Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

LMI-Based H_∞ State-Feedback Control for T-S Time-Delay Discrete Fuzzy Bilinear System

*Shun-Hung Tsai**
Graduate Institute of Automation Technology
National Taipei University of Technology
Taipei, Taiwan 10608, R.O.C.
shtsai@ntut.edu.tw

Ming-Ying Hsiao
Department of Electrical Engineering
Fortune Institute of Technology
Kaohsiung, Taiwan 831, R.O.C.
myhsiao@center.fotech.edu.tw

Tzuu.-Hseng S. Li, Kai.-Shiuan Shih, and Chan.-Hong Chao
Department of Electrical Engineering
National Cheng Kung University
Tainan, Taiwan 701, R.O.C.
thsli@mail.ncku.edu.tw
n2894115@mail.ncku.edu.tw
n2890109@mail.ncku.edu.tw

Chi-Hua Liu
General Education Center
Fortune Institute of Technology
Kaohsiung, Taiwan 831, R.O.C.
chliu@center.fotech.edu.tw

***Abstract*—This paper presents a robust H_∞ fuzzy controller for a class of T-S time-delay discrete fuzzy bilinear system (DFBS). Firstly, a discrete robust H_∞ fuzzy controller is proposed to stabilize the T-S time-delay DFBS with disturbance. Secondly, based on the Schur complement and some variable transformation, the stability conditions of the overall fuzzy control system are formulated by linear matrix inequalities (LMIs). Finally, a numerical example is utilized to demonstrate the validity and effectiveness of the proposed control scheme.**

***Keywords*—Discrete fuzzy bilinear system (DFBS), robust control, linear matrix inequalities (LMIs).**

I. INTRODUCTION

In recent years, bilinear systems and controls have been widely applied to a wide variety of fields, for example, bioengineering, biochemistry, nuclear engineering and socio-economics [1]-[5]. There are two advantages of using bilinear models to describe nonlinear models. One advantage is that bilinear systems are an adequate approximation than linear models for some real-world systems, including engineering applications in nuclear, thermal, and chemical processes, and many other nonengineering applications in biology, socioeconomics, and immunology [1], [2]. The other advantage is that many real physical processes may be appropriately modeled as bilinear systems when linear models are inadequate, for example, the population of biological specie [1]. For these two reasons, bilinear system is therefore important to design its controller, to investigate the stability, and to improve performance by applying various control techniques[1]–[3].

Besides bilinear systems, it is well known that fuzzy control has been attracting increasing attention to the stabilization of nonlinear systems [4]–[8]. Fuzzy control has been successfully applied to controller designs for nonlinear systems [5], [6], [8], [9]. In most of these papers, a so-called Takagi–Sugeno (T-S) fuzzy model is adopted to approximate a nonlinear plant, and then a fuzzy controller is designed via the parallel distributed compensation (PDC) scheme to stabilize the T-S fuzzy model. For stability analysis, LMI techniques [4], [7], [9], [10] have been developed to find solutions for the T-S fuzzy control system under consideration.

It is known that time-delay phenomenon commonly exists in dynamic systems due to measurement, transmission, transport lags and computational delays. Therefore, stabilization of time-delay systems is increasing being attentions in the literature [7], [9]. Recently, the T-S fuzzy model approach has been shown effective and powerful in dealing with nonlinear systems with time-delays. Besides, different methodologies have been also proposed for analysis and synthesis of T-S fuzzy systems with time-delay [8]–[11].

The main contributions of this paper are i) designing a fuzzy H_∞ controller for the T-S DFBS with disturbance; ii) investigating a robust H_∞ fuzzy controller for the T-S Time-Delay DFBS with disturbance; and iii) Describing all the stability conditions for the T-S Time-Delay DFBS with disturbance via LMIs.

This paper is organized as follows. In Section II, the DFBS is established and its fuzzy controller is also designed. The controller design method for robust H_∞ stabilization of the T-S DFBS with disturbance is derived in Section III. A numerical simulation is illustrated in Section IV. Finally, conclusions are given in Section V.

II. SYSTEM DESCRIPTION AND CONTROLLER DESIGN

The T-S fuzzy dynamic model is described by fuzzy IF-THEN rules, which locally represent input-output relations of nonlinear systems. Similar to [7, 10], the i th rule of the time-delay T-S DFBS with disturbance can be represented by the following form:

Plant Rule i :

IF $s_1(k)$ is F_{j1} and … and $s_g(k)$ is F_{jg}

THEN $x(k+1)=(A_i+\Delta A_i)x(k)+B_iu(k)+N_ix(k)u(k)$

$$+A_{di}x(k-\tau)+D_{1i}\omega(k) \quad (1a)$$

$$z(k)=C_ix(k)+D_{2i}\omega(k),\ i=1,2\cdots,r \quad (1b)$$

where k is iteration instant, F_{ij} is the fuzzy set and r is the number of IF-THEN rules; $x(k)\in R^{n\times 1}$ is the state, $u(k)\in R$ is the control input; $z(k)\in R^q$ is the controlled output; $\omega(k)\in R^{m\times 1}$ is the disturbance input which is belong to $l_2[0,+\infty)$; $s_1(k), s_2(k),\cdots s_g(k)$, are the premise variables. $A_i\in R^{n\times n}$, $B_i\in R^{n\times 1}$, $N_i\in R^{n\times n}$, $A_{di}\in R^{n\times n}$, $D_{1i}\in R^{n\times m}$, $C_i\in R^{1\times n}$, $D_{2i}\in R^{1\times m}$, and $\Delta A_i\in R^{n\times n}$ is real-valued unknown matrices representing time-varying parameter uncertainty, and is assumed to be of the form, in the following Assumption.

Assumption 1 [9]: The parameter uncertainties considered here are norm-bounded and presented by the form $\Delta A_i=G_iF_i(k)E_{1i}$, where G_i and E_{1i}, are known real constant matrices of appropriate dimensions, and $F_i(k)$ is an unknown matrix function with Lebesgue-measurable elements and satisfies $F_i^T(k)F_i(k)\le I$. The parameter uncertainty ΔA_i is said to be admissible if Assumption 1 hold.

The time-delay $\tau(t)$ may be unknown but is assumed to be a smooth function of time as consideration in [8] and [11],

$$\dot{\tau}(t)\le\alpha<1,\ \tau(t)\le\tau_0. \quad (2)$$

Similar to [7], [11], using a center average defuzzifier, product inference, and singleton fuzzifier, the time-delay DFBS with disturbance input (3) can be expressed at the bottom of next page, where $\mu_i(s(k))=\prod_{j=1}^{g}F_{ij}(s(k))$, $h_i=h_i(s(k))=\mu_i(s(k))/\sum_{j=1}^{r}\mu_j(s(k))$, $F_{ij}(s_j(k))$ is the grade of membership of $s_j(k)$ in F_{ij}, and $s(k)=\begin{bmatrix}s_1(k) & s_2(k) & \cdots & s_g(k)\end{bmatrix}^T$. Basic properties of $\mu_i(s(k))$ are $\mu_i(s(k))\ge 0$ and $\sum_{i=1}^{r}\mu_i(s(k))>0$. It is clear that $h_i(s(k))\ge 0$ and $\sum_{j=1}^{r}h_j(s(k))=1$.

Following the design concept in [7], the fuzzy controller for the T-S time-delay DFBS (2) is formulated as follows:

Control Rule j:

IF $s_1(k)$ is F_{j1} and … and $s_g(k)$ is F_{jg}

THEN $u(k)=\dfrac{\rho D_jx(k)}{\sqrt{1+x^T(k)D_j^TD_jx(k)}}$, $j=1,2\cdots,r$. (4)

The overall fuzzy control law can be represented by

$$u(k)=\sum_{j=1}^{r}h_j(s(k))\rho D_jx(k)/\sqrt{1+x^T(k)D_j^TD_jx(k)}$$

$$=\sum_{j=1}^{r}h_j(s(k))\rho\sin\theta_j \quad (5a)$$

$$=\sum_{j=1}^{r}h_j(s(k))\rho D_jx\cos\theta_j \quad (5b)$$

where $\sin\theta_j=D_jx(k)/\sqrt{1+x^T(k)D_j^TD_jx(k)}$,

$\cos\theta_j=1/\sqrt{1+x^T(k)D_j^TD_jx(k)}$, $\theta_j\in[-\pi/2\quad \pi/2]$,

$D_j\in R^{1\times n}$ is a vector, and $\rho>0$ is a scalar to be assigned.

The control objective is to design a T-S fuzzy H_∞ controller (5) to stabilize the T-S time-delay DFBS with disturbance (3). Substituting (5) into (3), one can get the closed-loop system,

$$x(k+1)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\Big(((A_i+\Delta A_i)+\rho N_i\sin\theta_j$$
$$+\rho B_iD_j\cos\theta_j)x(k)+A_{di}x(t-\tau(k))+D_{1i}\omega(k)\Big) \quad (6a)$$

$$z(k)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\left[C_ix(k)+D_{2i}\omega(k)\right]. \quad (6b)$$

Then the robust fuzzy H_∞ control problem to be addressed in this paper can be formulated as follows: give an fuzzy system (3) and a scalar $\gamma>0$, determine a fuzzy controller such that

R1) The closed-loop system (6) is robust asymptotically stable when $\omega(k)=0$.

R2) Under the zero initial condition, the controlled output $z(k)$ satisfies

$$\|z(k)\|_2<\gamma\|\omega(k)\|_2 \quad (7)$$

for all nonzero $\omega(k)\in l_2[0,+\infty)$ and all admissible uncertainties.

III. MAIN RESULT

In this section, an LMI approach will be developed to solve the problem of robust H_∞ feedback control of fuzzy system formulated in the previous section. Before discussing the proof of the theorem, we first give the following lemma which will be used in the proof of our main results.

Lemma 1: [7], [9]: Given any matrices X, Y and ε with appropriate dimensions such that $\varepsilon>0$ we have

$$X^TY+XY^T\le\varepsilon X^TX+\varepsilon^{-1}Y^TY. \quad (8)$$

Theorem 1: For the T-S time-delay DFBS with disturbance (1), there exists a robust H_∞ fuzzy feedback controller (5) such that the closed-loop system (6) is robust asymptotically stable and (7) is satisfied if there exist symmetric and positive definite matrices P and S, a scalar ρ, some vectors D_i and some scalars ε_{gij}, $g=1,\ldots,5$, $i,j=1,\ldots,r$, satisfying the following LMIs (9) and (10), shown at the bottom of this page.

Proof: Under the conditions of the theorem, we first establish the robust asymptotic stability of the system in (6). For this purpose, we consider (6) with $\omega(k)=0$; that is

$$x(k+1)=\sum_{i=1}^{r}h_i(s(k))\big((A_i+\Delta A_i)x(k)+B_iu(k)+N_ix(k)u(k)+A_{di}x(k-\tau(k))\big) \quad (11)$$

Now, we choose a Lyapunov function candidate for this system as follows:

$$V(x(k))=x(k)^TPx(k)+\sum_{\sigma=k-\tau}^{k-1}x^T(\sigma)Sx(\sigma). \quad (12)$$

Then it can be verified that

$$\Delta V=V(x(k+1))-V(x(k))$$

$$=x^T(k+1)Px(k+1)+x^T(k)(S-P)x(k)$$
$$-x^T(k-\tau(k))Sx(k-\tau(k)). \quad (13)$$

By substituting (11) into (13), we can get (14), shown at the bottom of next two page,

where $\Xi_{ij}=(((A_i+\Delta A_i)+(A_j+\Delta A_j)\rho B_iD_j\cos\theta_j+\rho N_i\sin\theta_j$
$+\rho B_jD_i\cos\theta_i+\rho N_j\sin\theta_i)x(k)$
$+A_{di}x(k-\tau(k))+A_{dj}x(k-\tau(k)))$.

$$x(k+1)=\sum_{i=1}^{r}h_i(s(k))\big((A_i+\Delta A_i)x(k)+B_iu(k)+N_ix(k)u(k)+A_{di}x(k-\tau(k))+D_{1i}\omega(k)\big) \quad (3)$$

$$LMI1=\begin{bmatrix} LMI1_1 & * \\ LMI1_2 & LMI1_3 \end{bmatrix}<0 \quad (1\le i\le r) \quad (9)$$

$$LMI1_1=\begin{bmatrix} S-P & * & * & * & * \\ PA_i & \frac{-1}{(\rho^2+1+\varepsilon_{5ii})}P & * & * & * \\ 0 & 0 & (\rho^2\varepsilon_{1ii}^{-1}+1+\varepsilon_{5ii}^{-1})A_{di}^TPA_{di}-S & * & * \\ PB_iD_i & 0 & 0 & \frac{-1}{(\rho^2+1+\varepsilon_{1ii}+\varepsilon_{2ii})}P & * \\ PN_i & 0 & 0 & 0 & \frac{-1}{(\rho^2+1+\varepsilon_{1ii}+\varepsilon_{2ii})}P \end{bmatrix}$$

$$LMI1_2=\begin{bmatrix} 0 & D_{1i}^TPA_{di} & 0 & 0 & 0 & -I \\ C_i & 0 & 0 & 0 & 0 & D_{2i} \\ E_{1i} & 0 & 0 & 0 & 0 & 0 \\ 0 & G_i^T & 0 & 0 & 0 & 0 \end{bmatrix},\ LMI1_3=diag\left[-(\gamma^2I-(\rho^2\varepsilon_{2ii}^{-1}+1+\rho^2\varepsilon_{4ii}^{-1})D_{1i}^TD_{1i}) \quad -\varepsilon_{3ii}I \quad -\varepsilon_{3ii}^{-1}I\right]$$

$$LMI2=\begin{bmatrix} LMI2_1 & * \\ LMI2_2 & LMI2_3 \end{bmatrix}<0 \quad (1\le i<j\le r) \quad (10)$$

$$LMI2_1=\begin{bmatrix} 4(S-P) & * & * & * & * & * & * & * \\ PA_i & \frac{-1}{2(2\rho^2+1+\varepsilon_{5ij})}P & * & * & * & * & * & * \\ PA_j & 0 & \frac{-1}{2(2\rho^2+1+\varepsilon_{5ij})}P & * & * & * & * & * \\ 0 & 0 & 0 & \begin{matrix}-S+(2\rho^2\varepsilon_{1ij}^{-1}+1+\varepsilon_{5ij}^{-1})\\ \times(A_{di}+A_{dj})^TP(A_{di}+A_{dj})\end{matrix} & * & * & * & * \\ PB_iD_j & 0 & 0 & 0 & \frac{-1}{(\rho^2+1+\varepsilon_{1ij})}P & * & * & * \\ PN_i & 0 & 0 & 0 & 0 & \frac{-1}{(\rho^2+1+\varepsilon_{1ij})}P & * & * \\ PB_jD_i & 0 & 0 & 0 & 0 & 0 & \frac{-1}{(\rho^2+1+\varepsilon_{1ij})}P & * \\ PN_j & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1}{(\rho^2+1+\varepsilon_{1ij})}P \end{bmatrix},$$

$$LMI2_2=\begin{bmatrix} 0 & (D_{1i}+D_{1j})^TP(A_{di}+A_{dj}) & 0 & 0 & 0 & 0 & 0 & 0 & -I \\ (C_i+C_j) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & (D_{1i}+D_{1j}) \\ E_{1i} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ E_{1j} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & G_i^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & G_j^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$LMI2_3=diag\left[(4\gamma^2I-\tfrac{1}{(2\rho^2\varepsilon_{2ij}^{-1}+\varepsilon_{4ij}^{-1}+1)^{-1}}(D_{1i}+D_{1j})^T(D_{1i}+D_{1j})) \quad -\varepsilon_{3ij}I \quad -\varepsilon_{3ij}I \quad -\varepsilon_{3ij}^{-1}I \quad -\varepsilon_{3ij}^{-1}I\right]$$

Clearly, if (13) is negative definite uniformly for all nonzero $x(k)$ and $k \ge 0$, then the controlled fuzzy system (11) is robustly asymptotically stable.

At first, we assume that the first sum of the equation in (14) is negative definite, that is,

$$\left[\left((A_i+\Delta A_i)+\rho B_i D_i \cos\theta_i+\rho N_i \sin\theta_i\right)x+A_{di}x(k-\tau)\right]^T P$$
$$\times\left[\left((A_i+\Delta A_i)+\rho B_i D_i \cos\theta_i+\rho N_i \sin\theta_i\right)x+A_{di}x(k-\tau)\right]$$
$$+x^T(S-P)x-x^T(k-\tau)Sx(k-\tau)<0 \qquad (15)$$

Using the Lemma 1, then (15) can be rewritten as $x^T(k)\left[(\rho^2+1+\varepsilon_{5ii})(A_i+\Delta A_i)^T P(A_i+\Delta A_i)\right.$

$$\left.+(\rho^2+1+\varepsilon_{1ii})(D_i^T B_i^T P B_i D_i+N_i^T P N_i)+S-P\right]x(t)$$
$$+x^T(k-\tau(k))((\rho^2\varepsilon_{1ii}^{-1}+1+\varepsilon_{5ii}^{-1})A_{di}^T P A_{di}-S)x(k-\tau(k))<0 \quad (16)$$

Applying the Schur complement to (16) results in (17), shown at the bottom of next page.

In order to use the convex optimization technique, we utilize the Assumption 1, and adopt the transformation matrix: $diag\left[I \quad P \quad I \quad P \quad P\right]$ and take a congruence transformation [7]. Through these procedures, then (18) can be yield, which is shown at the bottom of next page. From (9), it is easy to see that for $1 \le i \le r$, if (9) is held then (18) is negative.

Similarly, we assume that the second sum of the equation in (14) is negative definite and through the same procedure, then we can obtain (19), shown at the next page, From (10), it is easy to see that for $1 \le i \le r$, if (9) is held then (19) is negative.

Next, we will show that for any nonzero $\omega(k) \in l_2\left[0,+\infty\right)$ in the T-S time-delay DFBS with disturbance (6) satisfies (7) under zero initial condition. For this purpose, we introduce

$$J_N=\sum_{k=0}^{N}\left[z^T(k)z(k)-\gamma^2\omega^T(k)\omega(k)\right] \qquad (20)$$

where $N \in \mathbf{N}$. Substituting (6) into (20) with the zero initial condition, we can deduce

$$J_N=\sum_{k=0}^{N}\left[z^T(k)z(k)-\gamma^2\omega^T(k)\omega(k)\right]$$
$$+\left[V(x(k+1))-V(x(k))\right]\}-V(x(N+1))$$
$$\le\sum_{k=0}^{N}\left\{z^T(k)z(k)-\gamma^2\omega^T(k)\omega(k)+\left[V(x(k+1))-V(x(k))\right]\right\}$$
$$=\sum_{k=0}^{N}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{m=1}^{r}h_i h_j h_l h_m\left((C_i x+D_{2i}\omega)^T(C_l x+D_{2l}\omega)\right.\right.$$
$$+\left[((A_i+\Delta A_i)+\rho B_i D_j\cos\theta_j+\rho N_i\sin\theta_j)x(k)\right.$$
$$\left.+x(k-\tau(k))+D_{1i}\omega(k)\right]^T P$$
$$+\left[((A_l+\Delta A_l)+\rho B_l D_m\cos\theta_m+\rho N_l\sin\theta_m)x(k)\right.$$
$$\left.+x(k-\tau(k))+D_{1l}\omega(k)\right]$$
$$\left.-x^T(k)Px(k)-\gamma^2\omega^T(k)\omega(k)\right\} \qquad (21)$$

where $V(x(k))$ is given in (12). By Lemma 1, we have

$$\Delta V(x(k))+z^T(k)z(k)-\gamma^2\omega^T(k)\omega(k)$$
$$\le\sum_{i=1}^{r}h_i^2\xi^T(k)\Theta_{ii}\xi(k)+\frac{1}{2}\sum_{i<j}^{r}h_i h_j\xi^T(k)\Theta_{ij}\xi(k). \qquad (22)$$

Where $\xi(k)=\left[x(k) \quad x(k-\tau) \quad \omega(k)\right]$, Θ_{ii} and Θ_{ij} is shown at the bottom of next two page.

For obtaining the (9) and (10), we assume the (22) is negative definite. Firstly, we consider the (23) and utilize the Lemma 1 then (25), shown at the bottom of next two page, which can be obtained. Applying the Schur Complement, Assumption 1, and Lemma 1, then the (9) can be obtained from (25). Through a similar procedure, the (10) can also be obtained. Thus $J_N<0$, which implies $\|z(k)\|_2<\gamma\|\omega(k)\|_2$ for all nonzero $\omega(k)\in l_2\left[0,+\infty\right)$, if (9) and (10) are satisfied. This completes the proof of the theorem. Q.E.D.

IV. EXAMPLE

In this section, we apply the proposed method to design a robust H_∞ fuzzy controller for a T-S time-delay DFBS with disturbance. The T-S time-delay DFBS with disturbance is described as follows:

Rule 1: IF x_1 is about -1

THEN $x(k+1)=(A_1+\Delta A_1)x(k)+B_1u(k)+N_1x(k)u(k)$

$$+A_{d1}x(k-\tau(k))+D_{11}\omega(k) \qquad (26a)$$
$$z(k)=C_1x(k)+D_{21}\omega(k)$$

Rule 2: IF x_1 is about 1

THEN $x(k+1)=(A_2+\Delta A_2)x(k)+B_2u(k)+N_2x(k)u(k)$

$$+A_{d2}x(k-\tau(k))+D_{12}\omega(k) \qquad (26b)$$
$$z(k)=C_2x(k)+D_{22}\omega(k)$$

where $A_1=\begin{bmatrix}-0.4 & 0.06\\ 0.3 & -0.25\end{bmatrix}$, $A_2=\begin{bmatrix}0.38 & 0.08\\ -0.4 & 0.1\end{bmatrix}$,

$A_{d1}=\begin{bmatrix}0.12 & 0.1\\ 0.1 & 0.2\end{bmatrix}$, $A_{d2}=\begin{bmatrix}0.1 & 0.09\\ 0.4 & 0.05\end{bmatrix}$, $N_1=\begin{bmatrix}0.02 & 0\\ 0 & -0.03\end{bmatrix}$, $N_2=\begin{bmatrix}-0.01 & 0.02\\ 0 & -0.008\end{bmatrix}$, $B_1=\begin{bmatrix}0.3\\ -0.01\end{bmatrix}$, and $B_2=\begin{bmatrix}0.2\\ 0.08\end{bmatrix}$.

Based on Assumption 1, G_i, F_i, and E_{1i} are defined as follows, $G_i=\begin{bmatrix}0.3 & 0\\ 0 & 0.3\end{bmatrix}$, $F_i(t)=\begin{bmatrix}rand & 0\\ 0 & rand\end{bmatrix}$, $i=1,2$, where *rand* is a random number taken from a normal distribution over $[-1,1]$, $E_{11}=\begin{bmatrix}0.3 & 0\\ 0.3 & 0\end{bmatrix}$ and $E_{12}=\begin{bmatrix}0.2 & 0.2\\ 0 & 0.1\end{bmatrix}$. The membership functions of the state x_1 are defined as $\mu_1=0.5(1+x_1)$ and $\mu_2=0.5(1-x_1)$. Let $\rho=0.3$ and choose the controller gain matrices as $D_1=[-0.1 \quad -0.4]$ and $D_2=[-0.4 \quad -0.1]$.

In this example, we choose the H_∞ performance level $\gamma=1$ and applying ρ and all these matrices to inequalities (9) and (10) in Theorem 1, and utilizing the LMI tool box [7], one can figure out the common positive-definite matrices,

$P=\begin{bmatrix}19.649 & 5.0093\\ 5.0093 & 12.0597\end{bmatrix}$ and $S=\begin{bmatrix}7.3641 & 5.0042\\ 5.0042 & 6.7715\end{bmatrix}$.

The simulation results of applying the robust H_∞ fuzzy controller (5) to T-S time-delay DFBS with disturbance (26) with three different initial conditions $x(0)=[1.5 \quad -2]^T$, $x(0)=[-1.5 \quad -1.2]^T$, and $x(0)=[-1.3 \quad 1.2]^T$ and the exogenous $\omega(t)=\left[0.5e^{-0.001}\sin(k) \quad 0.3e^{-0.005t}\cos(2k)\right]^T$ are illustrated in Figure 1.

$$
\begin{aligned}
\Delta V &= V(x(k+1))-V(x(k))\\
&=\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{m=1}^{r}h_ih_jh_lh_m\Big\{\big(((A_i+\Delta A_i)+\rho B_iD_j\cos\theta_j+\rho N_i\sin\theta_j)x(k)+A_{di}x(k-\tau(k))\big)^T P\\
&\quad\times\big(((A_l+\Delta A_l)+\rho B_lD_m\cos\theta_m+\rho N_l\sin\theta_m)x(k)+A_{dl}x(k-\tau(k))\big)\Big\}+x^T(k)(S-P)x(k)-x^T(k-\tau(k))Sx(k-\tau(k))\\
&\le\sum_{i=1}^{r}h_i^2x^T(k)\Big\{\big[\big((A_i+\Delta A_i)+\rho B_iD_i\cos\theta_i+\rho N_i\sin\theta_i\big)x(k)+A_{di}x(k-\tau(k))\big]^T P\\
&\qquad\times\big[\big((A_i+\Delta A_i)+\rho B_iD_i\cos\theta_i+\rho N_i\sin\theta_i\big)x(k)+A_{di}x(k-\tau(k))\big]+x^T(k)(S-P)x(k)-x^T(k-\tau(k))Sx(k-\tau(k))\Big\}\\
&\quad+\frac{1}{2}\sum_{i<j}^{r}h_ih_j\,[\Xi_{ij}^TP\Xi_{ij}+4x^T(k)(S-P)x(k)-4x^T(k-\tau(k))Sx(k-\tau(k))]
\end{aligned}\tag{14}
$$

$$
\begin{bmatrix}
S-P & * & * & * & *\\
(A_i+\Delta A_i) & -(\rho^2+1+\varepsilon_{5ii})^{-1}P^{-1} & * & * & *\\
0 & 0 & (\rho^2\varepsilon_{1ii}^{-1}+1+\varepsilon_{5ii}^{-1})A_{di}^TPA_{di}-S & * & *\\
B_iD_i & 0 & 0 & -(\rho^2+1+\varepsilon_{1ii})^{-1}P^{-1} & *\\
N_i & 0 & 0 & 0 & -(\rho^2+1+\varepsilon_{1ii})^{-1}P^{-1}
\end{bmatrix}<0 \quad (1\le i\le r)\tag{17}
$$

$$
\begin{bmatrix}
S-P & * & * & * & * & * & *\\
PA_i & -(\rho^2+1+\varepsilon_{5ii})^{-1}P & * & * & * & * & *\\
0 & 0 & (\rho^2\varepsilon_{1ii}^{-1}+1+\varepsilon_{5ii}^{-1})A_{di}^TPA_{di}-S & * & * & * & *\\
PB_iD_i & 0 & 0 & -(\rho^2+1+\varepsilon_{1ii})^{-1}P & * & * & *\\
PN_i & 0 & 0 & 0 & -(\rho^2+1+\varepsilon_{1ii})^{-1}P & * & *\\
E_{1i} & 0 & 0 & 0 & 0 & -\varepsilon_{3ii}I & *\\
0 & G_i^T & 0 & 0 & 0 & 0 & -\varepsilon_{3ii}^{-1}I
\end{bmatrix}<0\tag{18}
$$

$$
lmi=\begin{bmatrix}lmi\_1 & *\\ lmi\_2 & lmil\_3\end{bmatrix}<0 \quad (1\le i<j\le r)\tag{19}
$$

$$
lmi\_1=\begin{bmatrix}
4(S-P) & * & * & *\\
PA_i & -2(2\rho^2+1+\varepsilon_{5ij})^{-1}P & * & *\\
PA_j & 0 & -2(2\rho^2+1+\varepsilon_{5ij})^{-1}P & *\\
0 & 0 & 0 & -S+(2\rho^2\varepsilon_{1ij}^{-1}+1+\varepsilon_{5ij}^{-1})(A_{di}+A_{dj})^TP(A_{di}+A_{dj})
\end{bmatrix},
$$

$$
lmi\_2=\begin{bmatrix}
(PB_iD_j)^T & (PN_i)^T & (PB_jD_i)^T & (PN_j)^T & (E_{1i})^T & (E_{1j})^T & 0 & 0\\
0 & 0 & 0 & 0 & 0 & 0 & G_i & 0\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & G_j\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}^T,
$$

$$
lmi\_3=-diag\left[(\rho^2+1+\varepsilon_{1ij})P \quad (\rho^2+1+\varepsilon_{1ij})P \quad (\rho^2+1+\varepsilon_{1ij})P \quad (\rho^2+1+\varepsilon_{1ij})P \quad \varepsilon_{3ij}I \quad \varepsilon_{3ij}I \quad \varepsilon_{3ij}^{-1}I \quad \varepsilon_{3ij}^{-1}I\right].
$$

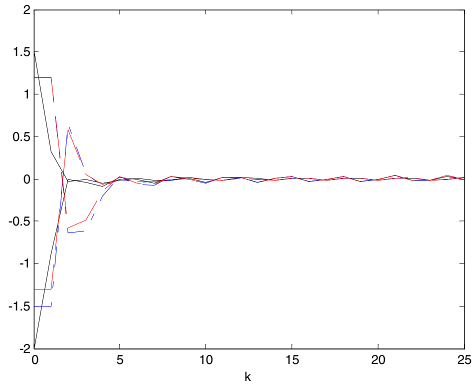

Figure 1. State responses of the of the T-S time-delay DFBS with disturbance.

From these simulation results, it can be seen the designed fuzzy controller ensures the robust asymptotic stability of the closed-loop system and guarantees a prescribed H_∞ performance level.

V. Conclusion

This paper has proposed a robust H_∞ fuzzy control scheme for a class of T-S time-delay DFBS with disturbance. For stabilizing the T-S time-delay DFBS, some sufficient conditions have been derived to guarantee the stability of the overall fuzzy control system via LMIs. Finally, the design approach has been applied to a numerical example to illustrate the validity and effectiveness of the proposed schemes.

Acknowledgment

This work was supported by the National Science Council of Taiwan, R.O.C., under Grant NSC97-2218-E-027-022.

References

[1] R. R. Mohler, "*Nonlinear systems: Vol. 2, Application to bilinear control*," Prentice-Hall, Englewood Cliffs, NJ, 1991.

[2] R. R. Mohler, *Bilinear Control Processes*. New York: Academic, 1973.

[3] R. Longchamp, "Stable feedback control of bilinear systems," *IEEE Trans. Automat. Contr.*, vol. 6, AC-25, pp. 37–45, Apr. 1980.

[4] T.-H. S. Li and S.-H. Tsai, "T-S fuzzy bilinear model and fuzzy controller design for a class of nonlinear systems," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 3, pp. 494–506, Jun. 2007.

[5] T.-H. S. Li, S.-H. Tsai, J.-Z. Lee, M.-Y. Hsiao, and C.-H. Chao, "Robust H_∞ fuzzy control for a class of uncertain discrete fuzzy bilinear systems," *IEEE Trans. Syst., Man, Cybern. Part B*. Vol. 38, No. 2, pp. 510–527, Apr. 2008.

[6] T.-H. S. Li, M.-Y. Hsiao, J.-Z. Lee, C.-H. Chao, and S.-H. Tsai, "Design of interval type-2 fuzzy sliding-mode controller," *Information Sciences*, Vol. 178, No. 6, pp. 1696–1716, March 2008.

[7] K. Tanaka and H. O. Wang, Fuzzy Control Systems Design and Analysis: *A Linear Matrix Inequality Approach*. New York: Wiley, 2001.

[8] S.-H. Tsai, and T.-H. S. Li "Robust fuzzy control of a class of fuzzy bilinear systems with time-delay," DOI:10.1016/j.chaos.2007.06.057, In Press, Corrected Proof, Available online 6 August 2007.

[9] X. Jiang and Q. L. Sano, "Robust H_∞ control for uncertain Takagi–Sugeno fuzzy systems with interval time-varying delay," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 2, pp. 321–331, Apr. 2007.

[10] X.-P Guan and C.-L Chen "Delay-dependent guaranteed cost control for T-S fuzzy systems with time delays" *IEEE Trans. Fuzzy Syst.*, vol. 14, pp. 386–397, Jun. 2006.

[11] B. Chen, X. P. Lin, S. C. Tong and C. Lin, "Observer-based stabilization of T-S fuzzy systems with input delay," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 3, pp. 652–663, Jun. 2008.

$$\Theta_{ii}=\begin{bmatrix} (\rho^2+1)(A_i+\Delta A_i)^T P(A_i+\Delta A_i)+S-P+C_i^T C_i \\ +(\rho^2+1+\varepsilon_{1ii}+\varepsilon_{2ii})(D_i^T B_i^T P B_i D_i+N_i^T P N_i) & * & * \\ A_{di}^T P(A_i+\Delta A_i) & (\rho^2\varepsilon_{1ii}^{-1}+1)A_{di}^T P A_{di}-S & * \\ D_{1i}^T P(A_i+\Delta A_i)+D_{2i}^T C_i & D_{1i}^T P A_{di} & -(\gamma^2 I-(\rho^2\varepsilon_{2ii}^{-1}+1)D_{1i}^T D_{1i}-D_{2i}^T D_{2i}) \end{bmatrix}<0 \quad (23)$$

$$\Theta_{ij}=\begin{bmatrix} (2\rho^2+1)(A_i+\Delta A_i+A_j+\Delta A_j)^T P(A_i+\Delta A_i+A_j+\Delta A_j)+(C_i+C_j)^T(C_i+C_j) \\ +(2\rho^2+1+\varepsilon_{1ij}+\varepsilon_{2ij})(D_j^T B_i^T P B_i D_j+N_i^T P N_i+D_i^T B_j^T P B_j D_i+N_j^T P N_j)+4(S-P) & * & * \\ (A_{di}+A_{dj})P(A_i+\Delta A_i+A_j+\Delta A_j) & (2\rho^2\varepsilon_{1ij}^{-1}+1)(A_{di}+A_{dj})^T P(A_{di}+A_{dj})-4S & * \\ (D_{1i}+D_{1j})^T P(A_i+\Delta A_i+A_j+\Delta A_j)+(D_{2i}+D_{2j})^T(C_i+C_j) & (D_{1i}+D_{2i})^T P(A_{di}+A_{dj}) & -(4\gamma^2 I-(2\rho^2\varepsilon_{2ij}^{-1}+1)(D_{1i}+D_{1j})^T(D_{1i}+D_{1j})-(D_{2i}+D_{2j})^T(D_{2i}+D_{2j})) \end{bmatrix}<0 \quad (24)$$

$$\begin{bmatrix} (\rho^2+1)(A_i+\Delta A_i)^T P(A_i+\Delta A_i)+S-P+C_i^T C_i \\ +(\rho^2+1+\varepsilon_{1ii}+\varepsilon_{2ii})(D_i^T B_i^T P B_i D_i+N_i^T P N_i) & * & * & * \\ 0 & (\rho^2\varepsilon_{1ii}^{-1}+1)A_{di}^T P A_{di}-S & * & * \\ 0 & D_{1i}^T P A_{di} & -(\gamma^2 I-(\rho^2\varepsilon_{2ii}^{-1}+1)D_{1i}^T D_{1i}) & * \\ C_i & 0 & D_{2i} & -I \end{bmatrix}<0 \quad (1\le i\le r) \quad (25)$$