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An approach for collaborative path planning in multi-robot systems

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***Abstract*—This paper presents a reactive algorithm for multirobot cooperative path planning. The algorithm integrates the real-time collision detection process with linear navigation functions. The collision detection strategy is based on an improved version of the swept volumes method that uses the relative kinematics of the robots in addition to the geometric considerations of the paths. The swept volumes determine the collision conditions, which are written in terms of the relative velocities and orientations. Speed and orientation collision intervals are then derived based on the swept volumes. A collision histogram is then constructed as a collection of the speed and orientation collision intervals. The collision histograms show the free directions and the direction corresponding to collision. The desired orientation and speed are chosen using the collision histograms. The strategy is illustrated using an extensive simulation study.**

***Index Terms*—multi-robot systems, path planning, collaboration.**

I. Introduction

Multi-agent robotic systems are used in various applications, such as industrial and home applications, military applications, etc. Cooperation between robots is necessary in various situations, in order to accomplish tasks which are difficult or time consuming for single robots. Examples of applications where multi-robot systems are used to accomplish a cooperative task include soccer robotics ([1], [2]), surveillance, exploration and tracking ([3], [4]), demining robots etc. During the last decade, the topic of cooperation between robots has seen important developments. Multi-robot formation control, where the aim is to establish and maintain some predetermined geometric shape is among the most important areas in multi-robot systems. Various techniques were suggested for this purpose. Reactive behaviors formation control for multi-robot systems is integrated with other navigational behaviors in [5]. Behavior-based formation control was also used in ([1], [6]). In [7], a global behavior for robot formation is achieved using local sensing and minimal communication. Modeling and controlling of multi-robot formation using graph theory is discussed in [8] for non-holonomic robots. Graph theory for multi-robot formation is also discussed in [9]. Various techniques from control theory, such as receding horizon control [10], Lyapunov theory [11] and multi-objective optimization control [12] were suggested for decentralized formation control. Other techniques based on geometric approaches [13], ant algorithms [14] and communication [15] were used for multi-robot formation and control.

Other important applications of multi-robot systems are target tracking using a team of robots [4] and dynamic coverage ([16], [17]). In dynamic coverage, the aim is to maximize the area covered by the sensors of the robots team. In [18], a multi-robot system for hunting a single target is discussed, where a model-based method is used. Multi-robot navigation is another important topic in multirobot systems, since navigation and collision avoidance are among the most important and elementary functions in robotics. In our opinion, multi-robot navigation with collision avoidance between robots has not received enough attention. In [19], local navigation strategies in unknown environment for a team of mobile robots are suggested based on line of sight communication capabilities. The paper by Fujimori et al. [20] is among the most important papers dealing with cooperative collision avoidance between robots, where collision is detected using the geometric considerations of the paths. The authors defined a priority function, based on which one robot acts. A similar approach to avoid moving obstacles is suggested in [21].

Our aim in this paper is to introduce a strategy for cooperative avoidance between mobile robots. Our strategy combines the swept volumes method for collision detection with the relative kinematics of the robots. The swept volume method is among the most discussed methods in the collision detection literature. The swept volume method presents the necessary conditions for collision, but not the sufficient conditions. To solve this problem, the swept volumes are expressed in terms of the kinematics equations. This allows to construct collision windows for the speeds and the orientations of the robots.

The paper is organized as follows: In section III, the navigation modes, the geometry and kinematics equations are discussed. In section IV, the collision course equations are derived based on the kinematics equations. In section V, we discus the control commands for the linear velocity and the orientation angle of the robots. In section VI, simulation study is suggested to illustrate the method.

II. Position of the problem

Consider N wheeled mobile robots $R = \{R_1, R_2, ..., R_N\}$ moving in the horizontal plane as shown in figure 1. The workspace contains a given number of stationary obstacles. In order to express the robots coordinates in a common reference frame of coordinates, we define a global system of coordinates W_O. O is the origin of W_O. Each robot starts from a given initial position $S_i, (i = 1, ..., N)$ and aims to reach a given final goal $G_i, (i = 1, ..., N)$ without collision with obstacles or other robots. The position of $R_i, (i = 1, ..., N)$ in the global Cartesian system of coordinates is given by (x_i, y_i). The orientation angle of R_i is $\theta_i \in [0, 2\pi]$. v_i is the linear velocity of R_i. The following assumptions are made

1) The control inputs of R_i are (v_i, θ_i).

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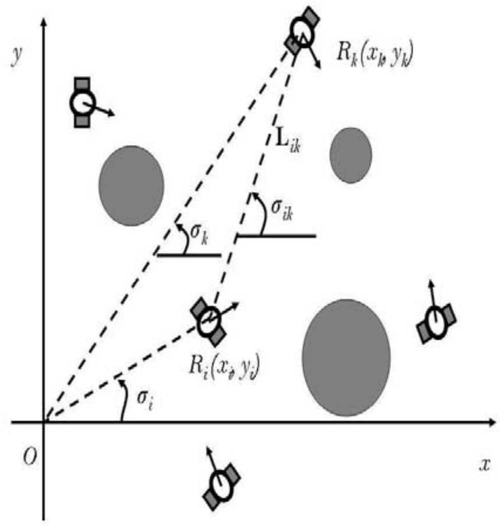

Fig. 1. An illustration of the geometry of the navigation problem

2) Every robot is characterized by a coverage area of its sensory system. The sensory system allows to measure in real time the speed and orientation of the objects moving in the coverage area.

The goal in this paper is to design local and global path planning methods that allow the robots to reach their final destination safely. The next section discusses the relative kinematics models.

III. Kinematics models

The cooperative collision avoidance mode is activated when the robots are in a collision course within a given distance l_0. l_0 is the switching distance to the collision avoidance mode. Consider the geometry between robots R_i and R_k as shown in figure 1.

1) The relative distance between R_i and R_k is $r_{ik} = r_{ki}$, with
$$r_{ik} = \sqrt{(y_i - y_k)^2 + (x_i - x_k)2} \quad (1)$$
2) $\vec{L}_{ik}$ is the imaginary straight line joining R_i and R_k.
3) The polar coordinates of robot R_i are (r_i, σ_i),$(i = 1, ..., N)$, where r_i is the radial variable and σ_i is the angular variable.
4) The direction angle σ_{ik} between R_i and R_k is the angle from the positive x-axis to L_{ik}. The direction angle is given by
$$\tan \sigma_{ik} = \frac{y_i - y_k}{x_i - x_k} \quad (2)$$

The polar kinematics model describing the motion of R_i are given by
$$\begin{aligned} \dot{r}_i &= v_i \cos(\theta_i - \sigma_i) \\ r_i \dot{\sigma}_i &= v_i \sin(\theta_i - \sigma_i) \end{aligned} \quad (3)$$

$\vec{v}_i$ and $\vec{v}_k$ are the velocity vectors for R_i and R_k, respectively; and $\vec{v}_{ik}$ is the relative velocity vector given by $\vec{v}_{ik} = \vec{v}_i - \vec{v}_k$. The relative velocity vector can be decomposed into two components along and across $\vec{L}_{ik}$ as follows
$$\vec{v}_{ik} = v_{ik}^r \vec{e}_r + v_{ik}^\sigma \vec{e}_\sigma \quad (4)$$
where $\vec{e}_r$ and $\vec{e}_\sigma$ are the unit vectors along and across $\vec{L}_{ik}$. v_i and v_k can also be decomposed into two components along and across $\vec{L}_{ik}$. By using the robots equations of motion in polar coordinates, we get
$$\begin{aligned} v_i^r = v_i \cos(\theta_i - \sigma_i), v_k^r = v_k \cos(\theta_k - \sigma_k) \\ v_i^\sigma = v_i \sin(\theta_i - \sigma_i), v_k^\sigma = v_k \sin(\theta_k - \sigma_k) \end{aligned} \quad (5)$$
where v_i^r and v_k^r are the components of the velocity vectors of R_i and R_k, respectively, along $\vec{L}_{ik}$. v_i^σ and v_k^σ are the components of the velocity vectors of R_i and R_k, respectively, across $\vec{L}_{ik}$. The values of v_{ik}^r and v_{ik}^σ can be easily obtained as follows
$$\begin{aligned} v_{ik}^r = \dot{r}_{ik} = v_i^r - v_k^r \\ v_{ik}^\sigma = r_{ik} \dot{\sigma}_{ik} = v_i^\sigma - v_k^\sigma \end{aligned} \quad (6)$$
By considering systems (5) and (6), we get
$$\begin{aligned} \dot{r}_{ik} = v_i \cos(\theta_i - \sigma_i) - v_k \cos(\theta_k - \sigma_k) \\ v_{ik}^\sigma = v_i \sin(\theta_i - \sigma_i) - v_k \sin(\theta_k - \sigma_k) \end{aligned} \quad (7)$$
This kinematics model gives the motion of R_i as seen by R_k. The range $r_{ik}(t)$ is decreasing on the time interval $[t_0, t_i]$ when $\dot{r}_{ik} < 0$. The robots are approaching from each other but they are not necessarily in a collision course. Similar to the swept volumes intersection, a negative sign of $\dot{r}_{ik}$ is not a sufficient condition for collision.

A. Backstepping control

Collision is avoided by controlling the linear velocity and/or orientation angle. For these commands, we use feedback linearization [22] in combination with backstepping or block control ([23], [24]), which gives for the linear velocity
$$v_i = -k_v(v_i - v_i^{des}) \quad (8)$$
and for the orientation angle
$$\theta_i = -k_\theta(\theta_i - \theta_i^{des}) \quad (9)$$
where v_i^{des} and θ_i^{des} are the values for the linear velocity and orientation angle that allow to avoid the collision. k_v and k_θ are positive gains.

B. Global path planning

To reach its final goal, R_i applies the following control strategy ([25])
$$\dot{\theta}_i = M \dot{\sigma}_{gi} + \frac{df(t)}{dt} \quad (10)$$
where σ_{gi} is the final goal's direction angle. $f(t)$ is a deviation function, and M is a positive number that represents the navigation parameter. M is used to tune the robots path to avoid collisions. This property offers very important flexibility, which is highly desired for path planning in dynamic environments.

IV. SWEPT VOLUMES AND COLLISION COURSE

The swept volumes method is one of the most powerful methods used for collision detection ([26], [27]) in robotics and computer graphics. Unfortunately, the swept volumes intersection is not a sufficient condition for collision. The goal in this section is to improve this aspect, and deduce the collision course condition between R_i and R_k. We define the swept volumes intersection and the collision course as follows

- Swept volumes intersection: It means that the swept volumes of the paths traced by R_i and R_k intersect in the future.
- Collision course: R_i and R_k are in a collision course if they will reach the intersection area at the same time. Swept volumes intersection is a necessary condition for the collision course.

The kinematic conditions for the swept volumes intersection and the collision course conditions are discussed in the next section based on the geometry of the paths and the robots kinematics equations.

A. Swept volumes intersection

The goal here is to establish a mathematical formulation for the kinematics conditions for the swept volumes intersection. The result will be used for the collision course detection. Consider the configuration of figure 2. Any other geometric configuration can be easily obtained from this configuration. The swept volumes intersection requires that $\theta_i - \sigma_{ik}$ and $\theta_k - \sigma_{ik}$ belong to the same interval $(-\pi, 0)$ or $(0, \pi)$. This means that the velocity vectors for both robots are above the line of sight (as shown in figure 2a) or below the line of sight (as shown in figure 2b). In figure 2a, we have $\theta_i - \sigma_{ik} \in (0, \pi)$ and $\theta_k - \sigma_{ik} \in (0, \pi)$ and in figure 2b, we have $\theta_i - \sigma_{ik} \in (-\pi, 0)$ and $\theta_k - \sigma_{ik} \in (-\pi, 0)$. The swept volumes intersection condition is stated as follows

- In the case when $\theta_i - \sigma_{ik} \in (0, \pi)$, the swept volumes intersect when the orientation angle for R_k satisfies

$$\theta_k \in (\sigma_{ik}, \theta_i) \tag{11}$$

  Clearly, θ_k in (11) satisfies $\theta_k - \sigma_{ik} \in (0, \pi)$. This case is shown in figure 2a.
- In the case when $\theta_i - \sigma_{ik} \in (-\pi, 0)$, the swept volumes intersect when the orientation angle for R_k satisfies

$$\theta_k \in (\theta_i, \sigma_{ik}) \tag{12}$$

  Clearly, θ_k in (12) satisfies $\theta_k - \sigma_{ik} \in (-\pi, 0)$. This case is shown in figure 2a.

The particular cases when $\theta_i - \sigma_{ik} = 0$ or $\pm\pi$ correspond to the pure pursuit or the pure escape. In the case when R_i is to the left of R_k, the swept volumes intersection conditions are obtained by switching the subscribes in equations (11) and (12).

B. Collision course

The collision course requires the swept volume intersection conditions to be satisfied. The result concerning the collision course is stated as follows.

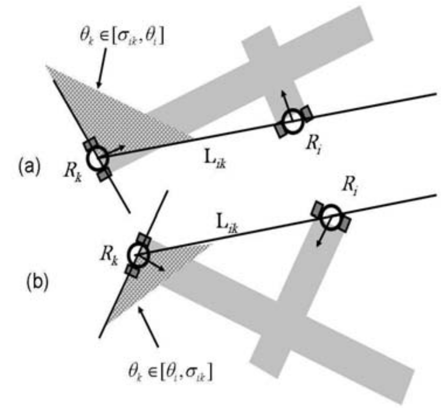

Fig. 2. An illustration of the kinematic conditions of the swept volumes intersection

Proposition 1: R_i and R_k are in a collision course if the intersection conditions (given by (11) or (12)) are satisfied and

$$\dot{\sigma}_{ik} \leq \varepsilon \tag{13}$$

which gives

$$v_k \sin(\theta_k - \sigma_{ik}) - \varepsilon \leq v_i \sin(\theta_i - \sigma_{ik}) \leq v_k \sin(\theta_k - \sigma_{ik}) + \varepsilon \tag{14}$$

where ε is a positive number that depends on the size of the robots. Systems (13) and (14) state that the turning rate of R_i with respect to R_k stays within given limits. The proof of proposition 1 is beyond the scope of the paper and is based on the relative kinematics equations. The collision course can also be expressed in terms of the robots orientation angles as follows.

Proposition 2: For given values of θ_i, v_i and v_k, with $v_k > v_i$, robots R_i and R_k are in a collision course when

$$\theta_k = \sigma_{ik} + \sin^{-1}\left(\frac{v_i}{v_k}\sin(\theta_i - \sigma_{ik})\right) \tag{15}$$

Proof:
The inverse sine function maps the interval $[-1, 1]$ to $[-\pi/2, \pi/2]$, and since $v_k > v_i$, we have

$$\sin^{-1}\left(\frac{v_i}{v_k}\sin(\theta_i - \sigma_{ik})\right) \in (-\pi/2, \pi/2) \tag{16}$$

as a result, the tangential velocity of R_k along $\vec{L}_{ik}$ is always positive

$$v_k^r = \cos\left(\sin^{-1}\left(\frac{v_i}{v_k}\sin(\theta_i - \sigma_{ik})\right)\right) > 0 \tag{17}$$

and since the relative range varies as follows

$$\dot{r}_{ik} = v_i^r - v_k^r \tag{18}$$

by replacing v_i^r and v_k^r by their formulae, it follows that $v_i^r < v_k^r$, and thus $\dot{r}_{ik} < 0$, $\forall \theta_i, \forall \sigma_{ik}, \forall v_k > v_i$. ■

Note that it is possible to express the collision course in

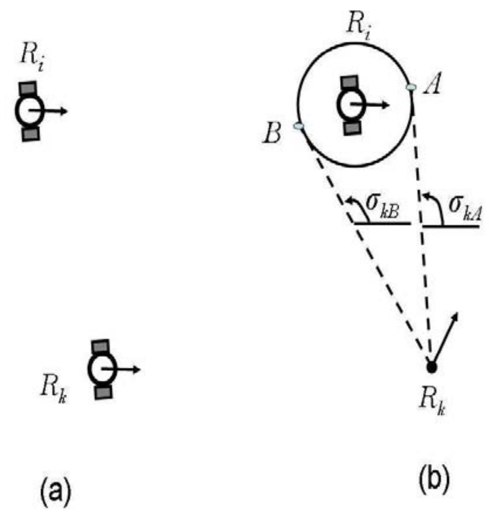

Fig. 3. Illustration of the limit angles

terms of the orientation angle of R_i instead of R_k. The result is similar to equation (15). Now, consider figure 3, where R_k is reduced to a single point and R_i is augmented in the size. We define the direction angles σ_{kA} and σ_{kB} as shown in the figure. We also define

$$v_k^a = v_i \frac{\sin(\theta_i - \sigma_{kA})}{\sin(\theta_k - \sigma_{kA})} \quad (19)$$

and

$$v_k^b = v_i \frac{\sin(\theta_i - \sigma_{kB})}{\sin(\theta_k - \sigma_{kB})} \quad (20)$$

Assuming that $v_k^a < v_k^b$. The speed collision interval (SCI) is characterized by

$$SCI_k^i = [v_k^a, v_k^b] \quad (21)$$

Collision with R_i takes place when $v_k \in SCI_k^i$. The collision course can be also expressed in terms of the orientation angles. We define

$$\theta_k^a = \sin^{-1}(\frac{v_i}{v_k} \sin(\theta_i - \sigma_{kA})) + \sigma_{kA} \quad (22)$$

and

$$\theta_k^b = \sin^{-1}(\frac{v_i}{v_k} \sin(\theta_i - \sigma_{kB})) + \sigma_{kB} \quad (23)$$

Assuming that $\theta_k^a < \theta_k^b$, the orientation collision interval (OCI) can be expressed in terms of the orientation angle of R_k when $v_i < v_k$ as follows

$$OCI_k^i = [\theta_k^a, \theta_k^b] \quad (24)$$

Collision with R_i takes place when $\theta_k \in OCI_k^i$. When $v_i > v_k$, the collision course is expressed in terms of the orientation angle of R_i instead of R_k. The equations are similar. The speed and orientation collision intervals given by (21) and (24) are combined together to form the collision histograms, defined as follows

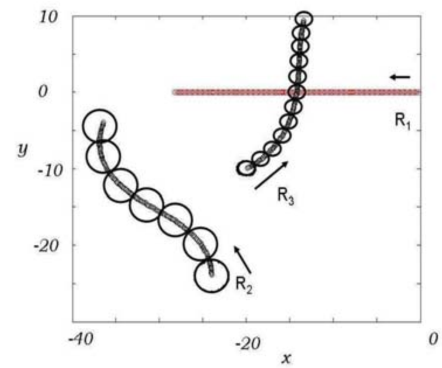

Fig. 4. Robots paths

$$SH_k = \bigcup_{i \neq k} SCI_k^i \quad (25)$$

$$OH_k = \bigcup_{i \neq k} OCI_k^i \quad (26)$$

where SH stands for the speed histogram and OH stands for the orientation histogram. The collision histograms consider all moving robots in the coverage area.

Example: The goal from this example is to illustrate the SCI and the OCI. This example shows 3 robots sharing the workspace. The initial positions for R_1, R_2, and R_3 are given by $(0,0)$, $(-20,-10)$ and $(-25,-25)$, respectively. $v_1 = 2m/s$ and $\theta_1 = \pi$. Figure 4 shows the paths traveled by the robots. Figure 5 shows the direction angles corresponding to the collision course between R_1 and R_2. Figure 6 shows the SCI of R_1 with R_2, and figure 7 shows the OCI for R_1 with R_2. The shaded area corresponds to collision. At time $t = 14s$, $v_1 \in SCI$ and $\theta_1 \in OCI$, this means that R_1 is in a collision course with R_2. To avoid collision, the velocity or the orientation has to be chosen outside the SCI or the OCI, respectively.

V. Collision avoidance mode: Linear velocity and orientation angle commands

Proposition 1 gives the collision course conditions. The robots can use two different strategies to avoid the collision, namely, velocity control and path deviation. Both strategies are discussed below.

A. Collision avoidance priorities

Two priorities are defined as follows

— Which robot must act? There exit different possibilities to assign priorities to the robots. In this paper, we give the higher priority of action to the fastest robot. In the reciprocal action approach, both robots perform the same task.

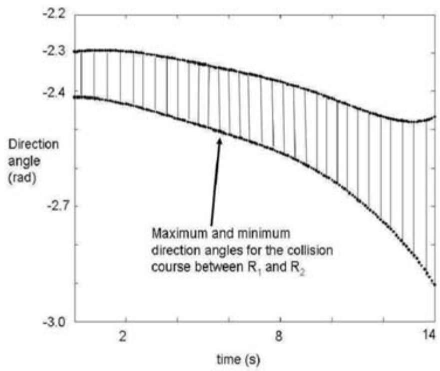

Fig. 5. SCI of R_1 with R_2

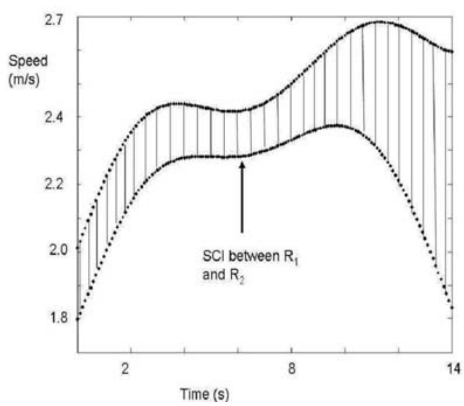

Fig. 6. Maximum and minimum direction angles for the collision course

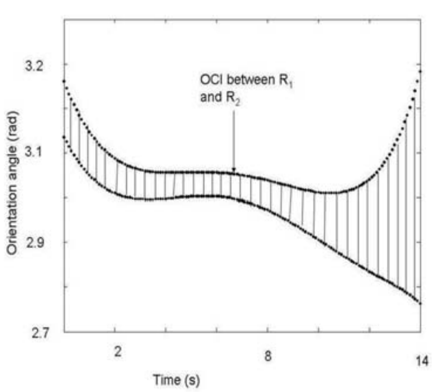

Fig. 7. OCI of R_1 with R_2

— Which action must the robot perform? The robot with higher priority action must choose between two actions: path deviation or velocity control.

B. Velocity control

When the robots are in a collision course, the robot with higher priority action (say R_k) must increase or decrease its velocity to avoid the collision. The switching control law satisfies (39), with $0 < v_k^{des} < v_{max}$, v_{max} is the robots maximum speed. k_v is a positive constant that depends on the dynamics constraints of the robots. In fact, k_v accounts for the linear acceleration of the robots, and thus it gives how fast the robot speed will reach its asymptotically stable desired value. Clearly, v_k^{des} is calculated so that

$$v_k^{des} \notin [v_k^a, v_k^b] \tag{27}$$

Robot R_k slows down when $v_k^{des} < v_k^a$, and accelerates when $v_k^{des} > v_k^b$.

C. Path deviation

The other approach to avoid collision is to use path deviation. When the collision course is detected, the robot with higher priority action deviates from its nominal path. The orientation angle command satisfies (28) where θ_k^{des} is the orientation angle that allows avoiding the collision. k_θ depends on the robot turning radius and must be chosen carefully to give the appropriate convergence to the desired value of k_θ. The desired value θ_k^{des} is calculated as follows

$$\theta_k^{des} \notin [\theta_k^a, \theta_k^b] \tag{28}$$

D. Reciprocal action

Reciprocal collision avoidance has been suggested recently [28]. Under this approach, both robots perform the same reasoning to avoid collision. For example if robot R_i deviates 5 degrees to the right, then R_k also deviates 5 degrees to the right to avoid collision.

E. Controlling the limit speeds and angles

As it can be seen from equations (19), (20), (22), and (23), the collision limit orientation angles θ_k^a and θ_k^b and speeds v_k^a and v_k^b depend on the speeds and orientation of R_i (i.e. θ_i, v_i). One possibility to avoid collision is by changing SCI_k or OCI_k. This can be easily accomplished by changing θ_i or v_i so that (27) or (28) are satisfied. VI.

VI. Simulation Example

Four robots are moving in this scenario. All robots are moving at the same speed but different orientations. Their initial positions are $R_1(0,0)$, $R_2(-24,-24)$, $R_3(-30,10)$, $R_4(-10,-10)$. It turns out that there is a collision course between R_1 and R_2. In order to avoid collision, R_1 changes its orientation using the OCI approach as shown on figure 8.

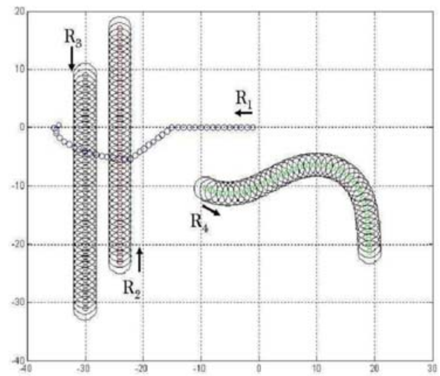

Fig. 8. An illustration of collision avoidance using the OCI

VII. Conclusion

We described a model-based strategy for path planning in multiagent systems. An integrated collision detection and navigation algorithm is suggested. The collision detection algorithm uses an improved version of the swept volume method, where the swept volume method is combined with the relative kinematics equations. Global path planning is accomplished by using linear navigation laws. The collision is characterized by the speed and orientation collision intervals, from which collision histograms are constructed. The appropriate values for the speeds and orientations are chosen from the collision histograms. Simulation is used to illustrate the method.

References

[1] R. Emery, K. Sikorski, and T. Balch, Protocols for collaboration, coordination and dynamic role assignment in robot team, in Proc. IEEE International Conference on Robotics and Automation, Washington DC, USA, May 2002, pp. 30083015.

[2] C. Castelpietra, L. Icchi, D. Nadri, M. Piaggio, A. Scalzo, and A. Sgorbissa, Coordination among heterogeneous robotic soccer players, in Proc. IEEE/RSJ International Conference on Intellligent Robots and Systems, Takamatsu, Nov. 2000, pp. 13851390.

[3] M. Mazo, A. Speranzon, K. Johansson, and X. Hu, Multi-robot tracking of a moving object using directional sensors, in Proc. IEEE International Conference Robotics and Automation, Apr. 2004, pp. 11031108.

[4] M. Hajjawi and A. Shirkhodaie, Cooperative visual team working and target tracking of mobile robots, in Proc. Southeastern Symposium on System Theory, Mar. 2002, pp. 376380.

[5] T. Balch and R. Arkin, Behavior-based formation control for multirobot teams, IEEE Transactions on Robotics and Automation, vol. 14, no. 6, pp. 926938, 1998.

[6] D. Dobroczynski, Multiagent behavioral control system for a group of mobile robots, in Proc. Second Workshop on Robot Motion and Control, Oct. 2001, pp. 115119.

[7] J. Fredslund and M. Mataric, A general algorithm for robot formation using local sensing and minimal communication, IEEE Transactions on Robotics and Automation, vol. 18, no. 5, pp. 837845, 2002.

[8] J. Desai, J. Ostrowski, and V. Kumar, Modeling and control of formations of nonholonomic mobile robots, IEEE Transactions on Robotics and Automation, vol. 17, no. 6, pp. 905908, 2001.

[9] Z. Jin and R. Murray, Double-graph control strategy of multi-vehicle formations, in Proc. IEEE International Conference on Decision and Control, Atlantis, Bahamas, Dec. 2004, pp. 19881994.

[10] W. Dunbar and R. Murray, Receding horizon control of multi-vehicle formations: A distributed implementation, in Proc. IEEE International Conference on Decision and Control, Atlantis, Bahamas, Dec. 2004, pp. 19952001.

[11] P. Ogren, M. Egerstedt, and X. Hu, A control lyapunov function approach to multiagent coordination, IEEE Transactions on Robotics and Automation, vol. 18, no. 5, pp. 847851, 2002.

[12] A. Guez, A multiple objectives optimization approach to robotic teams anlysis, design and control, in Proc. International Conference On Integration of Knowledge Intensive Multi-Agent Systems, Boston, USA, Sept. 2003.

[13] C. Balta and V. Kumar, Motion generation for formations of robots: a geometric approach, in Proc. IEEE International Conference on Robotics and Automation, Seoul, Korea, May 2001, pp. 12451249.

[14] D. Yingying, H. Yan, and J. Jingping, Multi-robot cooperation method based on the ant algorithm, in Proc. IEEE Swarm Intelligence Symposium, Indiana, USA, Apr. 2003, pp. 1418.

[15] H. Asama, Operation of cooperative multiple robots using communication in decentralized robotic system, in Proc. From Perception to Action Conference, Lausanne, Suisse, Sept. 1994, pp. 3646.

[16] I. Hussein and A. Bloch, Dynamic coverage optimal control for interferometric imaging spacecraft formations, in Proc. IEEE International Conference on Decision and Control, Atlantis, Bahamas, Dec. 2004, pp. 19821986.

[17] M. Batalin and G. Sukhatme, "Dynamic coverage via multi-robot cooperation," in Proc. Multi-robot Systems Workshop, Washington DC, USA, Mar. 2003, pp. 295296.

[18] H. Yamaguchi, A cooperative hunting behavior by mobile-robot troops, The internatinal Journal of Robotics Research, vol. 18, no. 8, pp. 931 940, 1999.

[19] A. Sgorbissa and R. C. Arkin, Local navigation strategies for a team of robots, Robotica, vol. 21, pp. 461473, 2003.

[20] A. Fujimori, M. Teramoto, P. Nikiforuk, and M. M. Gupta, Cooperative collision avoidance between multiple mobile robots, Journal of Robotic Systems, vol. 17, no. 7, pp. 347363, 2000.

[21] A.Fujimori and S. Tani, A navigation of mobile robots with collision avoidance for moving obstacles, in Proc. IEEE International Conference on Industrial Technology, Bangkok, Thailand, Dec. 2002, pp. 16.

[22] C. Byrnes and A. Isidory, New results and examples in nonlinear feedback stabilization, Systems and Control Letters, vol. 12, pp. 437 442, 1989.

[23] S. Drakunov, D. Izosimov, A. Lukjanov, V. Utkin, and V. Utkin, Block control principle, Automation and Remote Control, vol. 51, no. 6, pp. 737746, 1991.

[24] S. Drakunov, D. Izosimov, A. Lukjanov, A. Utkin, and V. Utkin, Block control principle, Automation and Remote Control, vol. 51, no. 5, pp. 601608, 1991.

[25] K. Bendjilali and F. Belkhouche, Kinematics-based navigation functions, Advanced Robotics, vol. 22, no. 11, pp. 12431264, 2008.

[26] P. Xavier, Fast swept- volume distance for robust collision detection, in Proc. IEEE International Conference on Robotics and Automation, Albuquerque, NM, USA, Apr. 1997, pp. 11621169.

[27] J. Kieffer and F. Litvin, Swept volume determination and interference detection for moving 3-d solids, Transactions of the ASME, Journal of Mechanical Design, vol. 113, pp. 456463, 1990.

[28] J. van den Berg, M. Lin, and D. Manocha, Reciprocal velocity obstacles for real-time multi-agent navigation, in Proc. IEEE International Conference on Robotics and Automation, Pasadena, California, May 2008, pp. 1928 1935.