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Skeletonization based on high-level markov random field

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***Abstract*—One essential challenge for some skeletonization methods are the skeletons are broken on the intersection regions or by noise. In order to connect the broken skeletons more reasonable and obtain good skeleton performance, we propose a method to repair the primary skeletons extracted by some recently methods. The primary skeletons are repaired using high-level Markov Random field (HLMRF) whose fetures are lines of primary skeletons. The final repaired skeletons are obtained by finding the optimal resolution of cost function defined by HLMRF. Experiments show the good performance of our method.**

Keywords—Skeleton; high-level MRF; lineon; wavelet maxima; primary skeleton

I. INTRODUCTION

Extraction of a skeleton from a ribbon-like shape is very important in pattern recognition. The skeleton of a shape is always defined as the locus of the symmetric points of the shape and it should be close to the human conception, one-pixel-wide and similar to the original shape.

In other word, skeletons should be a good substitution for the original shape and skeletonization becomes a very active area [1-2]. However, existing methods for extracting skeletons from shapes still face a variety of challenges. An essential challenge for skeletonization, are that skeletons are usually broken on intersection regions [3-9]. Thus how to connect primary skeletons or extracting no broken skeletons becomes a new subject.

Some efforts have been done for this subject [4-6, 10-11]. In [4, 6], authors proposed skeletonization based on primary skeletons and types of different feature points, which will be used for connect broken skeletons. Two disadvantage about this paper are: 1. It is dependent on "scale", which should be choosed previously by manual; 2. It used so many human experiential values in connecting primary skeletons that they can not be generalized easily.

One new technique of wavelet is proposed for replacing wavelet maxima in [10], which try to skeleton using wavelet minma. Although good performance is reported in this paper, it is still dependent on the "scale".

Some skeletonization scheme are based on singularity and regularity analysis [11]. Since the scheme is designed on the the constrained Delaunay triangulation technique, its comuptation burden can not be afforded for complex shapes.

In this paper, we argue that connecting broken primary skeletons should be consider as a optimal problem. The motivation is from [13], which reconstructs skeletons by designing a cost fuction of matching two line segments. But for skeletonization, it should consider more complex situation including noise or other uncertainties. HLMRF is a powerful tool for handling uncertainties, whose cost function is formulated by some energy terms and the optimal resolution is obtained from minimizing the cost function. Another advantage for MRF is its flexibility for different case and reqiurments through defining different energy terms.

To our knowledge, HLMRF has not been used for connecting broken skeletons. The main contributions for this paper is not only the new framework but also some details, such as defining new conceptions, designing new energy terms and finding optimal resoltions.

The reminder is arranged as: on the second section, we will introduce one method for obtaining primary skeletons based on wavelet maxima in order to preserve self contained nature for this paper; and then we will introduce the MRF theory and the energy formulations used in this paper on section 3; the whole framework will introduuce on section 4; and the experimental results and their discussion will be given on the final of this paper.

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II. SKELETONIZATION BASED ON WAVELET MAXIMA [5-6]

Skeletonization based on wavelet maxima is composed by two procedures: one is singlarity detection by wavelet maxima to decide the contours of a shape and the other is finding symmetric locus of contours. The discussion of this section will procede according to these two procedures. On this section, we will introduce basic ideas about the detection of singularities and contours using wavelet maxima. And then the skeleton extraction using will be given.

A. *Singularity located with 2-D wavelets:*

The singularity can be detected using wavele maxima. Main motivation is to project wavelet coefficients to their gradient direction, whose gray variantion is fastest in all dirctions, to improve the nature of direction location ability of wavelets.

We define two wavelet functions $\psi^1(x,y)$ and $\psi^2(x,y)$ such that

$$\psi^1(x,y)=\frac{\partial\theta(x,y)}{\partial x} \text{ and } \psi^2(x,y)=\frac{\partial\theta(x,y)}{\partial y} \tag{1}$$

Let $\psi_s^1(x,y)=\frac{1}{s^2}\psi^1(\frac{x}{s},\frac{y}{s})$ and $\psi_s^2(x,y)=\frac{1}{s^2}\psi^2(\frac{x}{s},\frac{y}{s})$. The wavelet transform of f(x,y) defined by.

$$W_s^1 f(x,y)=f*\psi_s^1(x,y) \text{ and } W_s^2 f(x,y)=f*\psi_s^2(x,y) \tag{2}$$

Then, we can easily induce that the gradient vector at point (x, y) is

$$\begin{pmatrix} W_s^1 f(x,y) \\ W_s^2 f(x,y) \end{pmatrix} = s\begin{pmatrix} \frac{\partial}{\partial x}(f*\theta_s)(x,y) \\ \frac{\partial}{\partial y}(f*\theta_s)(x,y) \end{pmatrix} = s\vec{\nabla}(f*\theta_s)(x,y) \tag{3}$$

The modulus of the gradient vector wavelet transform at point (x, y) is defined as:

$$|\nabla W_s f(x,y)|=\sqrt{|W_s^1 f(x,y)|^2+|W_s^2 f(x,y)|^2} \tag{4}$$

The direction of the gradient vector at point (x, y) is:

$$\varphi=\begin{cases} \arctan\frac{W_s^2 f(x,y)}{W_s^1 f(x,y)} & x>0 \\ \pi+\arctan\frac{W_s^2 f(x,y)}{W_s^1 f(x,y)} & x<0 \end{cases} \tag{5}$$

The direction of the gradient vector at a point indicates the direction in the image plane (x, y) which has the largest absolute value of the directional derivative of f(x, y). Singularity points can be defined as points where the modulus in (4) of the gradient vector is maxima in its direction in (5).

TABLE I. ALGORITHM OF WAVLET MAXIMA

1. Decide the scale s and threshold value k
2. Compute the Gradinet Vector according to (3)
3. Compute the Gradinet Direction according to (5)
4. Compute the wavelet modulus according to (4)
5. If $|\nabla W_s f(x, y)|-|\nabla W_s f(x1, y1)|>k$ in direction φ, $|\nabla W_s f(x, y)|$ is maximum modulus
6. Repeat 1-5 until all points are computed

B. *Skeletonization using wavelet maxima*

Since skeletons are locus of symmetric points, skeletonization converts to find symmetric locus of contours. In [4-6], skeletonization is carried out by "scale", which is previously defined by manual. In fact, in this method, scale is related to width of strokes. Therefore, if the "scale" can be tentatively decided as the width of strokes, each skeleton point is the middle point of a pair of contour points with inverse gradient directions. After locating the contour points using the algorithm on subsection 2.1, the skeltonization is as follow:

TABLE II. ALGORITHM OF SKELETONIZATION USING WAVLET MAXIMA

1. Decide the scale s.
2. For each contour point go ahead s pixels along its gradient direction:
If the pixel is a contour point, compute the middle point of two contour points, the middle point is a skeleton point;
If the pixle is not a contour point, give up this point.
3. repeat 1-2 until all contour points have been computed.

C. *Some comments*

Since the method in [4-6] depends scale seriously, in [5] we proposed a new method using Frenet frames and wavelet maxima to avoiding the dependence for scale. Skeletons extracted by both methods are broken in intersection regions. These two method can be used for extracting primary skeletons. Other methods, which will lead to skeletons broken, are also can be put in this section to form primary skeletons. Therefore, in generally speaking, our method is a wide used method for connecting broken line segments.

III. MARKOV RANDOM MODEL

Markov random field (MRF) provides a powerful model in visual processing. It allows us to develop optiaml results by designing some contextual contraints and finding its optimal solution. The basic theory of MRF and high-level MRF is present on this section.

A. *Markov randon field (MRF) [13]*

F={F_1,…,Fm}, is said to be a Markov random field on the site set S with respect to a neighborhood system N if and only if the following two conditions are satisfied:

$$P(f)>0,\forall\{f\in F\} \tag{6}$$

$$P(f_i\mid f_{s-\{i\}})=P(f_i\mid f_{N_i}) \tag{7}$$

where S-{i} is the site set which is different from i, $f_{s-\{i\}}$denotes the set of labels at the sites in S-{i} and

$$f_{N_i}=\{f_{i'}\mid i'\in N_i\}$$

stands for the set of labels at the sites neighboring i

A set of random variables F is said to be a Gibbs random field (GRF) on S with respect to N if and only if its configurations obey a Gibbs distribution. A Gibbs distribution takes the following form

$$P(f) = \frac{e^{-\frac{1}{T}U(f)}}{Z} \tag{8}$$

where

$$Z = \sum_{f \in F} e^{-\frac{1}{T}U(f)} \tag{9}$$

is a normalizing constant called the partition function, T is a constant called the temperature which shall be assumed to be 1 unless otherwise stated, and U(f) is the energy function. The energy

$$U(f) = \sum_{c \in C} V_c(f) \tag{10}$$

is a sum of clique potentials $V_c(f)$ over all possible cliques C. The value of $V_c(f)$ depends on the local configuration on the clique c.

Sometimes, it may be convenient to express the energy of a Gibbs distribution as the sum of several terms, each ascribed to cliques of size up two is,

$$U(f) = \sum_{\{i\} \in C_1} V_1(f_i) + \sum_{\{i,i'\} \in C_1} V_2(f_i, f_{i'}) \tag{11}$$

An MRF is characterized by its local property (the Markovianity) whereas a GRF is characterized by its global property (the Gibbs distribution). The Hammersley-Clifford theorem establishes the equivalence of these two types of properties. Thus searching the maximum of probabilty P(f) defined on (8) can be converted to minimization the energy of cliques, which can be defined on markov random field.

B. *High-level Markov random field (HLMRF)*

Let S index a discrete set of m sites S={1,…, m}, in which {1,…,m} are indices. A site often represents a point or a region in the Euclidean space. A set of sites may be categorized in terms of their "regularity". Sites on a lattice are considered as spatially regular. Sites which do not present spatial regularity are considered as irregular. This is the usual case corresponding to features extracted from images at a more abstract level, such as the detected corners and lines.

In this paper, the primary skeletons, which is composed by some skeleton lines, called as lineons, are considered as sites. The aim is to connect these sites according to a cost function, which is designed on MRF. The cost function evaluates the cost of connect two sites. Therefore, it is related to the corner difference, distance beween two lines and shift between to sites. These differences can be represented by the energy like the second term on (11).

$$U(f) = \sum_{\{i,i'\} \in C_1} \alpha C_{\{i,i'\}} + \beta S_{\{i,i'\}} + \lambda D_{\{i,i'\}} \tag{12}$$

where $C_{\{i,i'\}}$ represents the corner difference, $D_{\{i,i'\}}$ represnts the distance between two lines and $S_{\{i,i'\}}$ represnts the shift, α, λ and β are paramters used to banlance affect of two terms

In fact, we can design more complex cliques energy formula for this task, for example, the line length etc. However, there are two reasons for this only two terms cost: 1. more complex energy leads to more complex computation burden; 2. experiments presented in this paper are only primary results, which are used to proof validity of our method. Thus we only consider three main elements in constructing cost function.

IV. Framwork

The whole framework is:

1. extract primary skeletons

2. defineα, λ, β and a threshold value k

3. for each site, compute U(f) for other sites and find the minimum of U(f) and related line

4. if the minimum of U(f)<k, connect two line and renew the set of sites

otherwise, considering other sites

5. repeat 3-4, until can not find two sites could be connected

V. Experimental Results

Our method is compared with three state of arts method: one is wavelet maximum modulus, the second other is morphologically operation, the third is the method proposed in [5]. The disadvantages about wavelet maxima and morphologically operation are well-known facts.

The morphologically operation will produce some artifact in skeleton (see Fig. 1).

For wavelet maximum modulus, the detection results dependent seriously on the scale. The scale must coincide to the width of strokes. If this assumption does not meet, the skeleton points can not be located. This makes its detection must be in the images with same width strokes and the detection will be lost on intersectional regions (see Fig. 2).

The skeletonizaton method proposed in [5] tries to improve the dependence of scale for wavelet maxima. Although it improves the independence of scale for wavelet maxima, some skeletons on intersection regions are also broken (see Fig. 3).

Our method has prefect skeketonization results (see Fig. 4).

The black lines represent the original strokes while the black points represent the skeleton detected by different method.

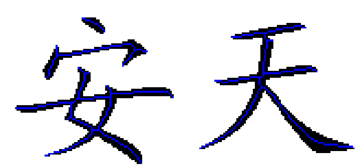

Figure 1. Skeleton by morphologically operation

Figure 2. Skeleton by wavelet maxima

Figure 3. Skeleton by method proposed in [5]

Figure 4. Skeleton by our new method

VI. SUMMARY AND FUTURE WORK

In this paper, we proposed a new skeletonization method based on HLMRF. This method can be used for contructing skeletons, which conform the human conception. Since the flexibility of HLMRF, our method also provides a framework for skeletonization according to different requirments.

The future work will focus on designing the new models in low quality character recognition.

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REFERENCE

[1] L. Lam, S.W. Lee, and C. Y. Suen, "Thinning methodologies—A comprehensive survey," *IEEE Trans. Pattern Anal. Machine Intell.*, vol. 14, pp. 869–885, 1992.

[2] Gisela Klette, "A comparative discussion of distance transforms and simple deformations in digital image processing", Machine Graphics & Vision International Journal, vol. 12, No. 2, Feb. 2003, Page(s): 235-256.

[3] Z. W. Liao, Z. S. Wang and S. X. Hu, "Skeletonize multi width ribbon-like shapes based on difference images and Frenet Frame", 2008 IEEE International Conference on Systems, Man, and Cybernetics, Singapore, 12 - 15 October 2008.

[4] Xinge You, and Yuan Yan Tang, "Wavelet-Based Approach to Character Skeleton", IEEE TRANSACTIONS ON IMAGE PROCESSING, VOL. 16, NO. 5, pp. 1220-1231, MAY 2007.

[5] Lihua Yang, Ching Y. Suen, and Yuan Yan Tang, "A Width-Invariant Property of Curves Based on Wavelet Transform With a Novel Wavelet Function", *IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS—PART B: CYBERNETICS,* VOL. 33, No. 3, pp. 541-548, JUNE 2003.

[6] Y. Y. Tang and X. G.You, "Skeletonization of ribbon-like shapes based on a new wavelet function," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 25, no. 9, pp. 1118–1133, Sep. 2003.

[7] S. Mallat*, Wavelet Tour of Signal Processing*. San Diego, CA: Academic, 1998.

[8] S. Mallat and S. Zhong, "Characterization of signals from multiscale edges," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 14, no. 7, pp. 710–732, Jul. 1992.

[9] S. G. Mallat and W. L. Hwang, "Singularity detection and processing with wavelets," *IEEE Trans. Inf. Theory*, vol. 38, no. 2, pp. 617–643, Mar. 1992.

[10] S. X. Hu, Yanming Xiong, Melody Z. W. Liao, Y. Y. Tang, " Skeletonization of Ribbon-like Shapes Based on Local Minimum Modules of Wavelet Transform", International Conference on Wavelet Analysis and Pattern Recognition 2007, Nov 1-3, Beijing.

[11] J. J. Zou and H. Yan, "Skeletonization of ribbon-like shapes based on regularity and singularity analyses," IEEE Trans. Syst. Man. Cybern. B, Cybern., vol. 31, no. 3, pp. 401–407, Jun. 2001.

[12] A.P. Whichello and Hong Yan, "Reconstruction of character skeletons using Gabor filter features", ELECTRONICS LETTERS, 26th October 1995, Vol. 37 No. 22, pp 1911-1912.

[13] Stan Z Li, "Markov Random Field Modeling in Image Analysis", Sprnger-Verlag Tokyo 2001.