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Synthesis of Tendon-Driven Manipulators with High Fault Tolerance

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Abstract— **The purpose of this work is the realization of a methodology for synthesizing tendon-driven manipulators with high fault tolerance. Characteristics of tendon-driven manipulators are briefly discussed. Criteria for the tendon-driven manipulators to have positive tension are then established. Constraints for such manipulator are subsequently derived from the null space of the structure matrix. With these constraints, manipulators can remain controllable when one of the tendons is failed to function. Finally, a procedure for determining the structure matrix that satisfies the constraints is developed via geometric method.**

***Keywords*—tendon-driven manipulator, fault tolerance, null space, controllability, structure matrix**

I. INTRODUCTION

Tendon-driven manipulators are manipulators that use tendons (or cables, chains, belts, artificial muscles) as the element for transmitting power and motion. They have been commonly applied in the design of small robotic manipulators such as mechanical fingers/hands and prosthetic arms. The main advantage of using such device is that tendons allow driving actuators to be tele-operated from the manipulator and therefore make the system lightweighted and compact design feasible. In the past decades, some issues regarding the design of tendon-driven manipulators have been thoroughly studied by many researchers. The kinematics of tendon-driven manipulators was investigated by Morecki [1], Salisbury [2], and Tsai and Lee [3]. Lee and Tsai [4] proposed a method for topological synthesis of tendon-driven manipulators, where rules for feasible tendon routings were derived. In recent, certain new issues have drawn the attention of researchers, i.e., the redundancy in the number of tendons. A tendon-driven manipulator with redundant tendons can posses additional flexibility in practical applications such as allowing fault tolerance, optimizing the performance of tendons, and reducing the burden of each tendon. Mruthyunjaya [5] defines that if a manipulator possesses more degrees of freedom (DOF) than the dimensions of the task space, the redundancy thus obtained is called "kinematic redundancy". On the other hand, if the number of the actuators is larger than that of the degrees of freedom, the resulting redundancy is called "actuation redundancy". A great number of research regarding parallel manipulators with actuation redundancy has been conducted [6-11]. However, not much literature can be found for the articulated tendon-driven manipulators with actuation redundancy. Kobayashi et al. [12] discussed several basic issues about tendon controllability and tendon redundancy of tendon-driven manipulators. They also summarized conditions for a symmetrical tendon-driven manipulator to remain controllable when some tendons are removed.

To provide better fault tolerance with respect to the failure of tendons while the manipulator is at work, it is important to establish the criteria for a tendon-driven manipulator with actuation redundancy. With these criteria, manipulators can remain controllable when one or more of the tendons is failed to function. In this paper, conditions for the tendon-driven manipulator with fault tolerance will be discussed. In addition, synthesis of the structure of the tendon-driven manipulator that allows any one tendon to fail will be performed. A two-dof robotic manipulator is used for illustration. Finally, a methodology for synthesizing such tendon-driven manipulators via geometric method is proposed.

II. PRINCIPLE OF OPERATION

A. Basic equations

Fig. 1 shows the planar schematic of a general n-DOF articulated manipulator with m ($m \geq n+1$) open-end control tendons. It has been shown [13] that the displacement relationship between the tendon space and the joint angle space can be written as

$$\mathbf{S} = \mathbf{A}\,\boldsymbol{\theta} \tag{1}$$

where $\mathbf{S} = [s_1, s_2, \ldots, s_m]^T$ is the (m x 1) tendon displacement vector, $\boldsymbol{\theta} = [\theta_1, \theta_2, \ldots, \theta_n]^T$ is the (n x 1) joint angle vector, and $\mathbf{A}$ is the (m x n) displacement structure matrix. On the other side, the force relationship between the tendon space and the joint angle space can be written as

$$\boldsymbol{\tau} = \mathbf{B}\,\boldsymbol{\xi} \tag{2}$$

where $\boldsymbol{\tau} = [\tau_1, \tau_2, \ldots, \tau_n]^T$ is the ($n\times1$) joint torque vector, $\boldsymbol{\xi} = [\xi_1, \xi_2, \ldots, \xi_m]^T$ is the ($m\times1$) tendon force vector, and $\mathbf{B} = \mathbf{A}^T$ is the ($n\times m$) force structure matrix, or simply called structure matrix. The element of **B,** b_{ij}, also indicates the adjacency condition of the tendon j with joint i. Element b_{ij} is nonzero if tendon j routes through joint i, otherwise it is zero.

In the force domain, once the tendon forces are given, the joint torques are uniquely determined. Inversely, given the joint torques, the tendon forces can be determined by the equation

$$\boldsymbol{\xi} = (\mathbf{B})^{+} \boldsymbol{\tau} + \mathbf{H}\,\boldsymbol{\lambda} \tag{3}$$

where $(\mathbf{B})^{+} = (\mathbf{B}^{T})[\mathbf{B}\,\mathbf{B}^{T}]^{-1}$ is the pseudo-inverse of **B**, **H** is an m x (m-n) dimensional matrix whose column vectors span the null space of matrix **B,** and $\boldsymbol{\lambda}$ is an (m-n) x 1 column vector whose elements are arbitrary constants. The first term on the right hand side of (3) is known as the particular solution while the second term is the homogeneous solution. The homogeneous solution is used to adjust the tension in tendons and hence must allow all tendons to remain positive tension.

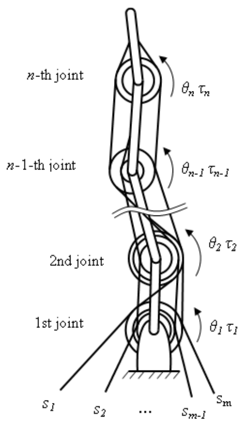

Figure 1. A general tendon-driven manipulator

For a tendon-driven robotic manipulator to be fully controllable, there must exists positive tension force for an arbitrary joint torque $\boldsymbol{\tau}$. Kobayashi [12] concluded that for an n-DOF tendon-driven manipulator with m tendons and symmetric routing, the number of m must be greater or equal to 2n. Yet this limitation is not applicable to tendon-driven manipulators with asymmetric tendon routing. Leijnse [14] found a similar result that the maximum number of non-redundant muscles in an n-DOF system with symmetric routing is 2n. Kobayashi[12] summarized that, for any n-DOF system with m tendons, if $m \geq 2n + \beta$, there can be at most β redundant tendons in the manipulator. These redundant tendons may provide fault tolerance for the manipulator once one of the driving tendons is broken or malfunctioned. In what follows, we shall develop a method to synthesize the structure of the tendon-driven manipulator that has high fault tolerance. In other word, the system still remains its full controllability when any one of the driving tendons is broken or mal-functioned.

B. Characteristics of the Null Space

From (3), it can be seen that the matrix **H** plays an important role for a tendon-driven manipulator to be controllable. Once the manner of tendon routing is determined, the matrix **H** is also determined. There are different ways of deriving the null space of a matrix, for example, orthonormal basis null space expression and rational basis null space expression. The former expression uses singular value decomposition to find the null space of a matrix [15], while the latter one uses Gauss-Jordan elimination to transfer the matrix to a reduced row echelon form [16]. In this work, we shall use the rational basis null space expression to derive the constraint for a tendon-driven manipulator with redundant tendons. As a result, for an n-DOF tendon-driven manipulator with m tendons, the null space of the structure matrix, **H**, when expressed in rational basis form, is as

$$\mathbf{H} = \begin{bmatrix} n_{11} & n_{12} & \cdots & n_{1(m-n)} \\ n_{21} & n_{22} & \cdots & n_{2(m-n)} \\ \cdots & \cdots & \cdots & \cdots \\ n_{n1} & \cdots & \cdots & n_{n(m-n)} \\ 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{N}_{nx(m-n)} \\ \mathbf{I}_{(m-n)x(m-n)} \end{bmatrix} \tag{4}$$

where submatrix $\mathbf{N}_{nx(m-n)}$ is the upper part of matrix **H** and $\mathbf{I}_{(m-n)x(m-n)}$ is the lower part of matrix **H** and is an identity matrix. Clearly, from (4), a sufficient and necessary condition for the tendon-driven manipulator to be fully controllable is the elements in each column vector of the submatrix, $\mathbf{N}_{nx(m-n)}$, must be of the same sign. Thus, by adjusting the coefficients in $\boldsymbol{\lambda}$, positive tendon tension can be achieved.

III. CONSTRAINTS TO MAINTAIN CONTROLLABILITY

A. Two-DOF manipulator with five tendons

In this section we will derive the constraints to maintain the full controllability of the tendon-driven manipulator when one of the tendons is broken or malfunctioned. For sake of clarity, a two-DOF manipulator system will be first illustrated. Since the minimum number of tendons for a two-DOF manipulator system to have redundant tendons is five [12], let the structure matrix **B** of the two-DOF system with 5 tendons be as

$$\mathbf{B} = \begin{bmatrix} r_1 & r_2 & r_3 & r_4 & r_5 \\ r_6 & r_7 & r_8 & r_9 & r_{10} \end{bmatrix} \tag{5}$$

where r_i's are the elements of structure **B**. In case one of the tendons is broken, **B** degenerates into a 2×4 dimensional matrix $\mathbf{B}'$ whose null space, when expressed in terms of rational basis form, is of the form

$$\mathbf{H} = \begin{bmatrix} n_{11} & n_{12} \\ n_{21} & n_{22} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{6}$$

where elements n_{ij}′s are to be determined. Since **H** is the null space of $\mathbf{B}'$, the relationship ($\mathbf{B}'\mathbf{H} = \mathbf{0}$) must hold for the system. Deploying $\mathbf{B}'\mathbf{H} = \mathbf{0}$ yields

$$b_{11}n_{11} + b_{12}n_{21} + b_{13} + 0 \cdot b_{14} = 0 \tag{7a}$$

$$b_{21}n_{11} + b_{22}n_{21} + b_{23} + 0 \cdot b_{24} = 0 \quad (7b)$$

$$b_{11}n_{12} + b_{12}n_{22} + 0 \cdot b_{13} + b_{14} = 0 \quad (7c)$$

$$b_{21}n_{12} + b_{22}n_{22} + 0 \cdot b_{23} + b_{24} = 0 \quad (7d)$$

where b_{ij}'s are the elements of $\mathbf{B}'$. The unknown parameters n_{ij}'s can be solved by Cramer's rule:

$$\begin{Bmatrix} n_{11} \\ n_{21} \end{Bmatrix} = [D_{23} / D_{12}, -D_{13} / D_{12}]^T \quad (8a)$$

$$\begin{Bmatrix} n_{12} \\ n_{22} \end{Bmatrix} = [D_{24} / D_{12}, -D_{14} / D_{12}]^T \quad (8b)$$

where D_{ij} is the determinant of a matrix formed by the i^{th} and j^{th} column in matrix $\mathbf{B}'$. As a whole, if $\mathbf{H_i}$ is denoted as the null space of the structure matrix when the i^{th} tendon is broken, then there may exist five possible $\mathbf{H_i}'$s , $\mathbf{H_1} \sim \mathbf{H_5}$ for each degenerated system after one tendon is broken. The five possible $\mathbf{H_i}'$s can be expressed as follows:

$$\mathbf{H_1} = \begin{bmatrix} D_{34}/D_{23} & D_{35}/D_{23} \\ -D_{24}/D_{23} & -D_{25}/D_{23} \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\mathbf{H_2} = \begin{bmatrix} D_{34}/D_{13} & D_{35}/D_{13} \\ -D_{14}/D_{13} & -D_{15}/D_{13} \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\mathbf{H_3} = \begin{bmatrix} D_{24}/D_{12} & D_{25}/D_{12} \\ -D_{14}/D_{12} & -D_{15}/D_{12} \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\mathbf{H_4} = \begin{bmatrix} D_{23}/D_{12} & D_{25}/D_{12} \\ -D_{13}/D_{12} & -D_{15}/D_{12} \\ 1 & 0 \\ 0 & 1 \end{bmatrix},$$

and $$\mathbf{H_5} = \begin{bmatrix} D_{23}/D_{12} & D_{24}/D_{12} \\ -D_{13}/D_{12} & -D_{14}/D_{12} \\ 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

In order to keep the system controllable if any one of the tendons is broken, at least one column vector in each $\mathbf{H_i}$ should be of the same sign. Therefore, to have the system remain controllable if any one of the tendons is broken, the following Boolean criteria must be true:

{[(D_{34} x $D_{23} > 0$ and D_{24} x $D_{23} < 0$) or (D_{35} x $D_{23} > 0$ and D_{25} x $D_{23} < 0$)],

and [(D_{34} x $D_{13} > 0$ and D_{14} x $D_{23} < 0$) or (D_{35} x $D_{13} > 0$ and D_{15} x $D_{13} < 0$)],

and [(D_{24} x $D_{12} > 0$ and D_{14} x $D_{12} < 0$) or (D_{25} x $D_{12} > 0$ and D_{15} x $D_{12} < 0$)],

and [(D_{23} x $D_{12} > 0$ and D_{13} x $D_{12} < 0$) or (D_{25} x $D_{12} > 0$ and D_{15} x $D_{12} < 0$)],

and [(D_{23} x $D_{12} > 0$ and D_{13} x $D_{12} < 0$) or (D_{24} x $D_{12} > 0$ and D_{14} x $D_{12} < 0$)]} (9)

There are nine unknowns (D_{12}, D_{13}, D_{14}, D_{15}, D_{23}, D_{24}, D_{25}, D_{34}, D_{35}) in (9). To further simplify the conditions in (9), assume that $D_{ij} \neq 0$. Then, the sign of each unknown D_{ij} could be either positive or negative. This will yield $2^9 = 512$ possible combinations from the 9 unknowns. A computer program has been developed to check all the 512 possible combinations and found that only four combinations satisfy the conditions in (9). Among the four combinations, two of them are merely contrast to the other two combinations by a sign change. Therefore, there are only two combinations satisfying (9), i.e.,

$$D_{12} > 0;\ D_{13} > 0;\ D_{14} < 0;\ D_{15} < 0;\ D_{23} < 0;$$
$$D_{24} > 0;\ D_{25} > 0;\ D_{34} < 0;\ D_{35} > 0 \quad (10)$$

or

$$D_{12} > 0;\ D_{13} > 0;\ D_{14} < 0;\ D_{15} < 0;\ D_{23} < 0;$$
$$D_{24} > 0;\ D_{25} > 0;\ D_{34} > 0;\ D_{35} < 0 \quad (11)$$

As a consequence, for a two-DOF tendon-driven manipulator if the structure is arranged such that (10) or (11) is satisfied, the mechanism can sustain controllability when any one of the tendons is broken. An example of such manipulator can be shown in Fig. 2 whose structure matrix is

$$\mathbf{B} = \begin{bmatrix} 1 & 1 & 1.5 & -1.4 & -2 \\ -1 & 1 & 1 & -1 & -1 \end{bmatrix}$$

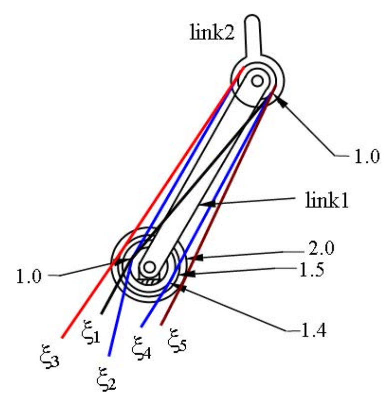

Figure 2. Schematic of a 2-DOF manipulator with 5 tendons

The same procedure can be applied to derive the criteria for the manipulator with 2-DOF and more tendons. When the number of tendons increases, the number of columns in the null space of $\mathbf{H_i}$ will increase but have similar format to that of five tendons. Therefore, similar criteria can be obtained by setting the nonzero elements in the null space $\mathbf{H_i}$ to be of the same sign. This can yield the result for the manipulator with 2-DOF and more tendons. It can be also noted that there will be no solution for m≤4. In other words, there will be no fault tolerance for a two-DOF system when the number of tendons is less than 5.

B. Three-DOF manipulators with seven tendons

The same procedure as in previous section can be applied to derive the criteria for the manipulator with three or more DOF's. Increasing the number of degrees of freedom, the constraints will become more complicated. Here, a three-DOF manipulator system will be illustrated to show the complexity. Since the minimum number of tendons for a three-DOF manipulator system to have redundant tendons is seven [12], let the structure matrix **B** of the three-DOF system with seven tendons be as

$$\mathbf{B} = \begin{bmatrix} r_1 & r_2 & r_3 & r_4 & r_5 & r_6 & r_7 \\ r_8 & r_9 & r_{10} & r_{11} & r_{12} & r_{13} & r_{14} \\ r_{15} & r_{16} & r_{17} & r_{18} & r_{19} & r_{20} & r_{21} \end{bmatrix} \quad (12)$$

where r_i's are the elements of structure **B**. In case one of the tendons is broken, **B** degenerates into a 3×6 dimensional matrix $\mathbf{B'}$ whose null space, when expressed in terms of rational basis form, is of the form

$$\mathbf{H} = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ n_{31} & n_{32} & n_{33} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (13)$$

The unknown parameters n_{ij}'s can be solved by Cramer's rule:

$$\begin{Bmatrix} n_{11} \\ n_{21} \\ n_{31} \end{Bmatrix} = [-D_{234} / D_{123}, D_{134} / D_{123}, -D_{124} / D_{123}]^T \quad (14a)$$

$$\begin{Bmatrix} n_{12} \\ n_{22} \\ n_{32} \end{Bmatrix} = [-D_{235} / D_{123}, D_{135} / D_{123}, -D_{125} / D_{123}]^T \quad (14b)$$

$$\begin{Bmatrix} n_{13} \\ n_{23} \\ n_{33} \end{Bmatrix} = [-D_{236} / D_{123}, D_{136} / D_{123}, -D_{126} / D_{123}]^T \quad (14c)$$

where D_{ijk} is the determinant of a matrix formed by the i^{th}, j^{th}, and k^{th} column in matrix $\mathbf{B'}$. If $\mathbf{H_i}$ is denoted as the null space of the structure matrix when the i^{th} tendon is broken, then there may exist seven possible $\mathbf{H_i}$'s, $\mathbf{H_1} \sim \mathbf{H_7}$ for each degenerated system after one tendon is broken. These seven possible $\mathbf{H_i}$'s can be expressed as follows:

$$\mathbf{H_1} = \begin{bmatrix} -D_{345}/D_{234} & -D_{346}/D_{234} & -D_{347}/D_{234} \\ D_{245}/D_{234} & D_{246}/D_{234} & D_{247}/D_{234} \\ -D_{235}/D_{234} & -D_{236}/D_{234} & -D_{237}/D_{234} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (15a)$$

$$\mathbf{H_2} = \begin{bmatrix} -D_{345}/D_{134} & -D_{346}/D_{134} & -D_{347}/D_{134} \\ D_{145}/D_{134} & D_{146}/D_{134} & D_{147}/D_{134} \\ -D_{135}/D_{134} & -D_{136}/D_{134} & -D_{137}/D_{134} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (15b)$$

$$\mathbf{H_3} = \begin{bmatrix} -D_{245}/D_{124} & -D_{246}/D_{124} & -D_{247}/D_{124} \\ D_{145}/D_{124} & D_{146}/D_{124} & D_{147}/D_{124} \\ -D_{125}/D_{124} & -D_{126}/D_{124} & -D_{127}/D_{124} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (15c)$$

$$\mathbf{H_4} = \begin{bmatrix} -D_{235}/D_{123} & -D_{236}/D_{123} & -D_{237}/D_{123} \\ D_{135}/D_{123} & D_{136}/D_{123} & D_{137}/D_{123} \\ -D_{125}/D_{123} & -D_{126}/D_{123} & -D_{127}/D_{123} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (15d)$$

$$\mathbf{H_5} = \begin{bmatrix} -D_{234}/D_{123} & -D_{236}/D_{123} & -D_{237}/D_{123} \\ D_{134}/D_{123} & D_{136}/D_{123} & D_{137}/D_{123} \\ -D_{124}/D_{123} & -D_{126}/D_{123} & -D_{127}/D_{123} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (15e)$$

$$\mathbf{H_6} = \begin{bmatrix} -D_{234}/D_{123} & -D_{235}/D_{123} & -D_{237}/D_{123} \\ D_{134}/D_{123} & D_{135}/D_{123} & D_{137}/D_{123} \\ -D_{124}/D_{123} & -D_{125}/D_{123} & -D_{127}/D_{123} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (15f)$$

$$\mathbf{H_7} = \begin{bmatrix} -D_{234}/D_{123} & -D_{235}/D_{123} & -D_{236}/D_{123} \\ D_{134}/D_{123} & D_{135}/D_{123} & D_{136}/D_{123} \\ -D_{124}/D_{123} & -D_{125}/D_{123} & -D_{126}/D_{123} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (15g)$$

In order to keep the system controllable if any one of the tendons is broken, at least one column vector in each $\mathbf{H_i}$ should be of the same sign. Therefore, we can have the following Boolean criteria to keep system controllable if any one of the tendons is broken

$\{[(D_{234} \times D_{345} < 0$ and $D_{234} \times D_{245} > 0$ and $D_{234} \times D_{235} < 0)$ or
$(D_{234} \times D_{346} < 0$ and $D_{234} \times D_{246} > 0$ and $D_{234} \times D_{236} < 0)$ or
$(D_{234} \times D_{347} < 0$ and $D_{234} \times D_{247} > 0$ and $D_{234} \times D_{237} < 0)]$ and
$[(D_{134} \times D_{345} < 0$ and $D_{134} \times D_{145} > 0$ and $D_{134} \times D_{135} < 0)$ or
$(D_{134} \times D_{346} < 0$ and $D_{134} \times D_{146} > 0$ and $D_{134} \times D_{136} < 0)$ or
$(D_{134} \times D_{347} < 0$ and $D_{134} \times D_{147} > 0$ and $D_{134} \times D_{137} < 0)]$ and
$[(D_{124} \times D_{245} < 0$ and $D_{124} \times D_{145} > 0$ and $D_{124} \times D_{125} < 0)$ or
$(D_{124} \times D_{246} < 0$ and $D_{124} \times D_{146} > 0$ and $D_{124} \times D_{126} < 0)$ or
$(D_{124} \times D_{247} < 0$ and $D_{124} \times D_{147} > 0$ and $D_{124} \times D_{127} < 0)]$ and
$[(D_{123} \times D_{235} < 0$ and $D_{123} \times D_{135} > 0$ and $D_{123} \times D_{125} < 0)$ or
$(D_{123} \times D_{236} < 0$ and $D_{123} \times D_{136} > 0$ and $D_{123} \times D_{126} < 0)$ or
$(D_{123} \times D_{237} < 0$ and $D_{123} \times D_{137} > 0$ and $D_{123} \times D_{127} < 0)]$ and
$[(D_{123} \times D_{234} < 0$ and $D_{123} \times D_{134} > 0$ and $D_{123} \times D_{124} < 0)$ or
$(D_{123} \times D_{236} < 0$ and $D_{123} \times D_{136} > 0$ and $D_{123} \times D_{126} < 0)$ or
$(D_{123} \times D_{237} < 0$ and $D_{123} \times D_{137} > 0$ and $D_{123} \times D_{127} < 0)]$ and
$[(D_{123} \times D_{234} < 0$ and $D_{123} \times D_{134} > 0$ and $D_{123} \times D_{124} < 0)$ or
$(D_{123} \times D_{235} < 0$ and $D_{123} \times D_{135} > 0$ and $D_{123} \times D_{125} < 0)$ or
$(D_{123} \times D_{237} < 0$ and $D_{123} \times D_{137} > 0$ and $D_{123} \times D_{127} < 0)]$ and
$[(D_{123} \times D_{234} < 0$ and $D_{123} \times D_{134} > 0$ and $D_{123} \times D_{124} < 0)$ or
$(D_{123} \times D_{235} < 0$ and $D_{123} \times D_{135} > 0$ and $D_{123} \times D_{125} < 0)$ or
$(D_{123} \times D_{236} < 0$ and $D_{123} \times D_{136} > 0$ and $D_{123} \times D_{126} < 0)]\}$ (16)

There are 22 unknowns (D_{123}, D_{124}, D_{125}, D_{126}, D_{127}, D_{134}, D_{135}, D_{136}, D_{137}, D_{145}, D_{146}, D_{147}, D_{234}, D_{235}, D_{236}, D_{237}, D_{245}, D_{246}, D_{247}, D_{345}, D_{346}, D_{347}) in (16). It can be found that (16) can still be satisfied when we disregard the following 5 variables : D_{127}, D_{146}, D_{237}, D_{247}, and D_{345}. Assume that $D_{ijk} \neq 0$. Then, the sign of each unknown D_{ijk} could be either positive or negative. This will yield 2^{17} = 131072 possible combinations from the 17 unknowns. A computer program has been developed to check all the 131072 possible combinations and found only four combinations satisfy the conditions in (16). Among the four combinations, two of them are merely contrast to the other two combinations by a sign change. Therefore, there are only two combinations satisfying (16), i.e.,

$$\begin{aligned}
&D_{123} > 0, D_{124} > 0, D_{145} > 0, D_{135} > 0, D_{136} > 0 \\
&D_{234} > 0, D_{246} > 0, D_{134} > 0, D_{147} > 0, \\
&D_{125} < 0, D_{126} < 0, D_{137} < 0, D_{235} < 0, D_{245} < 0, \\
&D_{236} < 0, D_{346} < 0, D_{347} < 0 \qquad (17)
\end{aligned}$$

or

$$\begin{aligned}
&D_{123} > 0, D_{124} > 0, D_{145} > 0, D_{135} > 0, D_{136} > 0 \\
&D_{234} > 0, D_{246} > 0, D_{137} > 0, D_{347} > 0, \\
&D_{125} < 0, D_{126} < 0, D_{235} < 0, D_{245} < 0, \\
&D_{236} < 0, D_{346} < 0, D_{134} < 0, D_{147} < 0 \qquad (18)
\end{aligned}$$

Following is a possible solution that satisfies (17) by trial and error:

$$\mathbf{B}=\begin{bmatrix} -0.4 & 1 & 0 & -0.7 & 0.7 & 0 & 0.8 \\ -2 & 1 & 1.2 & -1 & 2 & 1 & 1 \\ -1 & 1 & 1.2 & 0 & 0 & -3 & -0.5 \end{bmatrix}$$

Due to the complexity of the constraints, it is not easy to use an analytical method to find a structure matrix that satisfies (17) or (18). We will leave this to future study.

IV. Synthesis by Geometric Method

In order to obtain the elements in matrix **B**, some trial and error is conducted in Fig. 2 in section III *A*. In this section, a geometric method is used to determine the r_i's in matrix **B**. This method provides a quick solution for the synthesis of a two-DOF tendon-driven manipulator with redundant tendons. The column vectors in structure matrix **B** can be regarded as plane vectors. Let $\mathbf{B}=[\mathbf{v_1},\mathbf{v_2},\mathbf{v_3},\mathbf{v_4},\mathbf{v_5}]$, where $\mathbf{v_1}=[r_1, r_6]^T$, $\mathbf{v_2}=[r_2, r_7]^T$, and so on. Then $\mathbf{D_{ij}}$ (= $\mathbf{v_i} \times \mathbf{v_j}$) represents the area of a parallelogram formed by $\mathbf{v_i}$ and $\mathbf{v_j}$. Define that $\mathbf{D_{ij}} > 0$ if $\mathbf{v_i} \times \mathbf{v_j}$ is in counter-clock-wise direction; otherwise $\mathbf{D_{ij}} < 0$. With this in mind, the synthesis procedure for r_i's can be summarized as follows:

(a) Arbitrarily select a $\mathbf{v_1}$ and draw $\mathbf{v_1}$ to pass through the origin of a plane coordinate system. For example, $\mathbf{v_1}$ is chosen as directing to +X axis.

(b) The directions of $\mathbf{v_2}$ and $\mathbf{v_3}$ can be determined by the constraints in (10): $D_{12} > 0$, $D_{13} > 0$, and $D_{23} < 0$. This indicates that $\mathbf{v_2}$ must lie on the upper part of $\mathbf{v_1}$, $\mathbf{v_3}$ must lie on the upper part of $\mathbf{v_1}$, and $\mathbf{v_3}$ must lie on the upper part of $\mathbf{v_2}$, as shown in Fig. 3.

(c) With the constraint $D_{14} < 0$, $\mathbf{v_4}$ should be located in the third or fourth quadrant. With the constraint $D_{24} > 0$, $\mathbf{v_4}$ should form an angle with $\mathbf{v_2}$, and when measured from $\mathbf{v_2}$ in ccw direction, the magnitude is less than 180 degrees. With the constraint $D_{34} < 0$, $\mathbf{v_4}$ should form an angle with $\mathbf{v_3}$, and when measured from $\mathbf{v_3}$ in clock-wise direction, the magnitude is less than 180 degrees. As a result, $\mathbf{v_4}$ can be located in the cross-hatched area as shown in Fig. 3. Similarly, $\mathbf{v_5}$ can only be selected in the shaded area in Fig. 3.

(d) The structure matrix **B** can thus be obtained by selecting suitable vectors in the feasible area shown in Fig. 3. Following is another one feasible solution and the corresponding schematic drawing is shown in Fig. 4.

$$\mathbf{B}=\begin{bmatrix} 1 & 1 & 2 & -3 & -2 \\ 0 & 1 & 0.5 & -2 & -0.4 \end{bmatrix}$$

(e) The same procedure (a) through (d) can be applied to obtain the structure matrix with another set of constraints (11). Fig. 5 shows one feasible solution that subject to (11) and the corresponding schematic drawing is shown in Fig. 6.

$$\mathbf{B}=\begin{bmatrix} 0 & -1 & 3 & 0.5 & 1.5 \\ 2 & 1.5 & 3 & -2 & -3 \end{bmatrix}$$

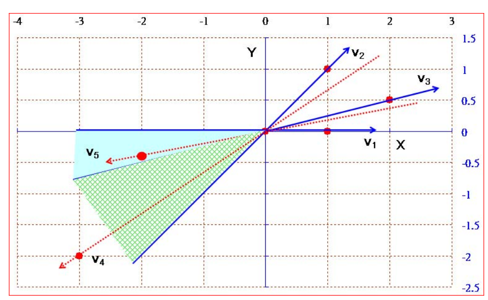

Figure 3. Geometric method used to synthesize the column vectors of structure matrix (subject to (10))

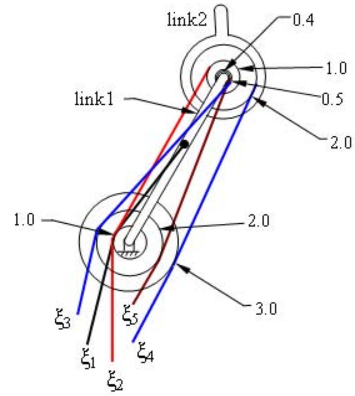

Figure 4. Schematic of the feasible solution (subject to (10))

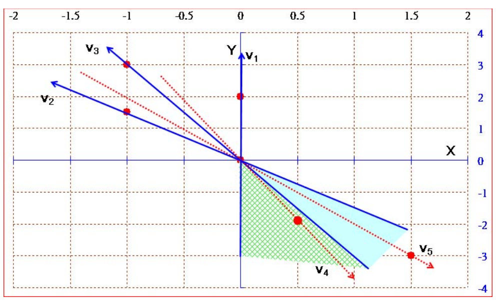

Figure 5. Geometric method used to synthesis the column vectors of structure matrix subject to (11))

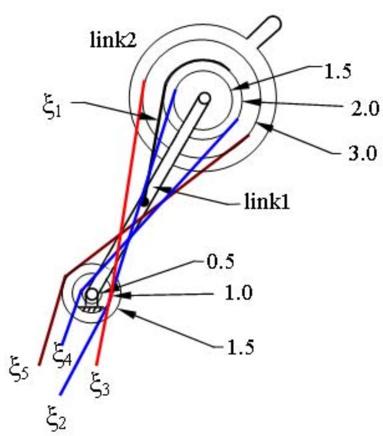

Figure 6. Schematic of the feasible solution (subject to (11))

V. CONCLUSION

In this work, a methodology for synthesizing articulated tendon-driven manipulators with high fault tolerance is developed. The fault tolerance allows any one of the tendons in the manipulator to be broken and the system still remains fully controllable. The constraints for such manipulator can be derived from the null space of the structure matrix. A procedure for determining the structure matrix is also developed via geometric method. As a whole, the two-DOF tendon-driven manipulator with five tendons is illustrated as the synthesis example. Future work will apply this method to high DOF manipulators with high number of tendons.

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