,j)$ at time $t$.

Fortunately, fuzzy theory allows modeling many forms of biological organization in a more realistic way [17]. For this reason, we characterize to $R$ using other kind of membership functions (Fig. 4). In the simulations we will use both definitions Fig. 3 and Fig. 4.

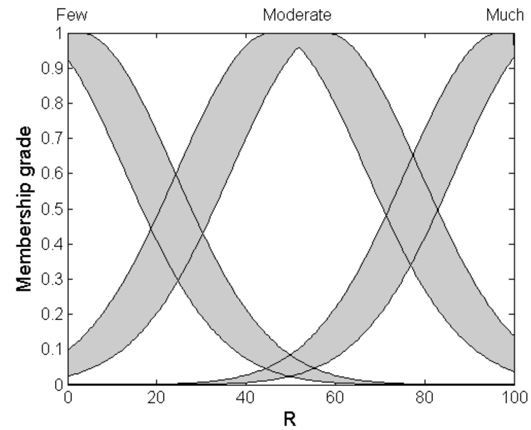


Figure 3. Upper and Lower membership functions for the antecedent corresponding to resources available in a cell at the time $t$.

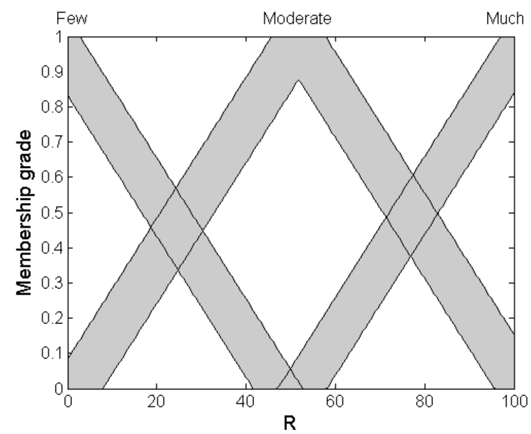


Figure 4. Upper and Lower membership functions for the antecedent corresponding to resources available in a cell at the time $t$

The output variable $T$ (Fig. 5) represents any output variable: $T_N$, $T_M$ or $T_E$, which are defined by five interval type-2 fuzzy sets: VeryLow, Low, Moderate, High, VeryHigh. Nevertheless, in our system each variable has different values range in its domain and can be changed according to the biologic behavior desired in the population dynamics.

In the dynamics of a population, indefinite increases in population size do not occur. A population may increase rapidly from a low level under favorable environmental conditions, but these increases in numbers will eventually approach the level where resources cannot support a continued increase, causing that the population size down as a consequence of resources being insufficient [1], [2]. Therefore, the resources consumption rate $T_C$, was included as an output variable into the IT2-FLS to simulate the decrement of the resources available in the cell $C(i,j)$ in each time $t$.

### B. *Definition of the fuzzy rules*

The IT2-FLS is characterized by IF-THEN fuzzy rules and its antecedents ($R$ and $K$) and consequents ($T_N$, $T_M$, $T_E$ and $T_C$). A total of nine fuzzy rules are obtained by the combination of the sets $F$ and $K$: Few-Few, Few-Moderate, Few-Much, Moderate-Few, Moderate-Moderate, Moderate-Much, Much-Few, Much-Moderate, Much-Much. Each rule determines the state of each output variable: VeryLow, Low, Moderate, High, VeryHigh. The definition of each fuzzy rule was derived from the interpretation of the Ecology laws established by [11], [12] and [18]. The same rules were used in model proposed by [10], but they were applied on type-1 fuzzy sets.

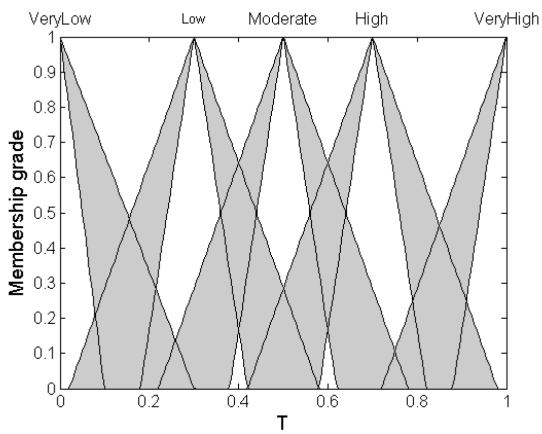


Figure 5. Upper and Lower membership functions for each consequent corresponding to the natality, mortality, emigration and consumption rates in a cell an the time *t*.

1) IF $K$ is few and $R$ is few THEN: $T_N$ is low, $T_M$ is moderate, $T_E$ is high, $T_C$ is very low.
2) IF $K$ is few and $R$ is moderate THEN: $T_N$ is moderate, $T_M$ is low, $T_E$ is moderate, $T_C$ is low.
3) IF $K$ is few and $R$ is much THEN: $T_N$ is high, $T_M$ is very low, $T_E$ is very low, $T_C$ is low.
4) IF $K$ is moderate and $R$ is few THEN: $T_N$ is low, $T_M$ is high, $T_E$ is high, $T_C$ is low.
5) IF $K$ is moderate and $R$ is moderate THEN: $T_N$ is moderate, $T_M$ is moderate, $T_E$ is moderate, $T_C$ is moderate
6) IF $K$ is moderate and $R$ is much THEN: $T_N$ is high, $T_M$ is low, $T_E$ is low, $T_C$ is high
7) IF $K$ is much and $R$ is few THEN: $T_N$ is very low, $T_M$ is very high, $T_E$ is very high, $T_C$ is very high
8) IF $K$ is much and $R$ is moderate THEN: $T_N$ is low, $T_M$ is moderate, $T_E$ is high, $T_C$ is high
9) IF $K$ is much and $R$ is much THEN: $T_N$ is moderate, $T_M$ is moderate, $T_E$ is moderate, $T_C$ is very high.

## IV. Simulation Results

The rate change under which the population size fluctuates is represented by the variable $r$ in the majority of the ecological mathematics models, for example in the Verhulst's model [12]:

$$N(t) = \frac{KN_0 e^{rt}}{K - N_0\left(e^{rt} - 1\right)} \tag{14}$$

The environment can only support a limited number of individuals (carrying capacity $K$) in a population before some resource limits the survival of those individuals. In Fig. 6, the population size reaches the carrying capacity for any value of $r$, and even though the carrying capacity changes, the population size also reaches its limits with any value of $r$, which means that there is not influence of the environment using (14), this is the environmental conditions always are optimal and the population only require time to arrive to the limit $K$.

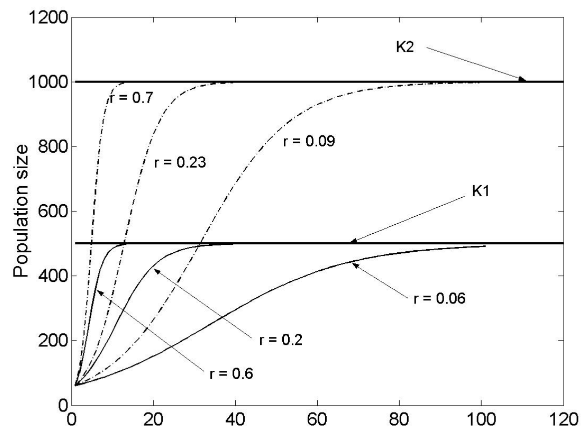


Figure 6. Population generated by the equation (16) using different values for *r* and *K*.

In most instances, resources are not unlimited and environmental conditions are not optimal. For this reason, in the model (1) the rates $\beta$, $\lambda$ and $\sigma$ are variables that depend on the environment resources located in certain space and time.

In our simulation program, the resources initial distribution and initial population size can present in several forms. These forms are classified in four study cases wherein the resources distribution and population size are combined (Table I). In all cases the initial population size distribution always is random. Its distribution is made on a random number of cells.

TABLE I. Case Studies

| Case | Parameters | | |
|---|---|---|---|
| | ***Resources*** | ***Population*** | ***Size*** |
| A | Uniform | Random | Much |
| B | Random | Random | Much |
| C | Uniform | Random | Few |
| D | Random | Random | Few |

The simulation results are presented per study cases. The trajectories described by model (1), reach an equilibrium level, and after this, the population size goes down as consequence of resources not available.

The trajectories presented as results are shown without reaching their extension. The main idea on the population dynamics is to establish the times on which the population reaches a level of stable equilibrium and the interval of time on which the population size fluctuates.

The simulation using case A starts with a uniform resources distribution on cellular space and many individuals in the population size. Therefore the population does not compete by resources at the beginning of the simulation (Fig. 7 and Fig. 8).

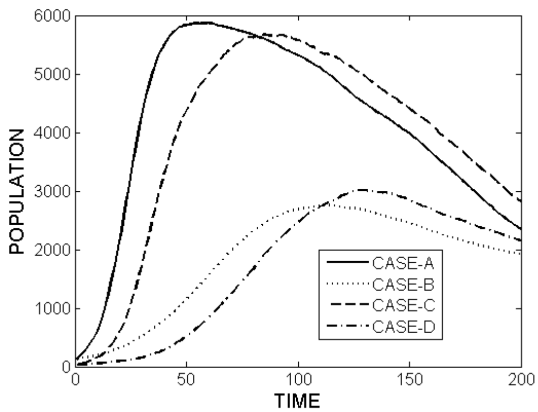


Figure 7. Population generated by model (1) using *R* characterized in Fig. 3.

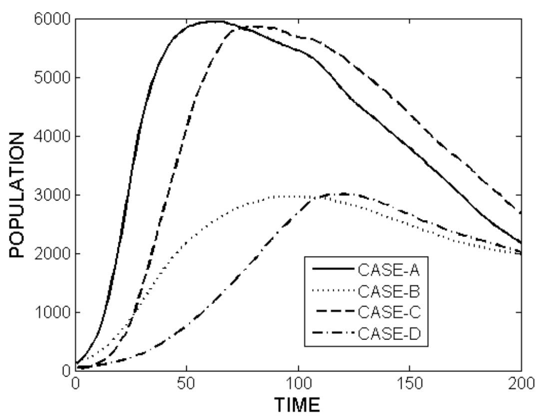


Figure 8. Population generated by model (1) using *R* characterized in Fig 4

Using case B, the simulation starts with a random initial resources distribution on cellular space and many individuals in the population size. In this case at the beginning the resources are limited therefore the population cannot increase as fast as the first case, however as at the beginning there are many individuals, the population can increase its size, describing a trajectory similar to one described using case A (Fig. 7 and Fig. 8).

Using case C or D, the resources are uniformly and randomly distributed respectively on cellular space, but the population size has few individuals. The population does not reach the equilibrium level as fast as the cases A and B (Fig. 7 and Fig. 8), because at the beginning there are few individuals on cellular space therefore the consumption rate is high; after of this, the population reach some equilibrium level even through the environment resources no unfavorable.

Some environmental factors can influence differently on the population size. If population density is high, such factors become increasingly limiting on the success of the population. A limiting factor on the populations is intraspecific competition which occurs when individuals within a population compete with one another to obtain the same resources. Sometimes intraspecific competition is direct, for example when two individuals competing for the same resource, or indirect, for example when one individual's action alters and possibly harms the environment of another individual.

Fig. 9 and Fig. 10 show a differentiation between the trajectories generated using the definition of *R* shown in Fig. 3 and Fig. 4.

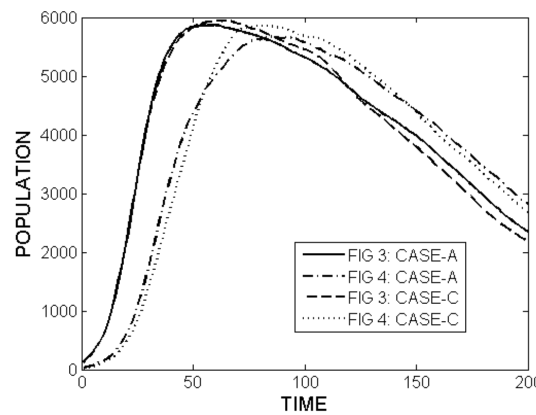


Figure 9. Population generated by model (1) using *R* characterized in Fig. 3 and Fig. 4, and using the initial conditions established in the cases A and C.

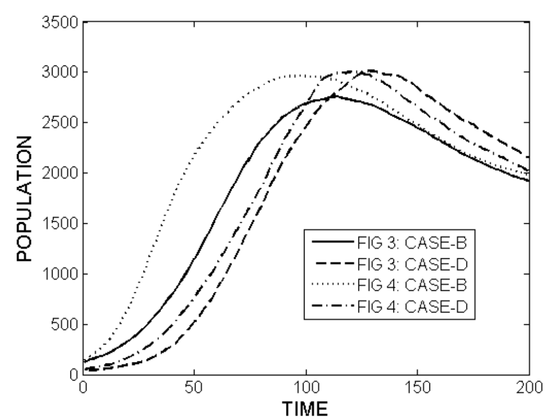


Figure 10. Population generated by model (1) using *R* characterized in Fig. 3 and Fig. 4, and using the initial conditions established in the cases B and D.

The time intervals wherein the population reached an equilibrium level are presented in Table II per study case, and these are compared against the time intervals presented by [10] using the same study cases, but they used a characterization different for the input variable *R*.

TABLE II. TIME INTERVALS IN EQUILIBRIUM

| Case | Characterization of *R* used in IT2-FLS | | |
|---|---|---|---|
| | ***R in Figure 3*** | ***R in Figure 4*** | ***R Model [10]*** |
| A | 50-75 | 50-75 | 80-120 |
| B | 80-135 | 75-125 | 100-130 |
| C | 75-100 | 75-100 | 100-120 |
| D | 110-145 | 100-125 | 120-150 |

The results show that using the case D the population required more time to reach some equilibrium level independently of the characterization of *R*. The resources initial distribution randomly (cases B and C) decreases the population density because the equilibrium levels reached by this are lower than the ones reached using the cases A and B. Both characterizations for *R* showed in Fig. 3 and Fig. 4 presents a growth similar shape. However, the characterization of *R* in Fig. 3 presents in the simulation results, interval time in equilibrium larger that the characterization of *R* in Fig. 4. This demonstrates that *R* can be characterized using different membership functions and the results will show growth shapes similar to the ones shown in this work. However, the population growth pattern is defined by the fuzzy rules. If the fuzzy rules change, then population growth pattern change. This can be demonstrated in a future research work.

## V. CONCLUSIONS

We maintain that interval type-2 fuzzy logic system integrated into the population growth model (1) represents a methodological advance for Biology. This model describes the population growth according with the fundamental and basic growth shape described by the early mathematical models made in Ecology to study the population dynamics [11] and [12]. Future work will be to find the appropriate FOU for the fuzzy sets defined in this model that can represent the uncertainty of environment resources applied to a real problem that guarantees the two aspects more important for the Biology inside population's dynamics study: high equilibrium levels and time intervals wherein the population size fluctuates on these equilibrium levels.

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