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Stabilization of network controlled system with multiple-packet transmission

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***Abstract*—This paper considers stabilization of network controlled systems (NCSs) with multiple-packet transmission. For NCSs node devices acting over a limited communication channel, we are particularly interested in the case that only one packet containing part of the state information can be transmitted at a time. Even if a state feedback is intended for the original system, control such a system is an output feedback problem. For NCSs transmitted in a periodic manner, sufficient conditions on stability and stabilization of the NCSs are presented. For NCSs transmitted in a stochastic manner, sufficient conditions on the mean square stabilization of NCSs are developed. Finally, numerical examples are given to show that the unstable system can be stabilized with part of state information transmitted over the network channel.**

***Index Terms*—Keywords goes here.**

I. INTRODUCTION

Due to the drastically reduced cost for cabling, installation, and maintenance to use network cables in control systems, recently, networked control systems (NCSs) have gained great attention from the control community and the network and communication community [12], [15]. NCSs are defined as the feedback control systems whose feedback control loops are closed through a real-time network. In the traditional control theory, the standard assumption on the state feedback control is that all the state information can be obtained by the controller. When an NCS is designed, a new constraint must be accommodated-the limited bandwidth of the communication network.

Control a plant via a limited communication channel has received much attention, see, e.g., [2]–[7]. This research is motivated by the fact that in many practical control systems, the communication among the devices on the network is through one limited communication channel. The problem of stabilization with finite communication bandwidth was introduced by [18], [19] and further pursued by [5], [7]. Ref. [9] proposed a dwell-time switching method to reduce the data rate of the network. Moreover, [8], [10] introduced signal quantization into the design of the systems, thus reduced the data rate. Ref. [14] made use of a model of the plant to reduce the network usage. A new framework for distributed control systems was introduced by [21], which used estimators at each node to achieve a significant savings in the required bandwidth. However, the research mentioned above is concerned primarily with analysis issues rather than control design. In [22], Yu et al considered stabilization of NCSs with node devices acting over a limited communication channel, which assumed zero order hold at the Register side: when a sensor fails to access the medium the value stored in a ZOH would be fed to the controller. In this work, we forgo the use of ZOH and let zero be fed into the controller when a sensor fails to access the medium. This setup will lead to a simpler model which enables one to design a stabilizing feedback gain.

We assume that the plant measurement is transmitted through a shared network channel with bandwidth constraint, and we are particularly interested in the case that the plant measurement is split into different packets and there is only one packet can be transmitted once a time. In order to reduce the complexity, we also assume that there is no network between the controller outputs and the plant inputs, and there are no transmission delays occur over the communication channel. First, we consider the packet transmitted in a periodic manner. Then, we consider the case that data packet transmitted in a stochastic manner.

For NCSs transmitted in a periodic manner, this paper is trying to design stabilizing feedback controllers which make the NCS stable in the Lyapunov sense. Inspired by the recent work [16], by using the well-known elimination lemma [17], we get further result of the periodic switched system. Sufficient conditions on the stabilization of the NCS, which is modelled into a periodic switched system in this paper, are derived. For NCSs transmitted in a stochastic manner, mean square stability conditions are developed with the feedback gain being obtained in terms of a group of LMIs.

The remainder of this paper is structured as follows. Section 2 models an NCS with multi-packet transmission. Section 3 presents stabilization and stability results for the state feedback and output feedback case. Section 4 develops mean square stability result for NCSs transmitted in a Markov chain. Section 5 presents a numerical simulation to illustrate the efficiency and feasibility of our proposed approach. Section 5 concludes this paper.

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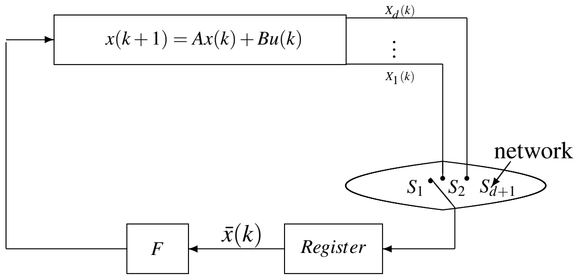

Fig. 1. An NCS controlled via state feedback

II. Modelling of NCSs with multiple packet transmission

In distributed NCSs, due to the wide location of sensors whose information length may surpass that of the network packet, multiple-packet transmission is necessary. In the multiple-packet transmission, plant/controller output is split into separate packets to be transmitted over different network channels and they may not arrive at the destination simultaneously. We consider the case that all the nodes act over a limited bandwidth communication channel. The NCS model of multi-packet transmission with state feedback is depicted in Fig. 1. The discrete-time, state-space model of the plant can be described as following:

$$\begin{array}{l} x(k+1) = Ax(k) + Bu(k), \\ u(k) = F(k)\bar{x}(k), \ \ k = 1, 2, \cdots, \end{array} \tag{1}$$

where $x(k) \in \mathbf{R}^n$, $u(k) \in \mathbf{R}^p$ are the plant state and the plant input, respectively. $\bar{x}(k)$ is the content of the register. $F(k)$ is the feedback gain to be designed.

Suppose the state is split into d packets $x(k) = \left[X_1^T(k), \cdots, X_d^T(k)\right]^T$, where $X_i(k) = \left[x_{r_{i-1}+1}(k) \ \cdots \ x_{r_i}(k)\right]^T$, $1 \le i \le d$, $0 = r_0 < r_1 < \cdots < r_d = n$.

$$\bar{X}_i(k) = \begin{cases} X_i(k) & \text{if the packet } i \text{ is transmitted} \\ 0 & \text{otherwise} \end{cases}$$

$F(k) = F_i$ if packet i is transmitted. Then

$$x(k+1) = \Lambda_i x(k), \tag{2}$$

for $i = 1, \cdots, d$, where

$$\Lambda_i = A + BF_iC_i, \tag{3}$$

with

$$C_i = \begin{bmatrix} 0 & & & & \\ & \ddots & & 0 & \\ & & I_i & & \\ & 0 & & \ddots & \\ & & & & 0 \end{bmatrix} \rightarrow \text{the } i\text{th block}.$$

Now, NCS (1) in multi-packet transmission is modelled as system (2), whose stability guarantees that of the original system.

III. Data packet transmitted in a periodic order

A. State feedback stabilization

We consider the case that all nodes transmitted in a toking-bus. With the toking bus protocol applied, the nodes are arranged logically into a ring and transmit their packets in a predetermined order. Notice that these equations are in the form of output feedback control, even if a state feedback control law is intended for the original NCS (1). It can be easily seen that system (2) is a switched linear system switching among the following subsystems

$$\{\Lambda_1, \cdots, \Lambda_d\},$$

in a periodic manner.

The following result gives a sufficient condition on the stability of NCS (1) with packets in different network channels transmitted in a periodic manner.

Theorem 1: Let the states of NCS (1) be split into multiple data packets and the transmission of these data packets be in a periodic manner. Then NCS (1) is uniformly asymptotically stable if all the eigenvalues of Ψ are contained within the unit circle, i.e., $|\lambda_i(\Psi)| < 1$ for $i = 1, 2, \cdots, n$, where $\Psi = \Lambda_d\Lambda_{d-1}\cdots\Lambda_1$.

Proof: Because of the relationship between system (1) and switched system (2), we only need to prove that system (2) is stable. The step response of system (2) is given by

$$w(k) = (\Lambda_s\Lambda_{s-1}\cdots\Lambda_1)\Psi^m w(1) \tag{4}$$

$$\text{for} \ \ k = dm+s, \ s = 1, \cdots, d, \ \ m = 1, 2, \cdots.$$

Therefore, $||w(k)|| \rightarrow 0$ iff each eigenvalue of Ψ lies within the unit circle. ■

The following lemma will play a key rule to design the feedback gain for NCS (1).

Lemma 1: [17] *Let the matrices U, W, and $\Phi = \Phi^*$ be given. Then the following statements are equivalent:*
(i) There exists a matrix V satisfying

$$UVW + (UVW)^* + \Phi < 0.$$

(ii) The following two conditions hold

$$N_u \Phi N_u^* < 0 \ \ \text{or} \ \ UU^* > 0,$$

$$N_w^* \Phi N_w < 0 \ \ \text{or} \ \ W^*W > 0,$$

where N_u and N_w^* are respectively orthogonal complement of N and W^*; that is

$$N_u U = 0, \quad N_w^* W^* = 0.$$

Here, we are in a position to give the stabilization result.

Theorem 2: If there exist a positive definite matrix P and matrices M, Y satisfying

$$PB = BM, \tag{5}$$

and the following LMI

$$\begin{bmatrix} -P & (PA+BY_1C_1)^T & 0 & \cdots & 0 & 0 \\ PA+BY_1C_1 & -2P & (PA+BY_2C_2)^T & \cdots & 0 & 0 \\ 0 & PA+BY_2C_2 & -2P & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -2P & (PA+BY_dC_d)^T \\ 0 & 0 & 0 & \cdots & PA+BY_dC_d & -P \end{bmatrix} < 0, \tag{6}$$

then NCS (1) can be asymptotically stabilized via the state feedback

$$u(k) = M^{-1}Y_i\bar{x}(k). \tag{7}$$

Proof: From Theorem 1, the stabilization problem is to compute the feedback gain F_i such that Ψ is Schur-stable. In the so-called Lyapunov framework, the Schur-stability of matrix Ψ can be guaranteed by the existence of a symmetric positive definite matrix P such that the following inequality holds:

$$-P + \Psi^T P \Psi < 0. \tag{8}$$

Condition (8) can be written as

$$\begin{bmatrix} I & \Lambda_1^T \end{bmatrix} \begin{bmatrix} -P & 0 \\ 0 & \Pi_{d-1}^T P \Pi_{d-1} \end{bmatrix} \begin{bmatrix} I \\ \Lambda_1 \end{bmatrix} < 0, \tag{9}$$

where

$$\Pi_{d-1} = \Lambda_d \Lambda_{d-1} \cdots \Lambda_2.$$

We define $N_u = \begin{bmatrix} I & \Lambda_1^T \end{bmatrix}$, $V = P$, and $W = \begin{bmatrix} 0 & I \end{bmatrix}$. By using Lemma 2, (9) is equivalent to the existence of symmetric positive definite matrix P such that the following inequality holds:

$$\begin{bmatrix} -P & 0 \\ 0 & \Pi_{d-1}^T P \Pi_{d-1} \end{bmatrix} + \begin{bmatrix} \Lambda_1^T \\ -I \end{bmatrix} P \begin{bmatrix} 0 & I \end{bmatrix} + \begin{bmatrix} 0 \\ I \end{bmatrix} P \begin{bmatrix} \Lambda_1 & -I \end{bmatrix} < 0.$$

Rearranging it, we obtain

$$\begin{bmatrix} -P & \Lambda_1^T P \\ P\Lambda_1 & -2P + \Pi_{d-1}^T P \Pi_{d-1} \end{bmatrix} < 0,$$

which can be written as

$$\begin{bmatrix} I & 0 & 0 \\ 0 & I & \Lambda_2^T \end{bmatrix} \begin{bmatrix} -P & \Lambda_1^T P & 0 \\ P\Lambda_1 & -2P & 0 \\ 0 & 0 & \Pi_{d-2}^T P \Pi_{d-2} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ 0 & \Lambda_2 \end{bmatrix} < 0,$$

where

$$\Pi_{d-2} = \Lambda_d \Lambda_{d-1} \cdots \Lambda_3.$$

Repeating this procedure, we can show that (9) can be guaranteed by the following inequality,

$$\begin{bmatrix} -P & \Lambda_1^T P & 0 & \cdots & 0 & 0 \\ P\Lambda_1 & -2P & \Lambda_2^T P & \cdots & 0 & 0 \\ 0 & P\Lambda_2 & -2P & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -2P & \Lambda_d^T P \\ 0 & 0 & 0 & \cdots & P\Lambda_d & -P \end{bmatrix} < 0. \tag{10}$$

Let $Y_i = MF_i$. Using the definition of Λ_i $(i = 1, \cdots, d)$, together with (5) and (3), we have that (10) is equivalent to (6). If this LMI is feasible, the explicit expression of the desired feedback gain is given by $F_i = M^{-1}Y_i$. ■

Above result shows that stabilization of NCS (1) can be achieved provided all the data packets can be transmitted in a rate $\frac{1}{d}$.

B. *Observer-based dynamic output feedback stabilization*

Now, we consider a model with output feedback controller as depicted in Fig. 2. The model is composed of discrete-time plant

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k), \\ y(k) &= Cx(k), \end{aligned} \tag{11}$$

controlled via observer-based dynamic output feedback

$$\begin{aligned} \hat{x}(k+1) &= (A - LC)\hat{x}(k) + Bu(k) + L\bar{y}(k), \\ u(k) &= F\hat{x}(k), \end{aligned} \tag{12}$$

where $y(k) \in \mathbf{R}^m$ is the plant output, $\bar{y}(k)$ is the content of the register. We assume that the output is split into d packets, that is, $y(k) = \begin{bmatrix} Y_1^T(k), \cdots, Y_d^T(k) \end{bmatrix}^T$, where $Y_i(k) = \begin{bmatrix} y_{s_{i-1}+1}(k) & \cdots & y_{s_i}(k) \end{bmatrix}^T$, $1 \le i \le d$, $0 = s_0 < s_1 < \cdots < s_d = m$. The content of the register is

$$\bar{y}(k) = \begin{bmatrix} \bar{Y}_1^T(k), \cdots, \bar{Y}_d^T(k) \end{bmatrix}^T,$$

where

$$\bar{Y}_i(k) = \begin{cases} Y_i(k), & \text{if packet } i \text{ is transmitted;} \\ 0, & \text{otherwise.} \end{cases}$$

Suppose all the packets are transmitted in a periodic manner, Y_i is transmitted after Y_{i-1} and $Y_1(0)$ is transmitted first.

Introducing the observer error

$$e(k) = x(k) - \hat{x}(k),$$

and letting

$$z(k) = [x^T(k), \; e^T(k)]^T,$$

we can easily obtain that

$$z(k) = \Gamma_i z(k-1), \tag{13}$$

for $i = 1, \cdots, d$, $n = 0, 1, \cdots$, where

$$\Gamma_i = \begin{bmatrix} A + BF & -BF \\ L(I - C_i)C & A - LC \end{bmatrix},$$

with

$$C_i = \begin{bmatrix} 0 & & & & 0 \\ & \ddots & & & \\ & & I_i & & \\ & & & \ddots & \\ 0 & & & & 0 \end{bmatrix} \rightarrow \text{the } i\text{th block.}$$

From the above discussion we know that system (13) is a switched system switching among subsystems

$$\{\Gamma_i\}_{i=1,\cdots,d}$$

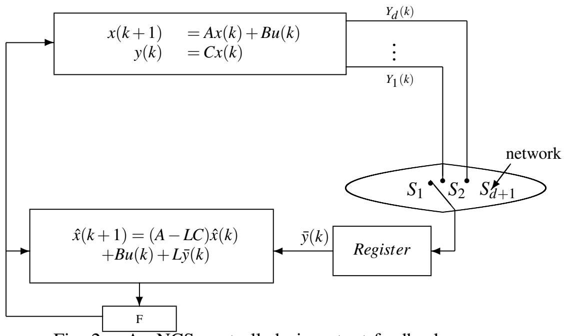

Fig. 2. An NCS controlled via output feedback

periodically.

It is clear that the stability of the system (11) can be obtained from that of system (13).

Likewise, Theorem 2 can also be extended to the output feedback form.

Theorem 3: If there exist matrices P_1, P_2, Y_1, Y_2 and a matrix M satisfying

$$P_1B = BM,$$

and the following LMI

$$\begin{bmatrix} -P & \Upsilon_1^T & 0 & \cdots & 0 & 0 \\ \Upsilon_1 & -2P & \Upsilon_2^T & \cdots & 0 & 0 \\ 0 & \Upsilon_2 & -2P & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -2P & \Upsilon_d^T \\ 0 & 0 & 0 & \cdots & \Upsilon_d & -P \end{bmatrix} < 0,$$

where

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, \Upsilon_i = \begin{bmatrix} P_1A + BY_1 & -BY_1 \\ Y_2(I - C_i)C & P_2A - Y_2C \end{bmatrix},$$

then system (11) can be asymptotically stabilized via dynamic output feedback controller (12) with

$$F = M^{-1}Y_1, \ L = P_2^{-1}Y_2.$$

Proof: The proof is similar to that of state feedback case and is thus omitted. ■

IV. Data packet transmitted in a stochastic order

Since packet reception at each time instance is often modelled as a stochastic process that is Bernoulli or a two-state Markov chain [4], we are particularly interested in the case that the data packet transmission is governed by a Markov chain in this section.

We give the definition of mean square for discrete-time system (2).

Definition 1: [11] System (2) is mean square stable if for every initial state (x_0, θ_0), $lim_{k\to\infty}E[\|x(k)\|^2] = 0$.

Below, we present necessary and sufficient matrix inequality conditions on mean square stable of system (2), which has been proved in [3].

Lemma 2: The system (2) is mean square stable iff there exist matrices $Q_i > 0$ for $i = 0, 1, \cdots, N$ satisfying any one of the following conditions:

$$1. \ A_m^T(\sum_{n=1}^{d} p_{mn}Q_n)A_m < Q_m, \ m = 1, \cdots, d; \tag{14}$$

$$2. \ \sum_{n=1}^{d} p_{mn}A_n^T Q_n A_n < Q_m, \ m = 1, \cdots, d. \tag{15}$$

We look for switched state feedback gain F_i such that the NCS (2) is mean square stable for data packet transmission governed by a Markov chain. Sufficient conditions are stated in the following theorem.

Theorem 4: The NCS (2) with data packet dropout can be stabilized in the MS sense if there exist matrices $Q_i > 0$ and matrices M, G, L_i for $i = 1, \cdots, d$ satisfying

$$GB = BM \tag{16}$$

and one of the following conditions

$$1. \ \begin{bmatrix} Q_m & \sqrt{p_{m1}}W_m^T & \sqrt{p_{m2}}W_m^T & \cdots & \sqrt{p_{md}}W_m^T \\ * & G+G^T-Q_1 & 0 & \cdots & 0 \\ * & * & G+G^T-Q_2 & \cdots & 0 \\ & & & \ddots & \\ * & * & * & & 0 \\ * & * & * & * & G+G^T-Q_d \end{bmatrix} > 0, \tag{17}$$

$m = 1, \cdots, d$, where $W_m = GA + BL_mC_m$;

$$2. \ \begin{bmatrix} Q_m & \sqrt{p_{m1}}W_1^T & \sqrt{p_{m2}}W_2^T & \cdots & \sqrt{p_{md}}W_m^T \\ * & G+G^T-Q_1 & 0 & \cdots & 0 \\ * & * & G+G^T-Q_2 & \cdots & 0 \\ & & & \ddots & \\ * & * & * & & 0 \\ * & * & * & * & G+G^T-Q_d \end{bmatrix} > 0, \tag{18}$$

$m = 1, \cdots, d$, where $W_m = GA + BL_mC_m$.
Furthermore, the state feedback gain is given by

$$F_m = M^{-1}L_m.$$

Proof: Let $L_m = MF_m$, we can deduce from (16) and (17) that

$$\begin{bmatrix} Q_m & \sqrt{p_{m1}}[G(A+BF_mC_m)]^T & \sqrt{p_{m2}}[G(A+BF_mC_m)]^T & \cdots \\ * & G+G^T-Q_1 & 0 & \cdots \\ * & * & G+G^T-Q_2 & \cdots \\ & & & \ddots \\ * & * & * & \\ * & * & * & * \end{bmatrix} \tag{19}$$

$$\begin{bmatrix} \sqrt{p_{md}}[G(A+BF_mC_m)]^T \\ 0 \\ 0 \\ 0 \\ G+G^T-Q_d \end{bmatrix} > 0,$$

Multiplying $[I \ \ -\sqrt{p_{m1}}(A+BF_mC_m)^T \ \ \cdots \ \ -\sqrt{p_{md}}(A+BF_mC_m)^T]$ and its transpose on the left and on the right side of (19), respectively, we know that (19) is equivalent to (14) with A_m replaced by $A+BF_mC_m$. Notice that the regularity of G is implied by the design $G^T + G > Q_i > 0$, which implies the nonsingular of the matrix M. The rest one can be deduced by analogy. ■

Remark 1: The introduction of the freedom matrix G reduces the conservative of the stability condition due to the presence of the extra degree of freedom. The existence of the controller depends on the feasibility of the LMIs in Theorem 4, which is affected by the probability transition matrix $P = [p_{mn}]$.

In this way, for the NCSs with multiple-packet transmission governed by a Markov chain, sufficient mean square stabilizing conditions have been developed and the state feedback controllers can be constructed in terms of LMIs.

V. Simulation Examples

In this section, Three numerical examples are given to demonstrate the effectiveness of the proposed method.

Example 1:

First, we consider the case that the state of the system is split into two parts and transmitted periodically.

$$x(k+1) = Ax(k) + Bu(k) \tag{20}$$

where

$$A = \begin{bmatrix} 1.4 & 0.1; \\ 0 & 0.3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

We will show that the unstable NCS can be stabilized with only half of the state information transmitted. From the discussion in Section 3.1, it can be shown that when the sensor data transmitted periodically, the NCSs switches between

$$x(k+1) = \Lambda_1 x(k) \text{ and } x(k+1) = \Lambda_2 x(k)$$

periodically, where

$$\Lambda_1 = A + BF_1C_1, \; \Lambda_2 = A + BF_2C_2,$$

with

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Then by solving the LMIs in Theorem 2 with LMI toolbox [1], we have

$$Y_1 = \begin{bmatrix} -0.7044 & 0 \end{bmatrix}, \; Y_2 = \begin{bmatrix} 0 & -0.0744 \end{bmatrix}, \; M = 0.5029,$$

$$P = \begin{bmatrix} 0.5029 & -0.0076 \\ -0.0076 & 0.708116.4898 \end{bmatrix}.$$

By Theorem 2, we can obtain the controllers as follows:

$$F_1 = M^{-1}Y_1 = [-1.4007 \;\; 0], \; F_2 = M^{-1}Y_2 = [0 \;\; -0.1480].$$

With the initial condition $x(0) = [2 \;\; -1]^T$, the step response of NCS (20) with multiple packet transmission is shown in Fig. 3. The unstable system can be effectively stabilized with the state of the system transmitted periodically.

Example 2:

Then, we consider output feedback stabilization of the following NCS with the output of the system split into two parts and transmitted periodically.

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) \end{aligned} \tag{21}$$

where

$$A = \begin{bmatrix} 0.3 & 0.1 \\ 0 & 1.1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \; C = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}.$$

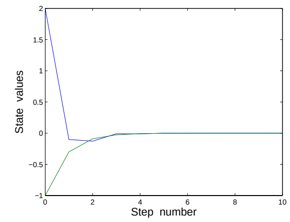

Fig. 3. The step response of NCS (20)

From the discussion in Section 3.2, it can be shown that when the sensor data transmitted periodically, the NCSs switches between

$$z(k+1) = \Gamma_1 z(k) \text{ and } z(k+1) = \Gamma_2 z(k)$$

periodically, where

$$\Lambda_1 = \begin{bmatrix} A+BF & -BF \\ L(I-C_1)C & A-LC \end{bmatrix}, \; \Lambda_2 = \begin{bmatrix} A+BF & -BF \\ L(I-C_2)C & A-LC \end{bmatrix}$$

with

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Then by solving the LMIs in Theorem 4 with LMI toolbox [1], we have

$$Y_1 = \begin{bmatrix} -0.0962 & -0.3195 \end{bmatrix}, \; Y_2 = \begin{bmatrix} 0.0048 & 0 \\ 0.2006 & 0 \end{bmatrix}, \; M = 0.5731,$$

$$P_1 = \begin{bmatrix} 0.7520 & -0.0263 \\ -0.0263 & 0.5731 \end{bmatrix}, \; P_2 = \begin{bmatrix} 0.7703 & -0.0715 \\ -0.0715 & 0.6078 \end{bmatrix}.$$

By Theorem 4, we can obtain the controllers as follows:

$$F = M^{-1}Y_1 = [-0.1678 \;\; -0.5575], \; L = M^{-1}Y_2 = \begin{bmatrix} 0.0373 & 0 \\ 0.3344 & 0 \end{bmatrix}.$$

For $x(0) = [2 \;\; -2]^T$, $e(0) = [1 \;\; -1]$, the step response and the error response of NCS (21) are shown in Fig. 4 and 5.

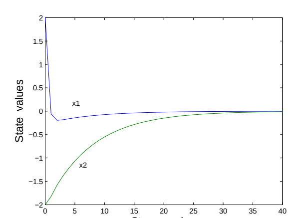

Fig. 4. The step response of NCS (21)

Example 3

Consider NCS (20) with state of the system split into two packets and transmitted in a Markov chain in a finite set $\{1, 2\}$.

The "expected" transition probability matrix of the quantity of data packet dropouts is given as follows

$$P = \begin{bmatrix} 0.5 & 0.5 \\ 0.3 & 0.7 \end{bmatrix}. \tag{22}$$

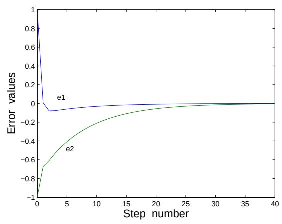

Fig. 5. The error response of NCS (21)

By solving the LMI problem (17) using LMI toolbox [1], we obtain

$$Q_1 = \begin{bmatrix} 5.2292 & -1.1323 \\ -1.1323 & 4.5670 \end{bmatrix}, \; Q_2 = \begin{bmatrix} 5.1041 & -0.6159 \\ -0.6159 & 1.4938 \end{bmatrix},$$

$$M = 3.7818, \; L_1 = [-0.88600], \; L_2 = [0 - 4.0545],$$

$$G = \begin{bmatrix} 5.0505 & -0.6619 \\ -0.6619 & 3.7818 \end{bmatrix}.$$

Then, by Theorem 1, we have $F_1 = [-0.2343 \;\; 0], F_2 = [0 - 1.0721]$.

With the initial condition $x(0) = [1 \;\; -1]^T$, the step response of NCS (20) with state of the system (20) transmitted in a Markov chain in a finite set is shown in Fig. 6, from which we can see that the unstable system can be effectively stabilized with the designed feedback controller.

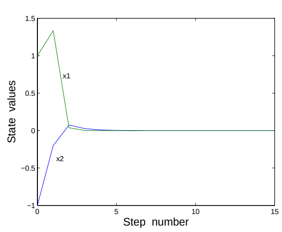

Fig. 6. The step response of NCS (20)

CONCLUSION

In this paper, we dealt with stability and stabilization of an NCS with multi-packet transmitted over a shared channel. For NCSs acted over a toking-bus, we modelled an NCS with multi-packet transmitted in a periodic manner as a periodically switched system, whose stability guaranteed that of the original system. For both of state feedback and observer-based output feedback case, sufficient conditions on stability and stabilization of the NCS were derived. For NCSs with packet transmitted in a Markovian manner, sufficient condition on the mean square stabilization was also obtained. The explicit express of the controllers were given in terms of LMIs. Finally, we gave numerical examples to illustrate the feasibility and effectiveness of our approach. The results obtained here suggest that data packet can be transmitted sparsely to save network bandwidth while preserving the stability of the NCS. This is of practical interest in the application of NCSs.

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