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Analysis of Networked Predictive Control Systems with Uncertainties

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Abstract — **This paper studies the robustness of networked predictive control systems (NPCS) with uncertainties. A networked predictive control strategy that compensates for delay actively rather than passively is introduced to cope with time-varying network delay and data dropout. The closed-loop networked predictive control system is described as a normal robust control system, which makes the control design and stability analysis convenient. The robustness analysis of the closed-loop networked predictive control system is discussed in details.**

Keywords — **Networked Control, predictive control, network delay, stability, robustness**

I. INTRODUCTION

The stability problem of closed-loop networked control systems (NCS) in the presence of network delays and data packet dropout has been addressed in [1]. Networked control systems under bounded uncertain access delay and packet dropout effects is formulated as discrete-time switched systems with arbitrary switching and then the stability and performance problems of the networked control systems has been reduced to the corresponding problems of switched systems [2, 3], which enables us to apply the existing theories of switched systems to networked control systems [4, 5]. To reduce network traffic load, a sampled-data NCS scheme has been presented and some conditions for global exponential stability of the closed-loop systems via state/output feedback, without/with network delays have been established in [6]. Some issues related to network bandwidth constraints and network traffic congestion in NCSs have been studied in [7, 8, 9]. Internet based control has also been considered for practical applications, for example, Internet-based process control [10], Internet based control systems as a control device [11], Internet robots [12], Internet based multimedia education [13], and process monitoring and optimization via the web [14].

The recent research of NCS mainly focuses on networked systems with some very strict assumptions on network delay (e.g., constant delay, less than one sampling period, or delay in either feedback or forward channel). Most stability conditions of closed-loop networked control systems have recently been obtained from direct applications of stability criteria of time-delay systems. They are usually only sufficient but not necessary, which are normally conservative. In fact, there are two challenging issues on networked control systems: one is how to actively compensate for time-varying network delays and overcome data dropouts, and the other is how to analyse the stability and robustness of closed-loop networked control systems with time-varying network delay and uncertainties in a less conservative way. These problems are very important in both theory and practice. This paper presents a networked predictive control strategy to compensate for time-varying network delay in both the feedback and forward communication channels and also to avoid data dropout. The robustness analysis of networked predictive control systems is given in details.

II. NETWORKED PREDICTIVE CONTROL

Based on the location of networks in a system, there are many different structures for networked control systems. For example, networks in a networked control system can be located between the sensor and controller, between the actuator and controller, and/or between the reference and controller. In this paper, the structure of networked predictive control system (NPCS) for study is shown in Fig. 1.

For the sake of simplicity, the following assumptions are made:

a) The network delay in the feedback channel (*i.e*, from the sensor to the controller) is bounded by n_b ;
b) The network delay in the forward channel (*i.e.*, from the controller to the actuator) is bounded by n_f ;
c) The number of consecutive data package drops in both the feedback and forward channels is bounded by n_d ;
d) The data transmitted through a network are with a time stamp.

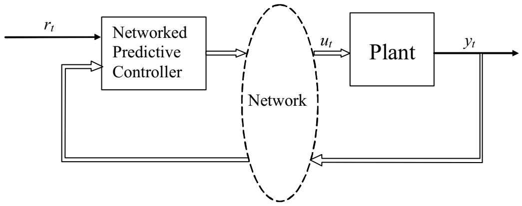

Figure 1. The networked predictive control system

In a practical NCS, there exists data loss. For instance, if the data packet does not arrive at a destination in a certain transmission time (*i.e.*, the upper bound of the network delay), it means this data packet is lost, based on commonly used network protocols. From the physical point of view, it is natural to assume that only a finite number of consecutive data dropouts can be tolerated in order to avoid the NCS becoming open-loop. The time stamp of the data transmitted through a network is very important for networked control systems. This is because a control sequence of a control system is based on time.

Although the network delay can be any real number, it is common that a computer based control system takes the information on the network delay into account once every sampling period. So, it is also assumed that the network delay is the integer multiple of the sampling period. In addition, the synchronisation is also an issue in networked control systems. There exist various ways to synchronise the time clocks in digital components (or computers). The problem of synchronization errors and their effects on feedback loops that are closed over communication networks has been studied by researchers, *e.g.*, [15]. As the paper mainly discusses the stability of networked control systems, it is assumed that the components in the system have been synchronised.

Consider the following linear discrete-time plant to be controlled:

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t \\ y_t &= Cx_t \end{aligned} \tag{1}$$

where $x_t \in \Re^n$, $y_t \in \Re^l$ and $u_t \in \Re^m$ are the state, output and input vectors of the system, respectively, and $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$ and $C \in \Re^{l\times n}$ the system matrices.

From assumptions b) and c), let $\tau = n_f + n_d$, which is the largest possible network delay in the forward channel, and k denote the time-varying network delay in the feedback channel, *i.e.,* $k \in \{0 \quad 1 \quad \cdots \quad n_b\}$. It is assumed that the states of the plant are not measurable. To obtain the state vector of the plant for the controller design on the controller side, an observer is designed as

$$\begin{aligned} \hat{x}_{t-k+1|t-k} &= A\hat{x}_{t-k|t-k-1} + Bu_{t-k} + L(y_{t-k} - \hat{y}_{t-k}) \\ \hat{y}_{t-k} &= C\hat{x}_{t-k|t-k-1} \end{aligned} \tag{2}$$

where $\hat{x}_{t-i|t-j} \in \Re^n \ (i < j)$ denotes the state prediction for time *t-i* on the basis of the information upto time *t-j* and $\hat{y}_t \in \Re^l$ is the output vector of the observer at time *t*, and the gain matrix $L \in \Re^{n\times l}$, which can be designed using standard observer design approaches.

Since there exists a network delay, the output signal *y* at time *t* is delayed for *k* steps on the controller side. Although the observer provides a one-step ahead prediction of the plant states using the output at time *t-k*, the state predictions from time *t-k+2* to *t+τ* are still not known. Based on the information available on the controller side, the other state predictions upto time *t+τ* can be constructed by

$$\hat{x}_{t-k+i|t-k} = A\hat{x}_{t-k+i-1|t-k} + Bu_{t-k+i-1} \tag{3}$$

for $i = 2, 3, ..., k+\tau$. When the states of the plant are estimated, there are many control methods available for the system. To illustrate the networked predictive control strategy, which was proposed in [18], the observer based state-feedback control method is employed. So, the control prediction to be generated on the controller side is

$$u_{t+\tau|t-k} = K\hat{x}_{t+\tau|t-k} \tag{4}$$

where $K \in \Re^{m\times n}$ is the controller gain matrix. On the actuator side, the control input will be taken as

$$u_t = u_{t|t-k-\tau} = K\hat{x}_{t|t-k-\tau} \tag{5}$$

It is clear that the network time delay can be compensated by the above control strategy. It has already been shown that the control performance of the closed-loop networked predictive control system is similar to the one without network (i.e., the closed-loop local control system) in [3]. From assumptions a) – b), it is clear that *τ* is fixed and *k* is measurable. Then all control inputs from *t-k* to *t+τ-1* are available on the controller side although some of them are not applied to the plant at time *t*. Thus, the state predictions given by (3) can be calculated, based on the available output y_{t-k} of the system.

There are several ways to deal with the data dropout in network communication protocols. For example, the lost data will be required to resend in the TCP/IP protocol. But, in real-time networked control systems, this TCP/IP mechanism will cause more network delay and is not acceptable for some control systems. For the real-time data transmission in networked control systems, the UDP/IP protocol is widely used because of the short network delay. Recently, there are three main methods to deal with the control input data dropout for real-time networked control systems. Method 1 is that if the control input data drop, the control input is set to zero [16]. Method 2 is that if the control input data drop, the control input keeps the previous control input until the new control input data arrive [17], Method 3 is that if the control input data drop, the control input uses the control prediction [18, 3]. These methods have advantages and disadvantages. Method 1 is simple but the control input causes an unsmooth switching, which may not be allowed in some control systems, and it is very difficult to provide the desired control performance. Method 2 has a smooth switching control input but it is hard to achieve the desired control performance. Method 3 provides the desired control performance but it costs a little communication efficiency. In this paper, to deal with the data dropout, the following mechanism is used. In case the output data in the feedback channel drop, the following data at time *t* are sent from the sensor side to the controller side:

$$\begin{bmatrix} y_t & y_{t-1} & \cdots & y_{t-n_d} \end{bmatrix} \tag{6}$$

Similarly, to prevent the control data dropout in the forward channel, the following control predictions at time *t* are sent from the controller side to the actuator side:

$$\begin{bmatrix} u_{t+\tau|t-k} & u_{t+\tau-1|t-k-1} & \cdots & u_{t+n_f|t-k-n_d} \end{bmatrix} \tag{7}$$

In terms of the time stamp of the transmitted data, two data buffers are needed to reorder the received data. One is for the control input data on the actuator side and the other for the output data on the controller side. So, under assumptions c) and d), the output on the controller side and the control input on the actuator side are always available for use.

It can be seen from (6) and (7) that some data (the control input and output) transmitted through network are not used for the networked control system. This will cost a little transmission efficiency, which is a disadvantage of the proposed strategy but is not a big issue because of fast communication network. On the positive side, the main issue in networked control systems, which is time-varying network delays and data dropouts, can be solved by the above networked predictive control strategy.

III. ROBUSTNESS ANALYSIS OF NPC SYSTEMS

In practice, there exist various uncertainties in control systems, for example, modelling errors, dynamics changes and external disturbances. In this section, the modelling uncertainties are taken into account. The robustness issue of the closed-loop networked predictive control system is discussed here. The plant with uncertainties is described by

$$\begin{aligned} x_{t+1} &= (A+\Delta A)x_t + (B+\Delta B)u_t \\ y_t &= (C+\Delta C)x_t \end{aligned} \tag{8}$$

where ΔA, ΔB and ΔC are the uncertainties of system matrices A, B and C, respectively.

It is clear from (2) that if the time is shifted for k steps forward, the observer can be rewritten as

$$\begin{aligned} \hat{x}_{t+1|t} &= A\hat{x}_{t|t-1} + Bu_t + L(y_t - \hat{y}_t) \\ \hat{y}_t &= C\hat{x}_{t|t-1} \end{aligned} \tag{9}$$

Define the state error be

$$e_t = x_t - \hat{x}_{t|t-1} \tag{10}$$

So, the observer can be expressed by

$$\begin{aligned} \hat{x}_{t+1|t} &= A\hat{x}_{t|t-1} + Bu_t + LCe_t + L\Delta Cx_t \\ \hat{y}_t &= C\hat{x}_{t|t-1} \end{aligned} \tag{11}$$

From the state prediction equation (3), it can be obtained that, the state predictions can be written as

$$\hat{x}_{t+\tau|t-k} = A^{k+\tau-1}\hat{x}_{t-k+1|t-k} + \sum_{i=2}^{k+\tau} A^{k+\tau-i}Bu_{t+i-k-1} \tag{12}$$

$$\begin{aligned} \hat{x}_{t+\tau|t-k+1} = A^{k+\tau-1}\hat{x}_{t-k+1|t-k} + \sum_{i=2}^{k+\tau} A^{k+\tau-i}Bu_{t+i-k-1} + \\ A^{k+\tau-2}LCe_{t-k+1} + A^{k+\tau-2}L\Delta Cx_{t-k+1} \end{aligned} \tag{13}$$

Subtracting (13) from (12) leads to the following:

$$\hat{x}_{t+\tau|t-k} = \hat{x}_{t+\tau|t-k+1} - A^{k+\tau-2}LCe_{t-k+1} - A^{k+\tau-2}L\Delta Cx_{t-k+1} \tag{14}$$

The above equation is recursively employed. Thus, it can be derived that

$$\hat{x}_{t+\tau|t-k} = \hat{x}_{t+\tau|t-1} - \sum_{i=0}^{k+\tau-2} A^i L(Ce_{t+\tau-i-1} + \Delta Cx_{t+\tau-i-1}) \tag{15}$$

Replacing $t+\tau$ by t yields

$$\hat{x}_{t|t-k-\tau} = \hat{x}_{t|t-1} - \sum_{i=0}^{k+\tau-2} A^i L(Ce_{t-i-1} + \Delta Cx_{t-i-1}) \tag{16}$$

Then, the control input (5) can be expressed by

$$\begin{aligned} u_t &= Kx_{t|t-k-\tau} \\ &= K\left(\hat{x}_{t|t-1} - \sum_{i=0}^{k+\tau-2} A^i L\left(Ce_{t-i-1} + \Delta Cx_{t-i-1}\right)\right) \\ &= K\left(x_t - e_t - \sum_{i=0}^{k+\tau-2} A^i L\left(Ce_{t-i-1} + \Delta Cx_{t-i-1}\right)\right) \end{aligned} \tag{17}$$

Combing plant (8), observer (11) and controller (17), the error between the plant and observer states can be written as

$$\begin{aligned} e_{t+1} &= x_{t+1} - \hat{x}_{t+1} \\ &= (A-LC)e_t + \Delta Ax_t + \Delta BKu_t - L\Delta Cx_t \\ &= (A-LC)e_t + (\Delta A + \Delta BK - L\Delta C)x_t \\ &\quad - \Delta BK\left(e_t + \sum_{i=0}^{k+\tau-2} A^i L\left(Ce_{t-i-1} + \Delta Cx_{t-i-1}\right)\right) \end{aligned} \tag{18}$$

Using controller (17), the plant state can be re-written as

$$\begin{aligned} x_{t+1} &= (A+\Delta A)x_t + (B+\Delta B)K\left(x_t - e_t - \sum_{i=0}^{k+\tau-2} A^i L(Ce_{t-i-1} + \Delta Cx_{t-i-1})\right) \\ &= -(B+\Delta B)K\left(e_t + \sum_{i=0}^{k+\tau-2} A^i L(Ce_{t-i-1} + \Delta Cx_{t-i-1})\right) \\ &\quad (A + BK + \Delta A + \Delta BK)x_t \\ &= (A+BK)x_t + (\Delta A + \Delta BK)x_t - BK\left(e_t + \sum_{i=0}^{k+\tau-2} A^i LCe_{t-i-1}\right) \\ &\quad - \Delta BK\left(e_t + \sum_{i=0}^{k+\tau-2} A^i LCe_{t-i-1}\right) - (B+\Delta B)K\left(\sum_{i=0}^{k+\tau-2} A^i L\Delta Cx_{t-i-1}\right) \end{aligned} \tag{19}$$

Combing (18) and (19) gives the following closed-loop equation:

$$\begin{bmatrix} X_{t+1} \\ E_{t+1} \end{bmatrix} = \left(\begin{bmatrix} H_{11} & H_{12} \\ 0 & H_{22} \end{bmatrix} + \begin{bmatrix} \Delta H_{11} & \Delta H_{12} \\ \Delta H_{21} & \Delta H_{22} \end{bmatrix}\right)\begin{bmatrix} X_t \\ E_t \end{bmatrix} \tag{20}$$

where

$$X_t = \begin{bmatrix} x_t^T & x_{t-1}^T & \cdots & x_{t-\tau-n_b+1}^T \end{bmatrix}^T$$

$$E_t = \begin{bmatrix} e_t^T & e_{t-1}^T & \cdots & e_{t-\tau-n_b+1}^T \end{bmatrix}^T$$

$$H_{11} = \begin{bmatrix} A+BK & & & \\ I & 0 & & \\ & \ddots & \ddots & \\ & & I & 0 \end{bmatrix} \qquad H_{22} = \begin{bmatrix} A-LC & & & \\ I & 0 & & \\ & \ddots & \ddots & \\ & & I & 0 \end{bmatrix}$$

$$H_{12} = \begin{bmatrix} -BK & -BKLC & -BKALC & \cdots & -BKA^{k+\tau-2}LC & 0 & \cdots \\ & & & \mathbf{0} & & & \end{bmatrix}$$

$$\Delta H_{11} = \begin{bmatrix} \Delta A+\Delta BK & -(B+\Delta B)KL\Delta C & -(B+\Delta B)KAL\Delta C & \cdots & -(B+\Delta B)KA^{k+\tau-2}L\Delta C & 0 & \cdots \\ & & & \mathbf{0} & & & \end{bmatrix}$$

$$\Delta H_{22} = \begin{bmatrix} -\Delta BK & -\Delta BKLC & -\Delta BKALC & \cdots & -\Delta BKA^{k+\tau-2}LC & 0 & \cdots \\ & & & \mathbf{0} & & & \end{bmatrix}$$

$$\Delta H_{21} = \begin{bmatrix} \Delta A+\Delta BK-L\Delta C & \Delta BKL\Delta C & \Delta BKAL\Delta C & \cdots & \Delta BKA^{k+\tau-2}L\Delta C & 0 & \cdots \\ & & & \mathbf{0} & & & \end{bmatrix}$$

$$\Delta H_{12} = \Delta H_{22}$$

Therefore, the stability of the closed-loop NPC system with uncertainties can be treated as the robustness problem of the following standard system:

$$Z_{t+1} = (H+\Delta H)Z_t \tag{21}$$

where

$$Z_t = \begin{bmatrix} X_{t+1} \\ E_{t+1} \end{bmatrix}, \quad H = \begin{bmatrix} H_{11} & H_{12} \\ 0 & H_{22} \end{bmatrix}, \quad \Delta H = \begin{bmatrix} \Delta H_{11} & \Delta H_{12} \\ \Delta H_{21} & \Delta H_{22} \end{bmatrix}$$

The robustness analysis on the above system can follow the results given by others, *e.g.*, [19]. Since the uncertainty *ΔH* is related to the controller gain *K* and observer gain *L,* which is different from the normal robust control problem, the design of the gains *K* and *L* should also make sure that the conditions on the uncertainty *ΔH* will not be violated. This means that this robust control problem will normally be solved in a numerical way because it is very difficult to have an analytical solution for K and L for most networked predictive control systems.

In addition, if there are no uncertainties, i.e., *ΔA=0, ΔB=0* and *ΔC=0*, the stability of the closed-loop networked predictive control system only depends on whether the eigenvalues of matrices *(A+BK)* and *(A-LC)* are within the unit circle.

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IV. SIMULATED EXAMPLE

To illustrate the stability and robustness of networked control systems, a servo control system is considered [20]. For the sampling period *0.04*s, the discrete-time model of the servo system is described by

$$A = \begin{bmatrix} 1.120 & 0.213 & -0.335 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0.0541 & 0.1150 & 0.0001 \end{bmatrix}$$

Let the desired poles of the closed-loop state feedback control system be [*-0.4, 0.7+0.6i, 0.7-0.6i*] and the desired poles of the observer be [*0.1, 0.3, 0.5*]. Using the pole assignment method, the control gain and observer gain are designed to be

$$K = \begin{bmatrix} \text{-0.1200} & \text{-0.5030} & \text{-0.0050} \end{bmatrix}$$

$$L = \begin{bmatrix} 4.9113 \\ \text{-0.4055} \\ 9.2845 \end{bmatrix}$$

The initial conditions of the system states and the observer states were set to be [*5, 5, -5*] and [*0, 0, 0*], respectively. Three cases are considered below.

Case 1: Networked control without network delay compensation

In this case, the network delay in the communication channels is not compensated, that is, the networked predictive control strategy is not employed but a normal feedback control is used. It is also assumed that there exists one-step network delay in the forward channel and no delay in the feedback channel, and there is also no uncertainty in the plant. So, the controller is given by $u_t = K\hat{x}_{t-1|t-2}$. The simulation results in Figure 2 show that the system is unstable.

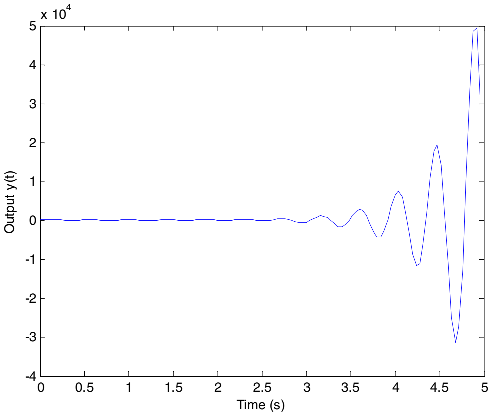

Figure 2. Networked control without compensating for network delay

Case 2: Networked predictive control without uncertainty

The networked predictive control strategy is used to compensate for the network delay. It is still assumed that there is no uncertainty in the plant. The network delay in the forward channel is *τ=4* (i.e., *4* sampling steps), where $n_f=3$ and $n_d=1$, and the time-varying network delay in the feedback channel is *k∈{0, 1, 2, 3}*, where $n_b=2$ and $n_d=1$. The simulation results given in Figure 3, where for the sake of comparison the output curve of the networked predictive control system is shifted for 7 sampling steps backward (7 is the maximum network delay in the system, which is the worst one), demonstrate that the closed-loop system is stable and the performance of the closed-loop networked predictive control system is the same as one of the local closed-loop control system (i.e., there is no network in the closed-loop system) except the first several steps. Actually, when the network delay increases, the performance and stability of the closed-loop networked predictive control do not change.

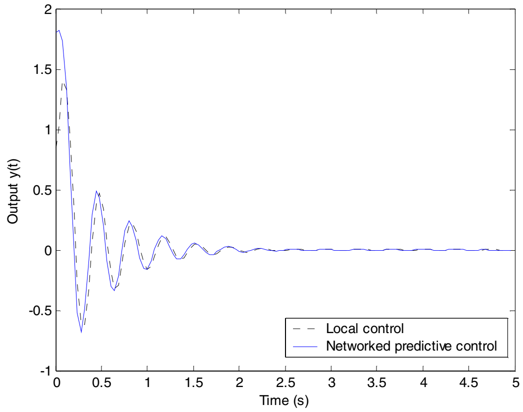

Figure 3. Networked predictive control of systems without uncertainty

Case 3: Networked predictive control with uncertainty

This case is similar to Case 2. But the uncertainties are introduced in the plant. The system matrices *A, B* and *C* are perturbed to be *1.05A, 1.05B* and *0.95C*, respectively. This means that ±5% uncertainty is added to the system matrices. The simulation results as shown in Figure 4, where the output curve of the networked predictive control system is shifted for 7 sampling steps backward, illustrate that the networked predictive control has good robustness.

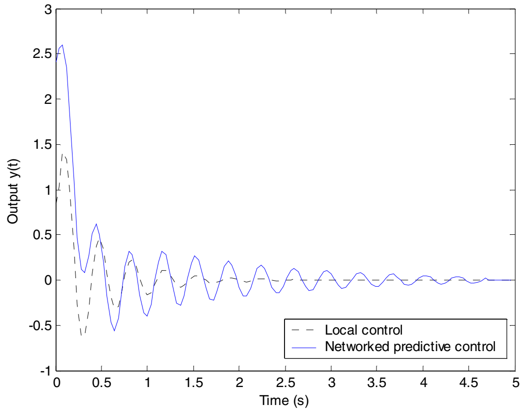

Figure 4. Networked predictive control of systems with uncertainty

V. CONCLUSIONS

This paper has analysed the robustness of closed-loop networked predictive control systems with uncertainties, variable network delay and data dropout. After the introduction to the proposed networked predictive control strategy, a compact form of describing the closed-loop NPC system was obtained. The robustness analysis shows that if there are no uncertainties in plants to be controlled the network delay is actively compensated and the control performance of NPC systems is not affected by network delay. In the case of uncertainties, the stability of NPC systems has been converted to a standard robust control problem.

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