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Single-Frequency GPS Receivers Ionospheric Time-Delay Approximation using Radial Basis Function Neural Network

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***Abstract* – Ionospheric errors are the most influential source of the GPS positioning errors. In an attempt to at least partially improve the GPS positioning process, a global ionospheric correction model is introduced within the standard GPS positioning service. Referred to as the Klobuchar model after its inventor, this model appeared as a compromise between computation complexity and correction accuracy. Neural Networks (NNs) are ideal tool for the approximation of single-frequency GPS receivers ionospheric time-delay behaviour, which has nature highly non-linear. A major advantage of using NNs for the approximation of single-frequency GPS receivers ionospheric time-delay behaviour over analytical methods is that no previous knowledge of the nature of the non-linear relationships is required. This paper presents the use of NN modeling to approximate single-frequency GPS receivers ionospheric time-delay to reduce GPS signal propagation error. The ionospheric time-delay is approximated utilizing the Radial Basis Function (RBF) NN approach. The NN-based approach reduces the computational burden respect to Klobuchar model. The method employed here is applicable on the L1 GPS receivers.**

***Keywords* — L1 GPS receivers, Ionospheric time-delay, Modeling, RBF Neural network**

I. INTRODUCTION

The Global Positioning System (GPS) is the navigation system based on a network of 24 satellites in 6 inclined orbits transmitting coded information continuously. At least 4 satellites send their individual information to the receiver on the earth to precisely indentify the position of the receiver. The GPS has become a global utility, and today, commercial applications are the driving force in the system's development. These applications include: machine control; land survey; intelligent transport systems. These are just a few of the numerous applications of GPS - the list could be long.

Ionosphere is the most important source of satellite navigation errors. Successful tackling of the GPS ionospheric time-delay not only improves the positioning performance of satellite navigation systems, but also enhances the quality of different technical systems based on satellite navigation [1].

As GPS signals travel through the ionosphere, the code phase is delayed and the carrier phase is advanced. The amount of delay is proportional to the Total Electron Content (TEC) in the ionosphere. The electron density is a function of local time, magnetic latitude, and sunspot cycle. It reaches a peak at about 2:00 PM local time. The delay is inversely proportional to signal-frequency and is elevation dependent and is typically on the order of several meters. In order to reduce the positioning error of single-frequency GPS receivers, it is imperative to have better ionosphere models. Ionosphere prediction under disturbed conditions still presents a challenge [2]. The main focus of this paper is to report a model that is capable of long-term predictions.

Theoretical work shows that Neural Networks (NNs) are powerful enough to uniformly approximate almost any arbitrary continuous function on a compact domain, similar to traditional universal approximation techniques based on Taylor function expansion, Fourier series, and so forth. However, in addition to their ability to represent complex non-linear functions, NNs can effectively construct approximations for unknown functions by learning from examples (known outcomes of the function). This ability to approximate unknown complex input-output mappings makes them attractive in practical applications where traditional computational structures have performed poorly (such as those with ambiguous data or large contextual influence). NN models can be complemented by other successful approximation techniques based on wavelets, kernel estimators, nearest neighbors, B-splines, hinging hyperplanes, projection pursuit regression, partial least squares, and fuzzy models [3].

The purpose of the present paper is to model single-frequency GPS receivers ionospheric time-delay using NNs. This paper is organized as follows. In section II, ionospheric effects and Klobuchar model are reviewed. The RBF NN employed in this paper is introduced in section III. Section IV presents results using RBF NN. Conclusions are given in section V.

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II. IONOSPHERIC EFFECTS AND KLOBUCHAR MODEL

The ionosphere is a part of the Earth's atmosphere, laying at the heights between 50 km and 2000 km above the Earth's surface, and consisting of several layers identified by differences in the level of ionisation. The GPS ionospheric error consists of the satellite signal propagation delay collected during the passage of the ionosphere. The GPS ionospheric time-delay is directly proportional to the number of electrons per unit area encountered during the passage of the ionosphere, usually referred to as the TEC. A global ionospheric correction model, referred to as the Klobuchar model, is deployed in standard GPS positioning service. It is capable of correction of up to 70% error caused by the ionosphere. However, the Klobuchar model presents the best result when the ionospheric conditions are stable, while performs poorly during the severe ionospheric disturbances [4].

In Klobuchar's model, the vertical ionospheric time-delay is expressed by the positive portion of a cosine wave plus a constant night-time bias, as follows [5]:

$$T_{ij} = Q_{ij}.[DC + A.\cos(\frac{2\pi(\tau - \tau_0)}{P})] \quad (1)$$

where

T_{ij} : ionospheric time-delay in vertical direction at intersection of ionosphere with line from $i-th$ station to $j-th$ satellite

$Q_{ij} = \sec[\sin^{-1}(\frac{R}{R+H}\cos\theta_{ij})]$: obliquity factor from $i-th$ station to $j-th$ satellite (R is radius of the Earth, H is mean ionospheric height, and θ_{ij} is elevation angle from the $i-th$ station to $j-th$ satellite)

$DC = 5\times 10^{-9}$ seconds (night-time value)

$A = \alpha_1 + \alpha_2\Phi_m + \alpha_3\Phi_m^2 + \alpha_4\Phi_m^3$ (amplitude)

$P = \beta_1 + \beta_2\Phi_m + \beta_3\Phi_m^2 + \beta_4\Phi_m^3$ (period)

Φ_m : geomagnetic latitude of ionosphere subpoint

τ : local time

$\tau_0 = 14:00$ local time (phase)

α_i, β_i : ionospheric parameters

The task of the master station is to generate the eight parameters, $[\alpha_1, \cdots, \alpha_4, \beta_1, \cdots, \beta_4]$, which will yield the best ionospheric time-delay estimate for a single-frequency GPS receivers. Klobuchar model is a compromise between computational complexity and corrections accuracy. Considering the computational capability of modern mobile devices and communication performance of modern mobile communication systems, a time has come to introduce the advanced models that more appropriately describe ionospheric time-delay in general and take into account the local state of the ionosphere.

III. IONOSPHERIC TIME-DELAY MODELING USING RADIAL BASIS FUNCTION NEURAL NETWORK

Radial Basis Functions (RBFs) are embedded into a two-layer feed-forward NN. Such a network is characterized by a set of inputs and outputs. In between the inputs and outputs there is a layer of processing units called hidden units. Each of them implements a RBF. The way in which the network is used for data modeling is different when approximating non-linear problems. By means of training, the NN models the underlying function of a certain mapping. In order to model such a mapping we have to find the network weights and topology [6].

Fig.1 shows block diagram of the inputs and output of proposed RBF NN for ionospheric time-delay modeling. The user's approximate geodetic latitude Φ_u , longitude λ_u , elevation angle E , azimuth A to each GPS satellite, and local time t are used as proposed RBF NN inputs variables. RBF NN output is ionospheric time-delay.

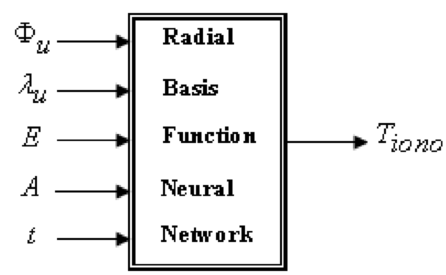

Figure 1. Block diagram illustrating the inputs and output to proposed RBF NN for single-frequency GPS receivers ionospheric time-delay approximation

The RBF NN activation function $\phi(.)$ must satisfy monotonic function conditions. The computation depends on the distance between the input feature vector X and the center of each hidden unit C_j , which is scaled by the metric R_j of the $j-th$ hidden unit:

$$h_j(X) = \phi[(X - C_j)^T R_j^{-1}(X - C_j)] \; ; \; j = 1, 2, \cdots, H \quad (2)$$

where H equals the number of hidden units (neurons). To reduce the number of parameters in R , we restrict it to be diagonal, with r_j equal to the radius vector of the $j-th$

hidden unit. In such cases a Gaussian function is generally used:

$$h_j(X) = \exp[-\sum_{i=1}^{i=I} (\frac{x_j - c_{ji}}{r_{ji}})^2] \ ; \ j = 1,2,\cdots,H \quad (3)$$

where vector C_j represents the location and $r_j = [r_{j1} \ \ r_{j2} \ \cdots \ r_{jI}]^T$ models the shape of the activation function. As the spread of the activation function becomes greater, the approximation by the function becomes smoother. Conventionally, all of the RBFs in the hidden layer units are of the some type. The RBF NN model input is not required to be linear while the output y, by requirement, is linear:

$$y(X) = \sum_{j=1}^{j=H} w_j h_j(X) \quad (4)$$

Fig.2 gives the structure scheme of the RBF NN model [7].

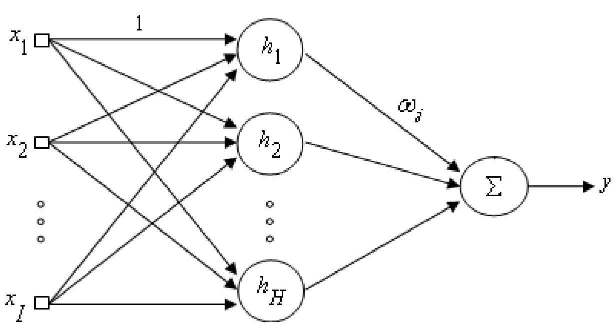

Figure 2. RBF NN architecture with $(I, H, 1)$ structure

In supervised learning, the weights w_j are adopted using the cost function criteria of the sum of squared errors minimization over the entire training set:

$$E = \frac{1}{2}(y - d)^2 \quad (5)$$

where d is response desired for y output. At the beginning, weights are initialized using small random values, then they are turned at each iteration of the supervised learning:

$$w_{j\ new} = w_{j\ old} + \Delta w_j \quad (6)$$

where:

$$\Delta w_j = -\eta \frac{\partial E}{\partial w_j} = -\eta(d - y)h_j(X) \quad (7)$$

where η is a non-fixed learning rate. These operations are repeated until the maximum number of iterations is reached or until the approximation error is less than some given threshold.

IV. SIMULATIONS AND RESULTS

Fig.3 represents the ionospheric time-delay approximation using Klobuchar model.

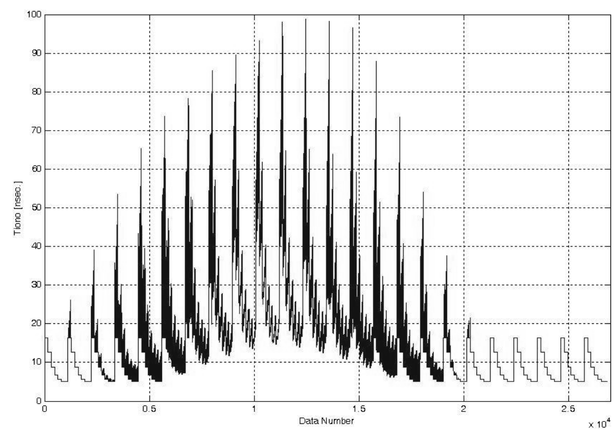

Figure 3. Ionospheric time-delay approximation using Klobuchar model for 24 hours

Computer simulations were performed to evaluate the RBF NN-based ionospheric time-delay approximation performance. The ionospheric time-delay approximation residual is defined as the difference between the ionospheric time-delay value by RBF NN approach and by Klobuchar model: $Tiono_{NN} - Tiono_K$. Increasing the number of training patterns also increases the memory for software implementation and increases the structure complexity for hardware implementation. Therefore, a trade-off should be made in selecting the number of training patterns. Fig.4 shows ionospheric time-delay approximation residual for 1:00 to 3:00 PM local time using RBF NN approach.

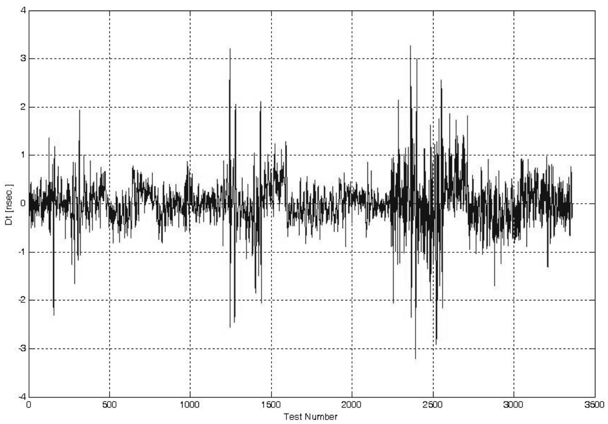

Figure 4. Ionospheric time-delay approximation residual for 1:00 to 3:00 PM local time using RBF NN approach

Six statistical measures (maximum, minimum, average, variance, standard deviation, and RMS), are used to evaluate approximation results [8]. Table 1 shows approximation errors (the difference between the RBF NN approach and Klobuchar model values) statistical significance characteristics.

TABLE I. APPROXIMATION PERFORMANCE EVALUATION USING PROPOSED RBF NN

Parameters	Error Value
Max	3.2693
Min	-3.2054
Average	0.000031
Variance	0.0234
Standard Deviation	0.1530
RMS	**0.1510**

V. CONCLUSIONS

The ionosphere affects the electromagnetic waves that pass through it by inducing an additional transmission time delay. The ionosphere influence has now become the largest error source in GPS positioning and navigation after the turn-off of the Selective Availability (SA). Our study shows that a RBF NN approach can be an effective tool to model the single-frequency GPS receivers ionospheric time-delay. The advantages of such an approach are simplicity and flexibility. When training a NN, we only need to specify the sequence of input parameters and target parameters. A RBF NN approach also allows updating the model without invoking previous data used, and models can be updated progressively. In this paper, RBF NN was applied for the L1 GPS receivers ionospheric time-delay approximation. RBF NN-based approximator was focused on. The RBF NN-based approach reduces the computational burden respect to Klobuchar model. The method employed here is applicable on the single-frequency GPS receivers. Simulations results were presented. The performances have been explored and discussed.

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