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A New Method for Forecasting the TAIEX Based on High-Order Fuzzy Logical Relationships

Chao-Dian Chen[1] and Shyi-Ming Chen[1, 2]
[1]Department of Computer Science and Information Engineering, National Taiwan University of Science and Technology, Taipei, Taiwan, R. O. C.
[2]Department of Computer Science and Information Engineering, Jinwen University of Science and Technology, Taipei County, Taiwan, R. O. C.

***Abstract*—In this paper, we present a new method to forecast the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) based on high-order fuzzy logical relationships. First, the proposed method fuzzifies the historical data into fuzzy sets to form high-order fuzzy logical relationships. Then, it calculates the value of the variable between the subscripts of adjacent fuzzy sets appearing in the antecedents of high-order fuzzy logical relationships. Then, it lets the high-order fuzzy logical relationships having the same antecedent to form a high-order fuzzy logical relationship group. Finally, it chooses a high-order fuzzy logical relationship group to forecast the TAIEX. The proposed method gets a higher average forecasting accuracy rate than the existing methods to forecast the TAIEX.**

***Keywords*—fuzzy sets, fuzzy time series, fuzzy forecasting, high-order fuzzy time series, high-order fuzzy logical relationships**

I. INTRODUCTION

In [12], [13], and [14], Song and Chissom presented the concepts of fuzzy time series based on the fuzzy set theory [21], where the values of fuzzy time series are represented by fuzzy sets. In recent years, some methods have been presented to handle forecasting problems based on fuzzy time series, such as enrollments forecasting [1], [2], [3], [4], [13], [14], [16], [20], temperature prediction [5], [11], [15], stock index forecasting [6], [8], [9], [10], [15], [17], [18], [19], [21], …, etc.

In this paper, we present a new method to forecast the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) based on high-order fuzzy logical relationships groups. First, the proposed method fuzzifies the historical data into fuzzy sets to form high-order fuzzy logical relationships. Then, it calculates the value of the variable between the subscripts of adjacent fuzzy sets appearing in the antecedents of high-order fuzzy logical relationships. Then, it lets the high-order fuzzy logical relationships having the same antecedent to form a high-order fuzzy logical relationship group. Finally, it chooses a high-order fuzzy logical relationship group to forecast the TAIEX. The proposed method gets a higher average forecasting accuracy rate than Chen's method [1], Chen and Wang's method [6], Huarng's method [8], Huarng and Yu's method [9], and Yu's method [18] for forecasting the TAIEX.

The rest of this paper is organized as follows. In Section II, we briefly review the concept of fuzzy time series from [12], [13] and [14]. In Section III, we present a new method to forecast the TAIEX based on high-order fuzzy logical relationships. In Section IV, we make a comparison of the forecasting result of the proposed method with the existing methods. The conclusions are discussed in Section V.

II. PRELIMINARIES

In [12], [13] and [14], Song and Chissom presented the concepts of fuzzy time series based on the fuzzy set theory [21], where the values in a fuzzy time series are represented by fuzzy sets. Let U be the universe of discourse, where $U = \{u_1, u_2, \ldots, u_n\}$. A fuzzy set A_i in the universe of discourse U is defined as follows:

$$A_i = f_{Ai}(u_1)/u_1 + f_{Ai}(u_2)/u_2 + \ldots + f_{Ai}(u_n)/u_n,$$

where f_{Ai} is the membership function of the fuzzy set A_i, $f_{Ai}: U \rightarrow [0,1]$, $f_{Ai}(u_j)$ is the degree of membership of u_j belonging to the fuzzy set A_i, $f_{Ai}(u_j) \in [0,1]$ and $1 \le j \le n$.

***Definition 2.1* [12]:** Let $Y(t)$ ($t = \ldots, 0, 1, 2, \ldots$) be the universe of discourse and be a subset of R. Assume that $f_i(t)$ ($i = 1, 2, \ldots$) are defined in the universe of discourse $Y(t)$, and assume that $F(t)$ is a collection of $f_i(t)$ ($i = 1, 2, \ldots$), then $F(t)$ is called a fuzzy time series of $Y(t)$ ($t = \ldots, 0, 1, 2, \ldots$).

If there exists a fuzzy relation $R(t-1, t)$, such that $F(t) = F(t-1) \circ R(t-1, t)$, where the symbol "$\circ$" represents the max-min composition operator, then $F(t)$ is called caused by $F(t-1)$ [12].

***Definition 2.2*:** Let $F(t-1) = A_i$ and let $F(t) = A_j$, where A_i and A_j are fuzzy sets, then the fuzzy logical relationship (FLR) between $F(t-1)$ and $F(t)$ can be denoted by $A_i \rightarrow A_j$, where A_i and A_j are called the left-hand side (LHS) and the right-hand side (RHS) of the fuzzy logical relationship, respectively.

Fuzzy logical relationships having the same left hand side can be grouped into a fuzzy logical relationship group (FLRG) [1]. For example, assume that the following fuzzy logical relationships exist:

$$\begin{gathered} A_i \rightarrow A_{ja}, \\ A_i \rightarrow A_{jb}, \\ \vdots \\ A_i \rightarrow A_{jm}, \end{gathered}$$

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then these fuzzy logical relationships can be grouped into a fuzzy logical relationship group, shown as follows:

$$A_i \rightarrow A_{ja}, A_{jb}, \ldots, A_{jm}.$$

Let $F(t)$ be a fuzzy time series. If $F(t)$ is caused by $F(t-1)$, $F(t-2)$, …, and $F(t-n)$, then the fuzzy logical relationship between them can be represented by the "nth-order fuzzy logical relationship"[2], shown as follows:

$$F(t-n), \ldots, F(t-2), F(t-1) \rightarrow F(t).$$

If $F(t-n) = A_{in}$, …, $F(t-2) = A_{i2}$, $F(t-1) = A_{i1}$, $F(t) = A_j$, where A_{in}, …, A_{i2}, A_{i1}, A_j are fuzzy sets, then the nth-order fuzzy logical relationship can be represented by

$$A_{in}, \ldots, A_{i2}, A_{i1} \rightarrow A_j,$$

where A_{in}, …, A_{i2}, and A_{i1} are called the antecedent fuzzy sets of the nth-order fuzzy logical relationship; "A_{in}, …, A_{i2}, A_{i1}" and "A_j" are called the left-hand side and the right-hand side of the nth-order fuzzy logical relationship, respectively.

If there exist the nth-order fuzzy logical relationships having the same left hand side, shown as follows:

$$A_{in}, \ldots, A_{i2}, A_{i1} \rightarrow A_{ja},$$

$$A_{in}, \ldots, A_{i2}, A_{i1} \rightarrow A_{jb},$$

$$\vdots$$

$$A_{in}, \ldots, A_{i2}, A_{i1} \rightarrow A_{jm},$$

then these nth-order fuzzy logical relationships form a nth-order fuzzy logical relationships group, shown as follows:

$$A_{in}, A_{i(n-1)}, \ldots, A_{i1} \rightarrow A_{ja}, A_{jb}, \ldots, A_{jm}.$$

III. A New Method for Forecasting the TAIEX Based on High-Order Fuzzy Logical Relationships

In this section, we present a new method for forecasting the TAIEX based on high-order fuzzy logical relationships. We divide the historical data of the TAIEX into two parts, where one part is used as the training data set and the other as the testing data set. The training data set and the testing data set of the TAIEX of every year from 1991 to 1999 are shown in Table I [7], respectively. The proposed method for forecasting the TAIEX based on high-order fuzzy logical relationships is shown as follows:

Step 1: Define the universe of discourse U, $U = [D_{min}-D_1, D_{max}+D_2]$ into intervals of equal length, where D_{min} and D_{max} are the minimum value and the maximum value of the historical data, respectively, and D_1 and D_2 are two proper positive real values to divide the universe of discourse U into n intervals u_1, u_2, …, u_n of equal length. For example from Table II, we can see that the minimum and the maximum stock indices of the year 1999 are 5474.79 and 8608.91, respectively. If we let $D_1 = 74.79$ and $D_2 = 91.09$, then the universe of discourse $U = [5400, 8700]$. Let the length of each interval in the universe of discourse U be 25. Then, the universe of discourse U can be divided into 132 intervals, which are defined as follows:

$$u_i = [5400 + (i-1) \times 25, 5400 + (i) \times 25], \quad (1)$$

where $i = 1, 2, \ldots, 132$.

TABLE I. Training Data Set and Testing Data Set of the TAIEX from 1991 to 1999 [7]

Year	1991	1992	1993	1994	1995	1996	1997	1998	1999
Training data set	1/1~ 10/30	1/4~ 10/30	1/5~ 10/30	1/5~ 10/29	1/5~ 10/30	1/4~ 10/30	1/4~ 10/30	1/3~ 10/31	1/5~ 10/30
Testing data set	11/1~ 12/28	11/2~ 12/29	11/2~ 12/31	11/1~ 12/31	11/1~ 12/30	11/1~ 12/31	11/3~ 12/31	11/2~ 12/31	11/1~ 12/28

TABLE II. Historical Training Data of the TAIEX of 1999 [7]

Date	TAIEX
1999/1/5	6152.43
1999/1/6	6199.91
1999/1/7	6404.31
1999/1/8	6421.75
⋮	⋮
1999/10/28	7681.85
1999/10/29	7706.67
1999/10/30	7854.85

TABLE III. Historical Testing Data of the TAIEX of 1999 [7]

Date	TAIEX
1999/11/1	7814.89
1999/11/2	7721.59
1999/11/3	7580.09
1999/11/4	7469.23
⋮	⋮
1999/12/24	8219.45
1999/12/27	8415.07
1999/12/28	8448.84

Step 2: Define the linguistic term A_i represented by fuzzy sets, shown as follows:

$$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + \ldots + 0/u_{n-1} + 0/u_n,$$

$$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + \ldots + 0/u_{n-1} + 0/u_n,$$

$$A_3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + \ldots + 0/u_{n-1} + 0/u_n,$$

$$\vdots$$

$$A_{n-1} = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + \ldots + 1/u_{n-1} + 0.5/u_n,$$

$$A_n = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + \ldots + 0.5/u_{n-1} + 1/u_n.$$

where A_1, A_2, …, and A_n are linguistic terms represented by fuzzy sets. For example, based on the obtained 132 intervals, we define the linguistic terms A_1, A_2, …, and A_{132}, shown as follows:

$$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4 + \ldots + 0/u_{131} + 0/u_{132},$$

$$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4 + \ldots + 0/u_{131} + 0/u_{132},$$

$$A_3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4 + \ldots + 0/u_{131} + 0/u_{132},$$

$$\vdots$$

$$A_{131} = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + \ldots + 1/u_{131} + 0.5/u_{132},$$

$$A_{132} = 0/u_1 + 0/u_2 + 0/u_3 + 0/u_4 + \ldots + 0.5/u_{131} + 1/u_{132}.$$

Step 3: Fuzzify each historical datum into a fuzzy set defined in **Step 2**. If the historical datum belongs to u_i and the maximum membership value of A_i occurs at u_i, then the historical datum is fuzzified into A_i, where $1 \leq i \leq n$. For example, the TAIEX of 1999/1/5 shown in Table II is 6152.43, which is fuzzified into A_{31}, as shown in Table IV. In the same way, the TAIEX of 1999/1/6 shown in Table II is 6199.91, which is fuzzified to A_{32}, as shown in Table IV. Table IV and Table V show the fuzzified TAIEX of the training data set and the testing data set of 1999, respectively.

TABLE IV. FUZZIFY TAIEX OF THE TRAINING DATA SET

Date	Fuzzy Set
1999/1/5	A_{31}
1999/1/6	A_{32}
1999/1/7	A_{41}
1999/1/8	A_{41}
⋮	⋮
1999/10/28	A_{92}
1999/10/29	A_{93}
1999/10/30	A_{99}

TABLE V. FUZZIFIED TAIEX OF THE TESTING DATA SET

Date	Fuzzy Set
1999/11/1	A_{97}
1999/11/2	A_{93}
1999/11/3	A_{88}
1999/11/4	A_{83}
⋮	⋮
1999/12/24	A_{113}
1999/12/27	A_{121}
1999/12/28	A_{122}

Step 4: Construct the nth-order fuzzy logical relationships from the fuzzified TAIEX of the training data set. For example, from Table IV, we can see that the fuzzified TAIEX of 1999/1/5 is A_{31}, the fuzzified TAIEX of 1999/1/6 is A_{32}, and the fuzzified TAIEX of 1999/1/7 is A_{41}. Thus, we can get the following second-order fuzzy logical relationship:

$$A_{31}, A_{32} \rightarrow A_{41}.$$

From Table IV, we also can see that the fuzzified TAIEX of 1999/1/6 is A_{32}, the fuzzified TAIEX of 1999/1/7 is A_{41}, and the fuzzified TAIEX of 1999/1/8 is A_{41}. Thus, we can get the following second-order fuzzy logical relationship:

$$A_{32}, A_{41} \rightarrow A_{41}.$$

In the same way, we can get the other fuzzy logical relationships from the fuzzified TAIEX of the training data set shown in Table IV, as shown in Table VI.

Step 5: Transform each nth-order fuzzy logical relationship "$A_{X1}, A_{X2}, A_{X3}, \ldots, A_{Xj}, \ldots, A_{Xn} \rightarrow A_{Xr}$" into the following form: $A_{X1}, A_{X1+V(X1)}, A_{X1+V(X1)+V(X2)}, \ldots, A_{X1+V(X1)+V(X2)+\ldots+V(Xi)}, \ldots, A_{X1+V(X1)+V(X2)+\ldots+V(Xi)+\ldots+V(Xm)} \rightarrow A_{X1+V(X1)+V(X2)+\ldots+V(Xi)+\ldots+V(Xm)+V(Xn)}$, where $V(X1)$, $V(X2)$, …, and $V(Xn)$ are integers. For example, if the second-order fuzzy logical relationship is "$A_{31}, A_{32} \rightarrow A_{41}$", then we can transform it into: "$A_{X1}, A_{X1+V(X1)} \rightarrow A_{X1+V(X1)+V(X2)}$", where the subscripts $X1$, $X2$, and $X3$ of the fuzzy sets A_{31}, A_{32}, and A_{41} are 31, 32, and 41, respectively, i.e; $X1 = 31$, $X2 = 32$ and $X3 = 41$. Thus, we can get $V(X1) = X2 - X1 = 32 - 31 = 1$ and $V(X2) = X3 - (X1+V(X1)) = X3 - X2 = 41 - 32 = 9$. Therefore, the second-order fuzzy logical relationship "$A_{31}, A_{32} \rightarrow A_{41}$" is represented as: "$A_{31}, A_{31+1} \rightarrow A_{31+1+9}$". Moreover, we can see that the second-order fuzzy logical relationship "$A_{32}, A_{41} \rightarrow A_{41}$" is represented as: "$A_{32}, A_{32+9} \rightarrow A_{32+9+0}$". In the same way, we can transform the second-order fuzzy logical relationships shown in Table VI into Table VII.

TABLE VI. SECOND-ORDER FUZZY LOGICAL RELATIONSHIPS

Date	Fuzzy Logical Relationships
1999/1/5,1999/1/6 → 1999/1/7	$A_{31}, A_{32} \rightarrow A_{41}$
1999/1/6,1999/1/7→ 1999/1/8	$A_{32}, A_{41} \rightarrow A_{41}$
1999/1/7,1999/1/8 → 1999/1/11	$A_{41}, A_{41} \rightarrow A_{41}$
1999/1/8,1999/1/11 → 1999/1/12	$A_{41}, A_{41} \rightarrow A_{39}$
⋮	⋮
1999/10/26,1999/10/27 → 1999/10/28	$A_{93}, A_{93} \rightarrow A_{92}$
1999/10/27,1999/10/28 → 1999/10/29	$A_{93}, A_{92} \rightarrow A_{93}$
1999/10/28,1999/10/29 → 1999/10/30	$A_{92}, A_{93} \rightarrow A_{99}$

TABLE VII. TRANSFORMED FUZZY VARIABLE LOGICAL RELATIONSHIPS

Date	Fuzzy Variable Logical Relationships
1999/1/5,1999/1/6 → 1999/1/7	$A_{31}, A_{31+1} \rightarrow A_{31+1+9}$
1999/1/6,1999/1/7→ 1999/1/8	$A_{32}, A_{32+9} \rightarrow A_{32+9+0}$
1999/1/7,1999/1/8 → 1999/1/11	$A_{41}, A_{41+0} \rightarrow A_{41+0+0}$
1999/1/8,1999/1/11 → 1999/1/12	$A_{41}, A_{41+0} \rightarrow A_{41+0-2}$
⋮	⋮
1999/10/26,1999/10/27 → 1999/10/28	$A_{93}, A_{93+0} \rightarrow A_{93+0-1}$
1999/10/27,1999/10/28 → 1999/10/29	$A_{93}, A_{93-1} \rightarrow A_{93-1+1}$
1999/10/28,1999/10/29 → 1999/10/30	$A_{92}, A_{92+1} \rightarrow A_{92+1+6}$

Step 6: Let the transformed nth-order fuzzy logical relationships obtained in **Step 5** having the same left-hand side form a nth-order fuzzy logical relationship group. For example, let us consider the following transformed third-order fuzzy logical relationships: "$A_{a1}, A_{a1+V(a1)}, A_{a1+V(a1)+V(a2)} \rightarrow A_{a1+V(a1)+V(a2)+V(a3)}$", "$A_{b1}, A_{b1+V(b1)}, A_{b1+V(b1)+V(b2)} \rightarrow A_{b1+V(b1)+V(b2)+V(b3)}$", …, and "$A_{k1}, A_{k1+V(k1)}, A_{k1+V(k1)+V(k2)} \rightarrow A_{k1+V(k1)+V(k2)+V(k3)}$", where $V(a1) = V(b1) = \ldots = V(k1)$ and $V(a2) = V(b2) = \ldots = V(k2)$, then these third-order fuzzy logical relationships can be grouped into a transformed third-order fuzzy logical relationships group, shown as follows:

$$A_{a1}, A_{a1+V(a1)}, A_{a1+V(a1)+V(a2)} \rightarrow A_{a1+V(a1)+V(a2)+V(a3)},$$

$$A_{b1}, A_{b1+V(b1)}, A_{b1+V(b1)+V(b2)} \rightarrow A_{b1+V(b1)+V(b2)+V(b3)},$$

$$\vdots$$

$$A_{k1}, A_{k1+V(k1)}, A_{k1+V(k1)+V(k2)} \rightarrow A_{k1+V(k1)+V(k2)+V(k3)}.$$

Moreover, this transformed third-order fuzzy logical relationships group can be represented as:

$A_X, A_{X+V(y1)}, A_{X+V(y1)+V(y2)} \rightarrow A_{X+V(y1)+V(y2)+V(a3)}, A_{X+V(y1)+V(y2)+V(b3)}, \ldots, A_{X+V(y1)+V(y2)+V(k3)}$. where $X = a1, b1, \ldots, k1$, $V(y1) = V(a1) = V(b1) = \ldots = V(k1)$ and $V(y2) = V(a2) = V(b2) = \ldots = V(k2)$. For example, let us consider the second-order fuzzy logical relationships "$A_{31}, A_{31+1} \rightarrow A_{31+1+9}$", "$A_{43}, A_{43+1} \rightarrow A_{43+1-4}$", and "$A_{37}, A_{37+1} \rightarrow A_{37+1-4}$" shown in Table VII, where their transformed second-order fuzzy logical relationships are "$A_x, A_{x+1} \rightarrow A_{x+1+9}$", "$A_x, A_{x+1} \rightarrow A_{x+1-4}$", and "$A_x, A_{x+1} \rightarrow A_{x+1-4}$", respectively. Because these transformed second-order fuzzy logical relationship have the same left-hand side "A_X, A_{X+1}", we can group them into a transformed second-order fuzzy logical relationship group "A_X, $A_{X+1} \rightarrow A_{X+1+9}$, A_{X+1-4}, A_{X+1-4}". Table VIII shows the transformed second-order fuzzy logical relationship groups derived from Table VII.

TABLE VIII. TRANSFORMED SECOND-ORDER FUZZY LOGICAL RELATIONSHIP GROUPS

Group	Transformed Second-Order Fuzzy Logical Relationship
Group 1	$A_X, A_{X-20} \rightarrow A_{X-20-2}$
Group 2	$A_X, A_{X-13} \rightarrow A_{X-13+1}, A_{X-13+1}$
Group 3	$A_X, A_{X-10} \rightarrow A_{X-10-13}$
Group 4	$A_X, A_{X-9} \rightarrow A_{X-9-2}, A_{X-9+5}$
Group 5	$A_X, A_{X-8} \rightarrow A_{X-8+3}, A_{X-8+3}, A_{X-8+1}, A_{X-8-7}$
Group 6	$A_X, A_{X-7} \rightarrow A_{X-7+1}, A_{X-7+1}, A_{X-7+0}, A_{X-7+1}$
Group 7	$A_X, A_{X-6} \rightarrow A_{X-6+0}, A_{X-6-6}, A_{X-6+9}$
Group 8	$A_X, A_{X-5} \rightarrow A_{X-5-5}, A_{X-5+0}, A_{X-5+2}, A_{X-5-1}, A_{X-5+4}, A_{X-5+0}, A_{X-5-9}, A_{X-5+2}, A_{X-5+6}, A_{X-5+0}, A_{X-5-2}$
Group 9	$A_X, A_{X-4} \rightarrow A_{X-4-2}, A_{X-4-8}, A_{X-4+3}, A_{X-4-2}, A_{X-4+1}, A_{X-4+3}, A_{X-4-10}, A_{X-4+1}, A_{X-4-1}, A_{X-4-5}, A_{X-4-2}, A_{X-4+5}$
Group 10	$A_X, A_{X-3} \rightarrow A_{X-3+9}, A_{X-3-3}, A_{X-3+0}, A_{X-3-3}, A_{X-3-8}, A_{X-3+1}, A_{X-3-6}, A_{X-3-6}, A_{X-3+1}$
Group 11	$A_X, A_{X-2} \rightarrow A_{X-2-2}, A_{X-2-3}, A_{X-2-1}, A_{X-2+10}, A_{X-2+6}, A_{X-2+1}, A_{X-2+2}, A_{X-2-1}, A_{X-2+4}, A_{X-2+1}, A_{X-2-5}, A_{X-2-1}, A_{X-2+4}, A_{X-2-8}, A_{X-2+4}, A_{X-2+2}, A_{X-2-1}$
Group 12	$A_X, A_{X-1} \rightarrow A_{X-1+1}, A_{X-1+0}, A_{X-1+0}, A_{X-1-1}, A_{X-1+5}, A_{X-1+4}, A_{X-1+5}, A_{X-1+0}, A_{X-1-1}, A_{X-1-1}, A_{X-1+7}, A_{X-1-7}, A_{X-1+0}, A_{X-1-4}, A_{X-1-4}, A_{X-1+4}, A_{X-1+6}, A_{X-1+0}, A_{X-1-7}, A_{X-1+4}, A_{X-1+1}, A_{X-1+0}, A_{X-1+1}$
Group 13	$A_X, A_{X+0} \rightarrow A_{X+0+0}, A_{X+0-2}, A_{X+0-5}, A_{X+0-9}, A_{X+0-2}, A_{X+0+1}, A_{X+0-2}, A_{X+0+2}, A_{X+0+5}, A_{X+0-1}, A_{X+0-4}, A_{X+0+0}, A_{X+0+0}, A_{X+0+1}, A_{X+0+0}, A_{X+0-1}, A_{X+0-1}, A_{X+0+4}, A_{X+0+1}, A_{X+0+5}, A_{X+0-3}, A_{X+0+9}, A_{X+0+4}, A_{X+0+3}, A_{X+0-2}, A_{X+0+2}, A_{X+0-4}, A_{X+0-1}$
Group 14	$A_X, A_{X+1} \rightarrow A_{X+1+9}, A_{X+1-4}, A_{X+1-4}, A_{X+1-3}, A_{X+1+0}, A_{X+1+1}, A_{X+1+2}, A_{X+1+5}, A_{X+1+1}, A_{X+1-7}, A_{X+1+5}, A_{X+1-3}, A_{X+1+8}, A_{X+1-1}, A_{X+1+0}, A_{X+1+1}, A_{X+1-3}, A_{X+1-4}, A_{X+1+2}, A_{X+1+0}, A_{X+1+9}, A_{X+1-5}, A_{X+1-20}, A_{X+1+17}, A_{X+1-5}, A_{X+1+3}, A_{X+1-4}, A_{X+1-1}, A_{X+1-5}, A_{X+1-4}, A_{X+1+0}, A_{X+1+6}$
Group 15	$A_X, A_{X+2} \rightarrow A_{X+2+2}, A_{X+2+1}, A_{X+2+0}, A_{X+2-4}, A_{X+2+4}, A_{X+2+4}, A_{X+2+7}, A_{X+2-3}, A_{X+2-2}, A_{X+2+9}, A_{X+2-3}$
Group 16	$A_X, A_{X+3} \rightarrow A_{X+3+1}, A_{X+3+11}, A_{X+3+4}, A_{X+3+1}, A_{X+3+4}, A_{X+3+1}, A_{X+3+3}, A_{X+3+1}, A_{X+3-4}, A_{X+3-8}, A_{X+3+5}$
Group 17	$A_X, A_{X+4} \rightarrow A_{X+4-4}, A_{X+4+5}, A_{X+4+0}, A_{X+4-5}, A_{X+4+2}, A_{X+4-1}, A_{X+4+3}, A_{X+4+8}, A_{X+4-13}, A_{X+4+0}, A_{X+4+12}, A_{X+4+4}, A_{X+4-1}, A_{X+4-2}, A_{X+4-1}, A_{X+4-2}$
Group 18	$A_X, A_{X+5} \rightarrow A_{X+5+3}, A_{X+5+4}, A_{X+5+9}, A_{X+5+0}, A_{X+5+1}, A_{X+5+1}, A_{X+5-1}, A_{X+5-5}, A_{X+5+1}, A_{X+5+1}$
Group 19	$A_X, A_{X+6} \rightarrow A_{X+6-1}, A_{X+6+8}, A_{X+6+2}, A_{X+6-2}$
Group 20	$A_X, A_{X+7} \rightarrow A_{X+7+0}, A_{X+7+3}$
Group 21	$A_X, A_{X+8} \rightarrow A_{X+8-1}, A_{X+8+6}, A_{X+8-5}$
Group 22	$A_X, A_{X+9} \rightarrow A_{X+9+0}, A_{X+9+1}, A_{X+9-2}, A_{X+9-2}, A_{X+9+0}, A_{X+9-1}, A_{X+9+0}$
Group 23	$A_X, A_{X+10} \rightarrow A_{X+10+4}, A_{X+10-5}$
Group 24	$A_X, A_{X+11} \rightarrow A_{X+11+10}$
Group 25	$A_X, A_{X+12} \rightarrow A_{X+12+15}$
Group 26	$A_X, A_{X+15} \rightarrow A_{X+15+3}$
Group 27	$A_X, A_{X+17} \rightarrow A_{X+17-1}$.

Step 7: Choose a transformed nth-order fuzzy logical relationship group for prediction. Assume that $F(t-n) = A_{in}$, $F(t-(n-1)) = A_{i(n-1)}$, …, $F(t-2) = A_{i2}$, and $F(t-1) = A_{i1}$, and assume that we want to predict $F(t)$, where A_{i1}, A_{i2}, …, and A_{in} are fuzzy sets. Based on the transformed nth-order fuzzy logical relationship groups obtained in **Step 6**, choose the corresponding transformed nth-order fuzzy logical relationship group for prediction. If the chosen transformed nth-order fuzzy logical relationship group is: "A_X, $A_{X+V(y1)}$, …, $A_{X+V(y1)+V(y2)+\ldots+V(y(n-1))} \rightarrow A_{X+V(y1)+\ldots+V(y(n-1))+V(an)}$, $A_{X+V(y1)+\ldots+V(y(n-1))+V(bn)}$, …, $A_{X+V(y1)+\ldots+V(y(n-1))+V(kn)}$", where $A_{in} = A_X$, $A_{i(n-1)} = A_{X+V(y1)}$, …, $A_{i1} = A_{X+V(y1)+V(y2)+\ldots+V(y(n-1))}$, then replace X by the subscript in of the fuzzy set A_{in} to get the derived fuzzy sets $A_{in+V(y1)+\ldots+V(y(n-1))+V(an)}$, $A_{in+V(y1)+\ldots+V(y(n-1))+V(bn)}$, …, and $A_{in+V(y1)+\ldots+V(y(n-1))+V(kn)}$ for prediction. Let $A_{j1} = A_{in+V(y1)+\ldots+V(y(n-1))+V(an)}$, let $A_{j2} = A_{in+V(y1)+\ldots+V(y(n-1))+V(bn)}$, …, and let $A_{jk} = A_{in+V(y1)+\ldots+V(y(n-1))+V(kn)}$. Then, the forecasted value FV of day t is calculated as follows:

$$FV = \frac{\sum_{i=1}^{k} m_{ji}}{k}, \tag{2}$$

where the maximum membership values of A_{j1}, A_{j2}, …, and A_{jk}, occur at the intervals u_{j1}, u_{j2}, …, and u_{jk}, respectively, and m_{j1}, m_{j2}, …, and m_{jk} are the midpoints of u_{j1}, u_{j2},…, and u_{jk}, respectively. For example, assume that we want to forecast the TAIEX of 1999/11/3 by the second-order fuzzy logical relationships. From Table V, we can see that the fuzzified TAIEX of 1999/11/1 and 1999/11/2 are A_{97} and A_{93}, respectively. Therefore, we can see that A_{97} and A_{93} can be transformed into A_x and A_{x-4}, respectively, where $X = 97$. Based on Table VIII, we choose Group 9 of the transformed second-order fuzzy logical relationships group for prediction, i.e., A_X, $A_{X-4} \rightarrow A_{X-4-2}$, A_{X-4-8}, A_{X-4+3}, A_{X-4-2}, A_{X-4+1}, A_{X-4+3}, A_{X-4-10}, A_{X-4+1}, A_{X-4-1}, A_{X-4-5}, A_{X-4-2}, A_{X-4+5}. By letting X be 97, we can see that A_{X-4-2}, A_{X-4-8}, A_{X-4+3}, A_{X-4-2}, A_{X-4+1}, A_{X-4+3}, A_{X-4-10}, A_{X-4+1}, A_{X-4-1}, A_{X-4-5}, A_{X-4-2}, and A_{X-4+5} become A_{91}, A_{85}, A_{96}, A_{91}, A_{94}, A_{96}, A_{83}, A_{94}, A_{92}, A_{88}, A_{91} and A_{98}, respectively. Then, the forecasted TAIEX of 1999/11/3 is equal to the average of the midpoints u_{91}, u_{85}, u_{96}, u_{91}, u_{94}, u_{96}, u_{83}, u_{94}, u_{92}, u_{88}, u_{91} and u_{98}, where the maximum membership values of the fuzzy sets A_{91}, A_{85}, A_{96}, A_{91}, A_{94}, A_{96}, A_{83}, A_{94}, A_{92}, A_{88}, A_{91} and A_{98} occur at u_{91}, u_{85}, u_{96}, u_{91}, u_{94}, u_{96}, u_{83}, u_{94}, u_{92}, u_{88}, u_{91} and u_{98}, respectively. From Eq. (1), we can see that the midpoints of the intervals u_{91}, u_{85}, u_{96}, u_{91}, u_{94}, u_{96}, u_{83}, u_{94}, u_{92}, u_{88}, u_{91} and u_{98} are 7662.5, 7512.5, 7787.5, 7662.5, 7737.5, 7787.5, 7462.5, 7737.5, 7687.5, 7587.5, 7662.5 and 7837.5, respectively. Thus, based on Eq. (2), the forecasted TAIEX of 1999/11/3 is calculated as follows:

$$\frac{7662.5 + 7512.5 + 7787.5 + 7662.5 + 7737.5 + 7787.5 + 7462.5 + 7737.5 + 7687.5 + 7587.5 + 7662.5 + 7837.5}{12} = 7677.08.$$

That is, the forecasted TAIEX of 1999/11/3 is 7677.08.

IV. EXPERIMENTAL RESULTS

In this section, we apply the proposed method for forecasting the TAIEX from 1991 to 1999 with different lengths of intervals. We evaluate the performance of the proposed method using the root mean square error (RMSE), which is defined as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (forecasted\ value_i - actual\ value_i)^2}{n}}, \tag{3}$$

where n denotes the number of dates needed to be forecasted, *forecasted value*$_i$ denotes the forecasted value of day i, *actual*

$value_i$ denotes the actual value of day i and $1 \leq i \leq n$. Table IX makes a comparison of the RMSE and the average RMSE of the proposed method to forecast the TAIEX for different lengths of intervals. From Table IX, we can see that when the length of interval is 25, the proposed method gets the smallest average RMSE. In Table X, we make a comparison of the RMSE and the average RMSE of the proposed method with Chen's method [1], Chen and Wang's method [6], Huarng's method [8], Huarng and Yu's method [9], and Yu's method [18]. From Table X, we can see that the proposed method gets the smallest average RMSE comparing to Chen's method [1], Chen and Wang's method [6], Huarng's method [8], Huarng and Yu's method [9], and Yu's method [18]. It means that the proposed method gets a higher average forecasting accuracy rate than Chen's method [1], Chen and Wang's method [6], Huarng's method [8], Huarng and Yu's method [9], and Yu's method [18] to forecast the TAIEX.

TABLE IX. A COMPARISON OF THE RMSES AND THE AVERAGE RMSE OF THE PROPOSED METHOD FOR DIFFERENT LENGTHS OF INTERVALS

Length of Interval \ RMSE \ Year	1991	1992	1993	1994	1995	1996	1997	1998	1999	Average RMSE
10	57.22	44.92	101.57	78.55	65.31	50.81	145.76	121.54	98.11	84.87
15	54.6	42.81	103.98	82.64	56.47	47.59	136.91	124.32	97.92	83.03
20	50.17	41.46	105.59	80.89	58.72	47.44	164.87	113.49	97.97	84.51
25	50.11	44.33	102.13	75.54	60.03	51.12	140.08	120.26	95.65	82.14
30	52.15	44.96	112.69	81.33	56.44	49.86	136.09	123.97	101.94	84.38
35	53.06	42.9	101.91	76.04	57.88	49.62	161.12	131.1	101.89	86.17
40	50.06	42.34	116.48	76.91	57.48	48.49	147.82	114.5	102.53	84.07
50	56.12	44.52	116.04	75.93	60.6	48.48	137.26	111.7	99.98	83.40
100	60.25	44.48	102.46	79.85	66.23	57.87	133.9	113.62	105.45	84.90

TABLE X. A COMPARISON OF THE RMSES AND THE AVERAGE RMSE FOR DIFFERENT METHODS

Method \ RMSE \ Year	1991	1992	1993	1994	1995	1996	1997	1998	1999	Average RMSE
Chen's Method [1]	80	60	110	112	79	54	148	167	149	107
Chen and Wang's Method [6]	42.89	43.48	103.37	89.81	52.2	52.83	140.75	116.88	104.88	83.01
Huarng's Method [8] Based on Average-Based Length Interval	79.4	59.9	105.2	132.4	78.6	52.1	148.8	159.3	159.1	108.3
Huarng's Method [8] Based on Distribution-Based Length Interval	80.2	60.3	110	111.7	78.6	54.2	148.0	167.3	148.7	106.5
Huarng and Yu's Method [9]	54.7	61.1	117.9	88.7	64.1	52.1	135.9	136.2	131.9	93.1
Yu's Method [18]	61	67	105	135	70	54	133	151	142	102
The Proposed Method	50.11	44.33	102.13	75.54	60.03	51.52	140.08	120.26	95.65	82.14

V. CONCLUSIONS

In this paper, we have presented a new method for forecasting the Taiwan Stock Exchange Capitalization Weighted Stock Index (TAIEX) based on high-order fuzzy logical relationships. From Table X, we can see that the proposed method gets the smallest average RMSE comparing to Chen's method [1], Chen and Wang's method [6], Huarng's method [8], Huarng and Yu's method [9], and Yu's method [18] for forecasting the TAIEX. That is, the proposed method gets a higher average forecasting accuracy rate than Chen's method [1], Chen and Wang's method [6], Huarng's method [8], Huarng and Yu's method [9] and Yu's method [18] for forecasting the TAIEX.

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