Temperature Prediction Based on Fuzzy Clustering and Fuzzy Rules Interpolation Techniques

Yu-Chuan Chang and Shyi-Ming Chen

Abstract—In this paper, we present a new method to deal with temperature prediction based on fuzzy clustering and fuzzy rules interpolation techniques. First, the proposed method constructs fuzzy rules from training samples based on the fuzzy C-Means clustering algorithm, where each fuzzy rule corresponds to a cluster and the linguistic terms appearing in the fuzzy rules are represented by triangular fuzzy sets. Then, it performs fuzzy inference based on the multiple fuzzy rules interpolation scheme, where it calculates the weight of each fuzzy rule with respect to the input observation based on the defuzzified values of triangular fuzzy sets. Finally, it uses the weight of each fuzzy rule to calculate the forecasted output. We also apply the proposed method to handle the temperature prediction problem. The experimental result shows that the proposed method gets higher average forecasting accuracy rates than Chen and Hwang’s method.

Keywords—fuzzy rules, temperature prediction, fuzzy clustering, fuzzy rules interpolation

I. INTRODUCTION

Forecasting activities play an important role in our daily life, where there are many kinds of forecasting activities, such as stock market forecasting, earthquake forecasting, traffic flow forecasting, weather forecasting, economic growth rate forecasting, enrollments forecasting, etc. If we can make a forecast as precise as possible, we can prevent damages from the coming disasters, such as economic recession, company loss, traffic jam, storms, typhoons, etc. In recent years, some forecasting methods have been presented based on fuzzy rules [2], [4], [5], [7], [10], [12], [13], [15].

However, fuzzy rule-based systems suffer from the problem of sparse fuzzy rule bases in which fuzzy rules incompletely cover the universe of discourse. Fuzzy rules in rule-based systems are usually limited to a few input variables, because a complete fuzzy rule base with K input variables and T fuzzy linguistic terms in each input variable needs $T^K$ fuzzy rules, where the complexity of the rule base is exponentially increasing with the number of input variables. In order to increase the efficiency of fuzzy rule-based systems with multiple variables, it is necessary to reduce bigger fuzzy rule bases into smaller fuzzy rule bases while keeping the essential fuzzy rules in the rule bases. However, reducing fuzzy rule bases will cause sparse fuzzy rule bases which contain blank areas uncovered by fuzzy rules in the universe of discourse while conventional fuzzy inference methods only can handle complete fuzzy rule bases [14]. In recent years, some fuzzy rules interpolation methods [6], [8], [11], [16], [17] have been presented to handle inferences in sparse fuzzy rule bases for sparse fuzzy rule-based systems.

In this paper, we present a new method to deal with temperature prediction based on fuzzy clustering and fuzzy rules interpolation techniques. First, the proposed method constructs fuzzy rules from training samples based on the fuzzy C-Means clustering algorithm [1], where each fuzzy rule corresponds to a cluster and the linguistic terms appearing in the fuzzy rules are represented by triangular fuzzy sets. Then, it performs fuzzy inference based on the multiple fuzzy rules interpolation scheme [6], where it calculates the weight of each fuzzy rule with respect to the input observation based on the defuzzified values [9] of triangular fuzzy sets and uses the weight of each fuzzy rule to calculate the forecasted output. We also apply the proposed method to handle the temperature prediction problem. The experimental result shows that the proposed method gets higher average forecasting accuracy rates than Chen and Hwang’s method [7].

II. PRELIMINARIES

A. Fuzzy C-Means Clustering Algorithm [1]

The fuzzy C-Means (FCM) clustering algorithm [1] is a widely used fuzzy clustering method in pattern recognition, which allows each data belonging to two or more clusters. The FCM clustering algorithm partitions data points $X_i$ ($i = 1, 2, ..., n$) into clusters $C_i$ ($i = 1, 2, ..., c$) based on the minimization of the following objective function [1]:

$$ J_u = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^m \|x_i - x_j\|^2, $$

where $\|x_i - x_j\|$ is the Euclidean distance between data point $X_i$ and the cluster center $V_i$ ($i = 1, 2, ..., c$), $u_{ij}$ is the membership grade of $X_i$ belonging to cluster $C_i$, $m$ is a fuzziness index [1], $m \geq 1$, $n$ is the number of data points, and $c$ is the number of clusters. The procedures of the FCM clustering algorithm are reviewed from [1] as follows:

Step 1: Randomize the membership grade $u_{ij}$, where $0 \leq u_{ij} \leq 1$, $\sum_{j=1}^{n} u_{ij} = 1$, $1 \leq i \leq c$ and $1 \leq j \leq n$. 


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Step 2: Calculate the cluster center $V_i$ of cluster $C_i$,

$$V_i = \frac{\sum_{j=1}^{c} (u_{ij})^m X_j}{\sum_{j=1}^{c} (u_{ij})^m},$$

where $1 \leq i \leq c$.

Step 3: Update the membership grade $u_{ij}$ of $X_j$ belonging to $C_i$, where

$$u_{ij} = \frac{1}{\sum_{j=1}^{c} \left( \frac{V_j - X_i}{\|X_i - X_j\|} \right)^{m-1}},$$

$1 \leq i \leq c$ and $1 \leq j \leq n$.

Step 4: Repeat Step 2 and Step 3 until the value of $J_m$ in Eq. (1) is no longer decreasing.

B. The Multiple Fuzzy Rules Interpolation Scheme [6]

A triangular fuzzy set $A$ can be represented by three characteristic points $(a, b, c)$, as shown in Fig. 1, where $b$ is called “the center point” whose membership value in $A$ is equal to 1, and $a$ and $c$ are called “the left point” and “the right point”, respectively, whose membership values in $A$ are equal to 0, respectively. The defuzzified value DEF($A$) of the triangular fuzzy set $A$ shown in Figure 1 is calculated as follows [9]:

$$\text{DEF}(A) = \frac{a + 2b + c}{4}. \quad (4)$$

![Figure 1. A triangular fuzzy set $A$.](image)

Let us consider the multiple fuzzy rules interpolation scheme, which is shown as follows:

- **Rule 1:** If $x_1 = A_{11}$ and $x_2 = A_{12}$ and ... and $x_i = A_{1i}$, then $y = B_1$.
- **Rule 2:** If $x_1 = A_{21}$ and $x_2 = A_{22}$ and ... and $x_i = A_{2i}$, then $y = B_2$.
- **Rule $p$:** If $x_1 = A_{pi}$ and $x_2 = A_{2p}$ and ... and $x_i = A_{pi}$, then $y = B_p$.

Observations: $x_1 = A_{1*}$ and $x_2 = A_{2*}$ and ... and $x_i = A_{i*}$

Conclusion: $y = B^*$

where **Rule $i$ ($i = 1, 2, ..., p$)** is the $i$th fuzzy rule in the sparse fuzzy rule base, $x_k$ denotes the $k$th antecedent variable ($k = 1, 2, ..., h$), $y$ denotes the consequence variable, $A_{ki}$ denotes the $k$th antecedent fuzzy set of Rule $i$, $B_i$ denotes the consequence fuzzy set of Rule $i$, $A_{i*}$ denotes the $i$th observation fuzzy set for the $k$th antecedent variable $x_k$ and $B^*$ denotes the interpolated consequence fuzzy set. Figure 2 shows an example of the multiple fuzzy rules interpolation scheme with two antecedent variables using triangular fuzzy sets.

![Figure 2. Multiple fuzzy rules interpolation scheme with two fuzzy rules using triangular fuzzy sets.](image)

III. THE PROPOSED METHOD FOR HANDLING FORECASTING PROBLEMS BASED ON FUZZY CLUSTERING AND FUZZY RULES INTERPOLATION TECHNIQUES

In this section, we present a new method to handle forecasting problems based on fuzzy clustering and fuzzy rules interpolation techniques. The flowchart of the proposed algorithm is shown in Figure 3.

![Figure 3. The flowchart of the proposed algorithm.](image)

Assume that there is a forecasting dataset having $n$ training samples $X_1, X_2, ..., X_n$, where the $j$th sample $X_j$ is represented by $(I_{j1}^{(1)}, I_{j2}^{(2)}, ..., I_{jn}^{(j)}, O_j)$, $I_{j1}^{(k)}$ is the $k$th input of $X_j$, $1 \leq k \leq h$, and $O_j$ is the desired output of $X_j$. Let $c$ denote the number of clusters. The proposed algorithm to construct fuzzy rules from training samples is now presented as follows:

**Step 1:** Apply the FCM clustering algorithm [1] to update the cluster center $V_i$ of cluster $i$ and the membership grade $u_{ij}$ of sample $X_j$ belonging to cluster $C_i$ based on Eq. (2) and Eq. (3), respectively, until the objective function $J_m$ in Eq. (1) is no longer decreasing, where $1 \leq i \leq c$, $1 \leq j \leq n$, and the fuzziness index $m = 2$.

**Step 2:** Based on the clusters $C_1, C_2, ..., C_c$ obtained in Step 1, construct fuzzy rules **Rule $1$, Rule $2$, ..., and Rule $c$** using triangular fuzzy sets, where **Rule $i$** corresponds to cluster $C_i$, shown as follows:

- **Rule $i$:** If $x_1 = A_{1i}$ and $x_2 = A_{2i}$ and ... and $x_h = A_{hi}$, then $y = B_i$.

where $x_k$ is the $k$th antecedent variable ($k = 1, 2, ..., h$), $A_{ki}$ is the $k$th antecedent fuzzy set of Rule $i$, $y$ is the consequence variable, $B_i$ is the consequence fuzzy set of Rule $i$, the center
point \( b_k \), the left point \( a_k \), and the right point \( c_k \) of the triangular fuzzy set \( A_k \) of Rule \( i \) are calculated as follows:

\[
b_k = I^{(k)}_1, \quad \text{where } u_k = \max_{i \in [1,c]} u_i, \quad b_k = I^{(k)}_1, \quad \sum_{j=1,2,...,c} u_j \times I^{(k)}_j, \quad a_k = \frac{\sum_{j=1,2,...,c} u_j \times O_j}{\sum_{j=1,2,...,c} u_j}, \quad c_k = \frac{\sum_{j=1,2,...,c} u_j \times O_j}{\sum_{j=1,2,...,c} u_j},
\]

where \( b_k \) is the center point having the membership value of 1 in \( A_k \); \( a_k \) and \( c_k \) are the left point and the right point having the membership value of 0 in \( A_k \); \( I^{(k)}_1 \) is the \( k \)th input of \( X_j \), \( 1 \leq k \leq h \) and \( 1 \leq j \leq c \); and similarly, the center point \( b_k \), the left point \( a_k \) and the right point \( c_k \) of the triangular fuzzy set \( B_i \) of Rule \( i \) are calculated as follows:

\[
b_i = O_j, \quad \text{where } u_i = \max_{i \in [1,c]} u_i, \quad a_i = \frac{\sum_{j=1,2,...,c} u_j \times O_j}{\sum_{j=1,2,...,c} u_j}, \quad c_i = \frac{\sum_{j=1,2,...,c} u_j \times O_j}{\sum_{j=1,2,...,c} u_j},
\]

where \( O_j \) is the desired output of \( X_j \) and \( 1 \leq i \leq c \). Based on Eqs. (6)-(11), we can obtain the triangular fuzzy sets of the fuzzy rules Rule 1, Rule 2, … and Rule \( c \), shown as follows:

Rule 1: If \( x_1 = A_{11} \) and \( x_2 = A_{12} \) and … and \( x_k = A_{1k} \) Then \( y = B_1 \); Rule 2: If \( x_1 = A_{21} \) and \( x_2 = A_{22} \) and … and \( x_k = A_{2k} \) Then \( y = B_2 \); …

Rule c: If \( x_1 = A_{c1} \) and \( x_2 = A_{c2} \) and … and \( x_k = A_{ck} \) Then \( y = B_c \).  

Step 3: If the inputs \( I^{(1)}_1, I^{(1)}_2, \ldots, \) and \( I^{(h)}_1 \) of the \( j \)th sample \( X_j \) activate some fuzzy rules, \( \min_{i \in [1,c]} \mu_{A_{ki}}(I^{(k)}_j) > \eta \cdot \mu_{A_{ki}}(I^{(k)}_j) \) is the membership value of the input \( I^{(k)}_j \) belonging to triangular fuzzy set \( A_{ki} \), \( 1 \leq i \leq p \), and \( p \) denotes the number of activated fuzzy rules, then calculate the inferred output \( O_j^* \) as follows:

\[
O_j^* = \frac{\sum_{i=1}^{p} \min_{i \in [1,c]} \mu_{A_{ki}}(I^{(k)}_j) \times \text{DEF}(B_i)}{\sum_{i=1}^{p} \min_{i \in [1,c]} \mu_{A_{ki}}(I^{(k)}_j)},
\]

where DEF\((B_i)\) denotes the defuzzified value of the consequence fuzzy set \( B_i \) of the activated fuzzy rule Rule \( i \), and \( 1 \leq i \leq p \). Otherwise, go to Step 4.

Step 4: Based on the fuzzy rules Rule 1, Rule 2, … and Rule \( c \) obtained in Step 2, we have the following multiple fuzzy rules interpolation scheme [6]:

Rule 1: If \( x_1 = A_{11} \) and \( x_2 = A_{12} \) and … and \( x_k = A_{1k} \) Then \( y = B_1 \); Rule 2: If \( x_1 = A_{21} \) and \( x_2 = A_{22} \) and … and \( x_k = A_{2k} \) Then \( y = B_2 \); …

Rule c: If \( x_1 = A_{c1} \) and \( x_2 = A_{c2} \) and … and \( x_k = A_{ck} \) Then \( y = B_c \). Observations: \( x_1 = I^{(1)}_1 \) and \( x_2 = I^{(2)}_1 \) and … and \( x_k = I^{(h)}_1 \) 

Conclusion: \( y = O_j^* \)

where \( x_k \) denotes the \( k \)th antecedent variable (\( k = 1, 2, \ldots, h \)), \( y \) denotes the consequence variable, \( A_{ki} \) denotes the \( k \)th antecedent fuzzy set of Rule \( i \) (\( i = 1, 2, \ldots, c \)). \( B_i \) denotes the consequence fuzzy set of Rule \( i \), \( I^{(k)}_j \) denotes the \( k \)th input (\( k = 1, 2, \ldots, h \)) of the \( j \)th sample \( X_j \). \( O_j^* \) denotes the inferred output with respect to the inputs \( I^{(1)}_j, I^{(2)}_j, \ldots, I^{(h)}_j \) of \( X_j \). Calculate the weight \( W_i \) of Rule \( i \) (\( i = 1, 2, \ldots, c \)) with respect to the input observations \( x_1 = I^{(1)}_j \) and \( x_2 = I^{(2)}_j \) and … and \( x_k = I^{(h)}_j \), where

\[
W_i = \left( \sum_{j=1}^{c} \left( \frac{r^* - r_j}{\|r^* - r_j\|} \right)^{1} \right)^{-1},
\]

\( r^* \) denotes the vector of the inputs (\( I^{(1)}_j, I^{(2)}_j, \ldots, I^{(h)}_j \)), \( r_j \) denotes the vector of the defuzzified values of antecedent fuzzy sets of Rule \( i \) (\( \text{DEF}(A_{1j}), \text{DEF}(A_{2j}), \ldots, \text{DEF}(A_{cj}) \)) based on Eq. (4), \( 1 \leq k \leq h \), \( 0 \leq W_i \leq 1 \) and \( \sum_{i=1}^{c} W_i = 1 \).

Step 5: Calculate the inferred output \( O_j^* \) where

\[
O_j^* = \sum_{i=1}^{c} W_i \times \text{DEF}(B_i),
\]

where DEF\((B_i)\) is the defuzzified value of consequence fuzzy set \( B_i \) based on Eq. (4), \( 0 \leq W_i \leq 1 \), and \( \sum_{i=1}^{c} W_i = 1 \).

IV. EXPERIMENTAL RESULTS

In this section, we apply the proposed method for temperature prediction based on the data set of the daily average temperature and the data set of the daily average cloud density from June 1996 to September 1996 in Taipei [3], as shown in Table I and Table II. If we want to forecast the daily average temperature of day \( i \), then we use the proposed method to get the forecasted variation (i.e., the inferred output) of day \( i \), and the forecasted daily average temperature of day \( i \) is equal to the daily average temperature of day \( i - 1 \) plus the forecasted variation of day \( i \). We partition each data set into four groups, i.e., June 1996, July 1996, August 1996 and September 1996, and apply the proposed method to each group by using the variations of the daily average temperature and the daily average cloud density between any two adjacent days. Table III shows the variations of the daily average temperature and the daily average cloud density in June 1996, respectively. Let us consider a widow basis \( w \) using the historical data of the past \( w \) days to predict the forecasted data of the day being considered, where \( w \) is a positive integer. That is, the historical variations
... $T_{i1}$ of the daily average temperature and the historical variations $D_{i0}$, ..., $D_{i3}$ of the daily average cloud density are used to predict the variation $T_i$ of the daily average temperature of day $i$, where $T_{i0}$, ..., $T_{i2}$, $T_{i3}$, $D_{i0}$, ..., $D_{i2}$, $D_{i3}$ and $T_i$ form a training sample ($T_{i0}$, $D_{i0}$, ..., $T_{i2}$, $D_{i2}$, $T_{i3}$, $D_{i3}$, $T_i$). Table IV shows the 27 training samples of June 1996 based on the window basis $w = 2$. For example, the historical variations $T_2 = 1.5$ and $T_3 = 1.4$ of the daily average temperature and the historical variations $D_2 = -13$ and $D_3 = 0$ of the daily average cloud density are used to predict the variation $T_4 = 1.5$ of the daily average temperature of June 4, 1996, where $T_2$, $T_3$, $D_2$, $D_3$ and $T_4$ form the training sample $X_i = (1.5, -13, 1.4, 0, 1.5)$. In the following, we apply the proposed method to forecast the daily average temperature of June 1996, where the number of clusters is set to five.

<table>
<thead>
<tr>
<th>Month Day</th>
<th>June</th>
<th>July</th>
<th>August</th>
<th>September</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{10}$</td>
<td>$T_{11}$</td>
<td>$T_{12}$</td>
<td>$T_{13}$</td>
<td>$T_{14}$</td>
</tr>
</tbody>
</table>

Table I. The Historical Data of the Daily Average Temperature From June 1, 1996 to September 30, 1996 in Taipei, Taiwan (Unit: °C) [3]

<table>
<thead>
<tr>
<th>Month Day</th>
<th>June</th>
<th>July</th>
<th>August</th>
<th>September</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{10}$</td>
<td>$D_{11}$</td>
<td>$D_{12}$</td>
<td>$D_{13}$</td>
<td>$D_{14}$</td>
</tr>
</tbody>
</table>

Table II. The Historical Data of the Daily Average Cloud Density From June 1, 1996 to September 30, 1996 in Taipei, Taiwan (Unit: %) [3]

Given the initial randomly generated membership grades shown in Table V, the proposed method to forecast the daily average temperature for June, 1996 is presented as follows:

**[Step 1]** After applying the FCM clustering algorithm [1], we can get the five clusters, as shown in Table VI, where the membership grades of the training samples belonging to each cluster are shown in Table VII.

**[Step 2]** Based on Eqs. (5)-(10), we can get the following five fuzzy rules Rule 1, Rule 2, Rule 3, Rule 4 and Rule 5 from the training samples, where

Rule 1: If $x_1$ is $A_{11}$ and $x_2$ is $A_{12}$ and $x_3$ is $A_{13}$ and $x_4$ is $A_{14}$, then $y$ is $B_1$.

Rule 2: If $x_1$ is $A_{21}$ and $x_2$ is $A_{22}$ and $x_3$ is $A_{23}$ and $x_4$ is $A_{24}$, then $y$ is $B_2$.

Rule 3: If $x_1$ is $A_{31}$ and $x_2$ is $A_{32}$ and $x_3$ is $A_{33}$ and $x_4$ is $A_{34}$, then $y$ is $B_3$.

Rule 4: If $x_1$ is $A_{41}$ and $x_2$ is $A_{42}$ and $x_3$ is $A_{43}$ and $x_4$ is $A_{44}$, then $y$ is $B_4$.

Rule 5: If $x_1$ is $A_{51}$ and $x_2$ is $A_{52}$ and $x_3$ is $A_{53}$ and $x_4$ is $A_{54}$, then $y$ is $B_5$.

The antecedent fuzzy sets $A_{1j}$ are $(-0.257, -0.1, 0.39)$, $A_{2j} = (-3.298, 4, 8.245)$, $A_{3j} = (-2.1, -2.1, -0.429)$ and $A_{4j} = (17.573, 36, 38.353)$, $A_{5j} = (-0.629, -0.3, 0.753)$, $A_{5j} = (-18.589, -10, -3.577)$, $A_{3j} = (-0.842, -0.6, 0.315)$, $A_{4j} = (-15.537, -9, -0.671)$, $A_{1j} = (-1.225, -0.6, -0.011)$, $A_{2j} = (9.253, 26, 31.664)$, $A_{3j} = (0.08, 1.2, 1.276)$, $A_{4j} = (-3.121, -26, -13.441)$, $A_{4j} = (-0.563, -0.1, 0.413)$, $A_{2j} = (8.094, 22, 28.972)$, $A_{3j} = (-0.841, -0.8, 0.222)$, $A_{4j} = (-11.465, -3, 5.737)$, $A_{3j} = (0.37, 2, 2)$, $A_{2j} = (-8.781, 1, 7.796)$, $A_{3j} = (-0.79, -0.7, -0.05)$ and $A_{4j} = (2.769, 16, 19.623)$ are calculated by Eqs. (5)-(7), respectively, and the consequence fuzzy sets $B_1 = (-1.032, -0.9, -0.058)$, $B_2 = (-0.297, 0.6, 1.237)$, $B_3 = (0.03, 1.2, 1.22)$, $B_4 = (-0.553, 0.2, 0.737)$ and $B_5 = (-0.315, 0.2, 0.611)$ are calculated by Eqs. (8)-(10), respectively.

Table III. The Variation of the Daily Average Temperature and the Daily Cloudy Density in June 1996

<table>
<thead>
<tr>
<th>Day</th>
<th>Daily Average Temperature</th>
<th>Variation of the Daily Average Temperature</th>
<th>Daily Average Cloud Density</th>
<th>Variation of the Daily Average Cloud Density</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table IV. The Training Samples for June 1996 Based on the Window Basis $w = 2$

<table>
<thead>
<tr>
<th>Training Samples</th>
<th>$A_{(i)}, A_{(i)}, A_{(i)}, A_{(i)}, A_{(i)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>$A_{i0}$</td>
</tr>
<tr>
<td>$X_2$</td>
<td>$A_{i1}$</td>
</tr>
<tr>
<td>$X_3$</td>
<td>$A_{i2}$</td>
</tr>
<tr>
<td>$X_4$</td>
<td>$A_{i3}$</td>
</tr>
<tr>
<td>$X_5$</td>
<td>$A_{i4}$</td>
</tr>
</tbody>
</table>

Table V. The Membership Grades of the Training Samples

$[Step 3]$ Let us consider to infer the output (i.e., the forecasted variation) with respect to the inputs of the training sample $X_i$ shown in Table IV. That is, we want to forecast the daily average temperature of June 4, 1996, we can see that no fuzzy rules can be activated by the inputs $I^{(1)}_i$, $I^{(2)}_i$, $I^{(3)}_i$ and $I^{(4)}_i$ of $X_i$ shown in Table IV, where $X_i = (I^{(1)}_i, I^{(2)}_i, I^{(3)}_i, I^{(4)}_i, O_i) = (1.5, -13, 1.4, 0, 1.5)$. Because $\min \mu_{A_{i0}}(I^{(1)}) = 0$ for Rule 1, $\min \mu_{A_{i1}}(I^{(2)}) = 0$ for Rule 2, $\min \mu_{A_{i2}}(I^{(3)}) = 0$ for Rule 3, $\min \mu_{A_{i3}}(I^{(4)}) = 0$ for Rule 4, and $\min \mu_{A_{i4}}(O_i) = 0$ for Rule 5, we conclude that no fuzzy rules can be activated by the inputs $I^{(1)}_i$, $I^{(2)}_i$, $I^{(3)}_i$, and $I^{(4)}_i$ of $X_i$ shown in Table IV.
The training sample inputs are the inputs of the training sample. Step 2: Based on the five generated fuzzy rules obtained in Step 4, \( \min_{\mu_{A_i}(x_i)} \mu_{A_i}(x_i) = 0 \) for Rule 4 and \( \min_{\mu_{A_i}(x_i)} \mu_{A_i}(x_i) = 0 \) for Rule 5. We go to Step 4.

**TABLE V. Initial Randomly Generated Membership Grades of the Training Samples Belonging to Each Cluster**

<table>
<thead>
<tr>
<th>Clusters</th>
<th>( C_1 )</th>
<th>( C_2 )</th>
<th>( C_3 )</th>
<th>( C_4 )</th>
<th>( C_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.125</td>
<td>0.209</td>
<td>0.047</td>
<td>0.035</td>
<td>0.478</td>
</tr>
<tr>
<td>2</td>
<td>0.362</td>
<td>0.014</td>
<td>0.105</td>
<td>0.292</td>
<td>0.342</td>
</tr>
<tr>
<td>3</td>
<td>0.143</td>
<td>0.105</td>
<td>0.047</td>
<td>0.292</td>
<td>0.342</td>
</tr>
<tr>
<td>4</td>
<td>0.060</td>
<td>0.105</td>
<td>0.047</td>
<td>0.292</td>
<td>0.342</td>
</tr>
<tr>
<td>5</td>
<td>0.425</td>
<td>0.105</td>
<td>0.047</td>
<td>0.292</td>
<td>0.342</td>
</tr>
</tbody>
</table>

Following multiple fuzzy rules interpolation scheme:

| Rule 1: If \( x_1 = A_{11} \) and \( x_2 = A_{21} \) and \( x_3 = A_{31} \) and \( x_4 = A_{41} \) Then \( y = B_1 \). |
| Rule 2: If \( x_1 = A_{12} \) and \( x_2 = A_{22} \) and \( x_3 = A_{32} \) and \( x_4 = A_{42} \) Then \( y = B_2 \). |
| Rule 3: If \( x_1 = A_{13} \) and \( x_2 = A_{23} \) and \( x_3 = A_{33} \) and \( x_4 = A_{43} \) Then \( y = B_3 \). |
| Rule 4: If \( x_1 = A_{14} \) and \( x_2 = A_{24} \) and \( x_3 = A_{34} \) and \( x_4 = A_{44} \) Then \( y = B_4 \). |
| Rule 5: If \( x_1 = A_{15} \) and \( x_2 = A_{25} \) and \( x_3 = A_{35} \) and \( x_4 = A_{45} \) Then \( y = B_5 \). |

Observations: \( x_1 = I_1^{(1)} \) and \( x_2 = I_2^{(1)} \) and \( x_3 = I_3^{(1)} \) and \( x_4 = I_4^{(1)} \) (1). Conclusion: \( y = B_5 \). where \( A_{11} = (-0.257, -0.1, 0.39) \), \( A_{12} = (-0.297, 0.6, 1.237) \), \( A_{13} = (-1.225, -0.6, -0.011) \), \( A_{14} = (9.253, 26, 31.664) \), \( A_{15} = (-0.08, 1.2, 0.276) \), \( A_{21} = (-31.212, -26, -13.441) \), \( B_1 = (-0.03, 1.2, 0.225) \), \( A_{22} = (8.094, 22, 28.972) \), \( A_{23} = (-0.841, -0.8, 0.222) \), \( A_{24} = (-11.465, -3, 5.737) \), \( B_2 = (-0.553, 0.2, 0.737) \), \( A_{25} = (0.37, 2.2, 5.737) \), \( A_{31} = (-8.781, 1, 7.796) \), \( A_{32} = (-0.79, -0.7, -0.05) \), \( A_{33} = (2.769, 16, 19.623) \) and \( B_3 = (-0.315, 0.2, 0.611) \). The inputs \( I_1^{(1)}, I_2^{(1)}, I_3^{(1)} \) and \( I_4^{(1)} \) of \( X_i \) are input observations, \( X_i = (1.5, -13, 1.4, 0.15) \), \( I_1^{(2)} = 1.5 \), \( I_2^{(2)} = -13 \), \( I_3^{(3)} = 1.4 \), \( I_4^{(4)} = 0 \), and \( O_i^* \) denotes the inferred output. Based on Eq. (12), we can get the weights \( W_1, W_2, W_3, W_4 \) and \( W_5 \) of Rule 1, Rule 2, Rule 3, Rule 4 and Rule 5 with respect to the input observations \( x_1 = 1.5, x_2 = -13, x_3 = 1.4 \) and \( x_4 = 0 \), respectively, where \( W_1 = 0.0462, W_2 = 0.7042, W_3 = 0.0315, W_4 = 0.0535 \) and \( W_5 = 0.1646 \). [Step 5] Based on Eq. (13), we can calculate the inferred output \( O_i^* \) with respect to the training sample \( X_i \), where \( O_i^* = 0.409 \).

Therefore, the forecasted variation of June 4, 1996 is 0.409. Because the forecasted daily average temperature of June 4, 1996 is equal to the daily average temperature of June 3, 1996 (i.e., 29) plus the forecasted variation of June 4, 1996 (i.e., 0.409), the forecasted daily average temperature of June 4, 1996 is equal to 29 + 0.409 = 29.409. Table VIII shows the forecasted variations and the forecasted daily average temperature from June 4, 1996 to June 30, 1996.

In this paper, we use the average error rate (AER) to evaluate the forecasted result for temperature prediction, where

\[
AER = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{T_{\text{Forecasted}(i)} - T_{\text{Actual}(i)}}{T_{\text{Actual}(i)}} \right) \times 100\% ,
\]

where \( T_{\text{Forecasted}(i)} \) and \( T_{\text{Actual}(i)} \) denote the forecasted temperature and the actual temperature of day \( i \), respectively. The average error rate of the forecasted result of the proposed method shown in Table VIII is 2.68%. Because the FCM clustering algorithm [1] might produce different fuzzy clustering results depending on the initial randomly generated membership grades of the training samples belonging to each cluster, in this paper, we execute the proposed method 30 times and take the average of the average error rates of the forecasting results as the average error rate.

Table IX shows the average error rates of the forecasting results from June 1996 to September 1996 for different window bases based on Chen and Hwang’s method [7]. Table X, Table XI and Table XII show the average error rates of the forecasting results from June 1996 to September 1996 for different window basis based on the proposed method with five, ten and fifteen generated fuzzy rules, respectively. From the forecasting results shown in Table IX, Table X, Table XI and Table XII, respectively, we can see that the proposed method gets smaller average error rates than Chen and Hwang’s method [7].
In this paper, we have presented a method to deal with temperature forecasting based on fuzzy clustering and fuzzy rules interpolation techniques. First, the proposed method constructs fuzzy rules from training samples based on the fuzzy C-Means clustering algorithm [1], where each fuzzy rule corresponds to a fuzzy cluster and the linguistic terms appearing in the fuzzy rules are represented by triangular fuzzy sets. Then, it performs fuzzy inference based on the multiple fuzzy rules interpolation scheme. It calculates the weight of each fuzzy rule with respect to the input observation and uses the weight of each fuzzy rule to get the inferred output. We also have applied the proposed method to handle the temperature prediction problem. From the experimental result, we can see that the proposed method gets higher average forecasting accuracy rates than Chen and Hwang’s method [7].

V. CONCLUSIONS

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