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Output Planning Based on Data Envelopment Analysis and Extended Cournot Model

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***Abstract*—Based on data envelopment analysis (DEA) and extended Cournot model, this paper studies how each decision making unit (DMU) makes its most preferred output plans for the next production period when the market demands for one or more outputs are forecasted to change by a certain amount. Taking the perspective of noncooperative competition among all DMUs, this study proposes a method to search for the ideal production plans satisfying both market demand and production possibility set. Both numerical example and real world application to fast-food industry have showed that the proposed method is reasonable and executive.**

***Keywords*—data envelopment analysis (DEA), Cournot model, noncooperative competition, maximal output increase, equilibrium output (increase), ideal output**

I. INTRODUCTION

In recent years, more and more attention has been paid to applying data envelopment analysis (DEA) to the production planning problem. Developed by Charnes et al. in [1], DEA is an effective and widely used method for evaluating the relative efficiency of a group of decision making units (DMUs) with multiple inputs and outputs. Up to now, the DEA method has been applied to various settings and been integrated with many other methods, including the production planning problem discussed in this paper. Kumar and Sinha develop in [2] two DEA-based multi-period non-linear production planning models which regard each time period as a DMU and link them through inventory carrying constraints, with the efficiency sum of all DMUs as the objective function. In [3], Beasley also presents non-linear resource allocation models to jointly decide inputs and outputs for each DMU for the next period at the same time with the objective of maximizing the average efficiency of all. Besides dealing with non-linear models, the approach explicitly sets upper limits on the total amount of each input and of each output.

Consider that there are a set of n homogenous individual units in a competitive market in the sense that they all produce the same set of outputs at the cost of the same set of inputs. Each unit aims at as large profit gained from production as possible by competing on market shares with one another. Note that most DEA-based works concerning with input-output planning and resource allocation, such as [4] and [5], view all individual units under discussion as cooperative within a centralized decision-making environment. Different from such a cooperative perspective, this paper focuses on noncooperative competition. With the forecasted demand changes for outputs, our study develops a method based on DEA and extended Cournot model to make new output plans for all individual units in such a way that each unit makes its own total profit from production as large as possible while subjected to production capacity limitation. Each DMU's maximal production capacity for each output that is technically feasible can be obtained from the production possibility set. Meanwhile the equilibrium output (or equilibrium output increase) can be calculated from the extended Cournot model. Then the ideal output plan will be obtained by taking both maximum production capability and equilibrium output into consideration.

According to different situations about the forecasted market demand changes in the next production period, the discussion in this paper is divided into the following three cases: (1) one or more output demands are forecasted to increase; (2) one or more output demands are forecasted to decrease; (3) some increasing output demands coexist with some other decreasing output demands.

The rest of this paper is organized as follows. Section II, III and IV discuss the ideal output plans in the above-mentioned three cases, respectively, using a same simple numerical example to demonstrate the idea. Section V applies the approach to real world data set consisting of 16 fast food restaurants and Section VI concludes.

II. IDEAL OUTPUT PLANS UNDER INCREASING DEMANDS

Assume that there are a set of *n* decision making units (DMUs) and unit *j* is denoted by $DMU_j\left(j=1,\ldots,n\right)$. The *i*th input and *j*th output of DMU_j are denoted by $x_{ij}\left(i=1,\ldots,m\right)$ and $y_{rj}\left(r=1,\ldots,s\right)$, respectively. Since the situation at present can be a relatively reliable reference for future decisions, the current input-output mixes are used to estimate the production possibility set which describes all technically feasible input-output plans. The original production possibility set is defined by

$$T=\left\{\left(x_i,y_r\right)\middle| x_i\geq\sum_{j=1}^{n}\lambda_j x_{ij}, y_r\leq\sum_{j=1}^{n}\lambda_j y_{rj}, \lambda_j\geq 0, i=1,\ldots,m, r=1,\ldots,m\right\}. (1)$$

A. Ideal Output Plan with One Increasing Demand

1) Maximal output increase

Assume that via market investigation and some forecasting models, the market demand for output r_0 can be forecasted to increase by D_{r_0}, while the demands for all the other outputs remain unchanged. In this case, all DMUs attempt to maximize their respective profit by raising output r_0 production while consuming the current input levels. Under the original input levels and production possibility set, the maximal output r_0 increase for $DMU_j\,(j=1,\ldots,n)$ can be obtained from the following linear programming (LP) model (2).

$$\begin{cases} \max \Delta y_{r_0 j} \\ s.t. \sum_{k=1}^{n} \lambda_k x_{ik} \le x_{ij}, i=1,\ldots,m \\ \sum_{k=1}^{n} \lambda_k y_{rk} \ge y_{rj}, r=1,\ldots,s, r\ne r_0 \\ \sum_{k=1}^{n} \lambda_k y_{r_0 k} \ge y_{r_0 j} + \Delta y_{r_0 j} \\ \lambda_k \ge 0, k=1,\ldots,n \end{cases} \tag{2}$$

Let $\Delta y^*_{r_0 j}$ represent the optimal solution for $\Delta y_{r_0 j}$ in (2). Then with unchanged consumption levels, the maximal output r_0 increase for DMU_j which is technically feasible is $\Delta y^*_{r_0 j}\left(j=1,\ldots,n\right)$. If there is $\sum_{j=1}^{n} \Delta y^*_{r_0 j} \le D_{r_0}$, which indicates that the total maximal output r_0 production capacity of all DMUs still does not exceed the forecasted market demand for output r_0, then each DMU_j could raise its production on output r_0 to $\left(y_{r_0 j} + \Delta y^*_{r_0 j}\right)\left(j=1,\ldots,n\right)$ as the ideal production plan in the next production period. However, in most cases, if not all, there is $\sum_{j=1}^{n} \Delta y^*_{r_0 j} > D_{r_0}$, which means that total supply of output r_0 will go beyond the total forecasted demand if all DMUs exert their respective maximal output r_0 production capacity. In this situation, how should the ideal output r_0 plan for each DMU be made in the next production period? Here we take Cournot model into consideration.

2) Equilibrium output increase

Although the conventional Cournot model involves only two participants, its idea can be extended to the situation of multiple participants under several similar assumptions which are stated in [6]. It is common sense that when the market demand for output r_0 increases, no one would be willing to decrease its output r_0 production. Therefore, here we only consider the possible increasing part of output r_0 for each DMU, and the increasing part of market demand. Assume that the common linear demand curve faced by all *n* DMUs takes the following form

$$P_{r_0} = P^0_{r_0} - \frac{P^0_{r_0}}{D_{r_0}} \sum_{j=1}^{n} \tilde{y}_{r_0 j}, \tag{3}$$

where $P^0_{r_0}$ and P_{r_0} represent the unit prices to output r_0 in the current and next production period, respectively. Although observable in the current market, we do not need to actually obtain this current unit price due to its absence in the equilibrium output calculated later. $\tilde{y}_{r_0 j}$ represents the increasing output r_0 amount chosen by $DMU_j\,(j=1,\ldots,n)$, and then output r_0 will increase by $\sum_{j=1}^{n} \tilde{y}_{r_0 j}$ in total on the market.

Under the assumption upon unchanged input levels, profit maximization is equivalent to revenue maximization since costs are constant for all DMUs. The revenue in the increasing part of output r_0 for DMU_j is expressed as

$$R_{r_0 j} = \tilde{y}_{r_0 j}\left(P^0_{r_0} - \frac{P^0_{r_0}}{D_{r_0}} \sum_{k=1}^{n} \tilde{y}_{r_0 k}\right), j=1,\ldots,n. \tag{4}$$

To maximize increased revenue for each DMU, we let $\frac{\partial R_{r_0 j}}{\partial \tilde{y}_{r_0 j}} = 0$, and get the response function for DMU_j as

$$\tilde{y}^*_{r_0 j} = \frac{1}{2} D_{r_0} - \frac{1}{2} \sum_{\substack{k=1 \\ k\ne j}}^{n} \tilde{y}_{r_0 k}, j=1,\ldots,n, \tag{5}$$

By solving the group of equations represented by (5), we get the Nash equilibrium as

$$\tilde{y}^*_{r_0 j} = \frac{D_{r_0}}{n+1}, j=1,\ldots,n. \tag{6}$$

Equation (6) indicates that when the market capacity of output r_0 increases by D_{r_0}, the equilibrium output r_0 increase for DMU_j via Cournot model is $\tilde{y}^*_{r_0 j} = \frac{D_{r_0}}{n+1}$.

3) Ideal output plan

The equilibrium output r_0 increase for each DMU_j represented by (6) does not consider the maximal production capacity under the original input levels and production possibility set. However, there exists no possibility for any DMU to produce output that exceeds its own maximal output capacity. Therefore the ideal output r_0 increase for each DMU_j, denoted by $y^*_{r_0 j}$, should be no greater than its maximal output increase $\Delta y^*_{r_0 j}$, i.e. $y^*_{r_0 j} \le \Delta y^*_{r_0 j}$. If there is $\tilde{y}^*_{r_0 j} > \Delta y^*_{r_0 j}$, then DMU_j is unable to realize this equilibrium output r_0 increase under the current input levels, and to maximize its own interest, DMU_j must choose the maximal output increase $\Delta y^*_{r_0 j}$ as its ideal output increase in the next production period. When all DMUs satisfying the above situation choose their respective ideal output increase, all the other DMUs should accordingly adjust their equilibrium output increases via Cournot model, and compare the new equilibrium output increase with the corresponding maximal output increase to decide for ideal output increase or another new equilibrium increase. This process will go on until all newly gained equilibrium output increases are no greater than their corresponding maximal output increases.

Let J represent $\{1,...,n\}$. We divide all DMUs into the following two categories. One category, denoted by $DMU_{j_1}(j_1 \in J_1; J_1 \subset J)$, consists of all DMUs that have already decided the ideal output r_0 increase. For each $DMU_{j_1}(j_1 \in J_1; J_1 \subset J)$, its ideal output r_0 increase is equal to its maximal output r_0 increase, i.e. $y^*_{r_0 j_1} = \Delta y^*_{r_0 j_1}, j_1 \in J_1$. The other category, denoted by $DMU_{j_2}(j_2 \in J_2; J_2 \subset J)$, consists of all the rest DMUs in J. It is obvious that $J_1 \bigcup J_2 = J$ while J_1 and J_2 have no shared element. Assume that there exist l_2 DMUs in J_2, and then given the output r_0 increase chosen by each DMU in J_1, which is $y^*_{r_0 j_1} = \Delta y^*_{r_0 j_1}(j_1 \in J_1)$, the new equilibrium output r_0 increase for $DMU_{j_2}(j_2 \in J_2)$ can be calculated as $\tilde{y}^*_{r_0 j_2} = \dfrac{D_{r_0} - \sum_{k_1 \in J_1} \Delta y^*_{r_0 k_1}}{l_2 + 1}$, where $\tilde{y}^*_{r_0 j_2} \le \Delta y^*_{r_0 j_2}, j_2 \in J_2$. This newly gained equilibrium output r_0 increase is the ideal output r_0 increase for DMU_{j_2} in J_2, i.e. $y^*_{r_0 j_2} = \dfrac{D_{r_0} - \sum_{k_1 \in J_1} \Delta y^*_{r_0 k_1}}{l_2 + 1}, j_2 \in J_2$. Accordingly, the ideal output r_0 production plan for DMUs in J_1 and J_2 are $y_{r_0 j_1} + \Delta y^*_{r_0 j_1}, j_1 \in J_1$ and $y_{r_0 j_2} + \dfrac{D_{r_0} - \sum_{k_1 \in J_1} \Delta y^*_{r_0 k_1}}{l_2 + 1}, j_2 \in J_2$, respectively, which is the sum of current output r_0 level and ideal output r_0 increase.

B. Ideal Output Plans with Multiple Increasing Demands

Assume that the market demands for the following *t* outputs $r_p\,(p=1,...,t)$ are forecasted to increase by $D_{r_p}\,(p=1,...,t)$ in the next production period, respectively, while the market demands for all the rest $(s-t)$ outputs $r_q\,(q=t+1,...,s)$ remain unchanged. Then in order to realize maximal profit under the original input levels and production possibility set, the maximal output $r_p\,(p=1,...,t)$ increase for $DMU_j\,(j=1,...,n)$ can be calculated by the following LP model (7)

$$
\begin{cases}
\max \sum_{p=1}^{t} P^0_{r_p} \Delta y_{r_p j} \\
s.t. \sum_{k=1}^{n} \lambda_k x_{ik} \le x_{ij}, i = 1,...,m \\
\quad \sum_{k=1}^{n} \lambda_k y_{r_q k} \ge y_{r_q j}, q = t+1,...,s \\
\quad \sum_{k=1}^{n} \lambda_k y_{r_p k} \ge y_{r_p j} + \Delta y_{r_p j}, p = 1,...,t \\
\quad \lambda_k \ge 0, k = 1,...,n
\end{cases}, \tag{7}
$$

where $P^0_{r_p}\,(p=1,...,t)$ represents the current unit price to output r_p that is observable in the market.

Let $\Delta y^*_{r_p j}\,(p=1,...,t)$ represent the optimal solution for $\Delta y_{r_p j}$ in (7). Then under the current input levels, the maximal output $r_p\,(p=1,...,t)$ increase for DMU_j is $\Delta y^*_{r_p j}$ in terms of technical feasibility and profit maximization.

View each output $r_p\,(p=1,...,t)$ as output r_0 in Section II (Part *A*), and calculate its equilibrium output increase and ideal output increase. When searching for the ideal output r_p increase, the maximal output increase $\Delta y^*_{r_p j}$ for each DMU obtained from (7) is the upper limit for output $r_p\,(p=1,...,t)$ increase. Similar to the situation of one increasing demand, all DMUs can make their respective ideal output $r_p\,(p=1,...,t)$ production plan for the next production period. This process will be demonstrated later in Section V, the application part.

III. IDEAL OUTPUT PLANS UNDER DECREASING DEMANDS

A. Ideal Output Plan with One Decreasing Demand

Now we consider the situation with decreasing market demands. Assume that the market demand for output r_0 can be forecasted to decrease by D_{r_0}, while the demands for all the rest outputs remain unchanged. In this case, all DMUs should decrease their respective output r_0 production by a certain amount in order to satisfy the shrinking market demand. However, on the other hand, each DMU attempts to decrease as little output production as possible to maximize its profit when consuming the current input levels.

Assume that the common linear demand curve faced by all n DMUs takes the following form

$$P_{r_0} = P_{r_0}^0 - \frac{P_{r_0}^0}{\sum_{j=1}^{n} y_{r_0 j} - D_{r_0}} \sum_{j=1}^{n} \hat{y}_{r_0 j}, \tag{8}$$

where $P_{r_0}^0$ and P_{r_0} represent the unit prices to output r_0 in the current and next production period, respectively. Also we do not need to actually obtain the current unit price $P_{r_0}^0$. $\hat{y}_{r_0 j}$ represents the output r_0 production chosen by DMU_j, and then output r_0 will amount to $\sum_{j=1}^{n} \hat{y}_{r_0 j}$ in total on the market.

Under the assumption upon unchanged input levels, profit maximization is equivalent to revenue maximization. The revenue in output r_0 for $DMU_j\,(j=1,...,n)$ is expressed as

$$R_{r_0 j} = \hat{y}_{r_0 j}\left(P_{r_0}^0 - \frac{P_{r_0}^0}{\sum_{k=1}^{n} y_{r_0 k} - D_{r_0}} \sum_{k=1}^{n} \hat{y}_{r_0 k}\right), j=1,...,n. \tag{9}$$

To maximize revenue for each DMU, we let $\frac{\partial R_{r_0 j}}{\partial \hat{y}_{r_0 j}} = 0$, and get the response function for DMU_j as.

$$\hat{y}_{r_0 j}^* = \frac{1}{2}\left(\sum_{k=1}^{n} y_{r_0 k} - D_{r_0}\right) - \frac{1}{2}\sum_{\substack{k=1\\k\neq j}}^{n} \hat{y}_{r_0 k}, j=1,...,n. \tag{10}$$

Then the Nash equilibrium can be obtained as

$$\hat{y}_{r_0 j}^* = \frac{\sum_{k=1}^{n} y_{r_0 k} - D_{r_0}}{n+1}, j=1,...,n, \tag{11}$$

which means that when the market capacity of output r_0 equals $\left(\sum_{j=1}^{n} y_{r_0 j} - D_{r_0}\right)$, the equilibrium output r_0 production for DMU_j via Cournot model is $\hat{y}_{r_0 j}^* = \frac{\sum_{k=1}^{n} y_{r_0 k} - D_{r_0}}{n+1}$.

However, the equilibrium output r_0 production for each DMU_j represented by (11) does not take the current output production $y_{r_0 j}$ into consideration. There exists slim possibility for any DMU to produce output that exceeds its current production level in a shrinking market prospect. Therefore the ideal output r_0 production for each DMU_j should be no greater than its current output level.

Let J represent $\{1,...,n\}$. If DMU_j's equilibrium output r_0 production is greater than its current output r_0 level, i.e. $\hat{y}_{r_0 j}^* > y_{r_0 j}$, then to maximize its own interest, DMU_j will not choose the equilibrium output production but the current output r_0 level $y_{r_0 j}$ as its ideal output plan in the next production period. When all DMUs satisfying the above situation have chosen their respective ideal output production, all the rest DMUs in J should accordingly adjust their equilibrium output production via Cournot model. By comparing the new equilibrium output with the corresponding current output level, all the rest DMUs decide between ideal output production and another new equilibrium output. This process will go on until all newly gained equilibrium outputs are no greater than the corresponding current output levels.

Then we divide all DMUs into the following two categories. One category, denoted by $DMU_{j_1}\,(j_1 \in J_1; J_1 \subset J)$, consists of all DMUs that have already decided the ideal output r_0 production plan. For each $DMU_{j_1}\,(j_1 \in J_1; J_1 \subset J)$, its ideal output r_0 plan is equal to its current output r_0 level $y_{r_0 j_1}\,(j_1 \in J_1)$. The other category, denoted by $DMU_{j_2}\,(j_2 \in J_2; J_2 \subset J)$, consists of all the rest DMUs in J. It is obvious that $J_1 \cup J_2 = J$ while J_1 and J_2 have no shared element. Assume that there exist l_2 DMUs in J_2, and then given the output r_0 plan chosen by each DMU in J_1, the new equilibrium output for $DMU_{j_2}\,(j_2 \in J_2)$ can be calculated as $\hat{y}_{r_0 j_2}^* = \frac{\sum_{k_2 \in J_2} y_{r_0 k_2} - D_{r_0}}{l_2+1}$, where $\hat{y}_{r_0 j_2}^* \le y_{r_0 j_2}, j_2 \in J_2$. This newly gained equilibrium output is the ideal output r_0 plan for DMU_{j_2} in J_2.

B. Ideal Output Plans with Multiple Decreasing Demands

Assume that the market demands for the following d outputs $r_g\left(g=1,..,d\right)$ are forecasted to decrease by D_{r_g} in the next production period, respectively, while the market demands for all the rest $\left(s-d\right)$ outputs $r_h\left(h=d+1,...,s\right)$ remain unchanged. View each output $r_g\left(g=1,..,d\right)$ as output r_0 in Section III (Part *A*), and calculate its ideal output production plan.

IV. IDEAL OUTPUT PLANS UNDER COEXISTING INCREASING AND DECREASING DEMANDS

Assume that in the next production period, the market demands for the following t outputs $r_p\left(p=1,...,t\right)$ are forecasted to increase by D_{r_p}, respectively; meanwhile the market demands for the following d outputs $r_g\left(g=t+1,...,t+d\right)$ are forecasted to decrease by D_{r_g}, respectively. For all the rest $\left(s-t-d\right)$ outputs $r_h\left(h=t+d+1,...,s\right)$, the market demands remain unchanged.

Firstly viewing each output $r_g\left(g=t+1,...,t+d\right)$ with decreasing demand as output r_0 in Section III (Part *A*), we can make ideal output plans for each DMU on those demand-shrinking outputs. Denote the ideal output r_g production plan for DMU_j by $y^*_{r_g j}\left(g=t+1,...,t+d\right)$. As for those demand-increasing outputs $r_p\left(p=1,...,t\right)$, in order to realize maximal profit under the original input levels and production possibility set, the maximal output r_p increase for $DMU_j\left(j=1,...,n\right)$ can be calculated by the following LP model (12).

$$\begin{cases} \max \sum_{p=1}^{t} P^0_{r_p} \Delta y_{r_p j} \\ s.t. \sum_{k=1}^{n} \lambda_k x_{ik} \le x_{ij}, i=1,...,m \\ \sum_{k=1}^{n} \lambda_k y_{r_h k} \ge y_{r_h j}, h=t+d+1,...,s \\ \sum_{k=1}^{n} \lambda_k y_{r_g k} \ge y^*_{r_g j}, g=t+1,...,t+d \\ \sum_{k=1}^{n} \lambda_k y_{r_p k} \ge y_{r_p j} + \Delta y_{r_p j}, p=1,...,t \\ \lambda_k \ge 0, k=1,...,n \end{cases} \quad (12)$$

where $P^0_{r_p}\left(p=1,...,t\right)$ represents the current unit price to output r_p that is observable in the market. Let $\Delta y^*_{r_p j}\left(p=1,...,t\right)$ represent the optimal solution for $\Delta y_{r_p j}$ in (12). Then under the current input levels and the ideal output $r_g\left(g=t+1,...,t+d\right)$ production plans, the maximal output $r_p\left(p=1,...,t\right)$ increase for DMU_j is $\Delta y^*_{r_p j}$ in terms of technical feasibility and profit maximization. View each output r_p as output r_0 in Section I (Part A), and calculate its equilibrium output increase and ideal output increase. When searching for the ideal output r_p increase, the maximal output increase $\Delta y^*_{r_p j}$ for each DMU obtained from (12) is the upper limit for output $r_p\left(p=1,...,t\right)$ increase. Then all DMUs can make their respective ideal output plans on those demand-increasing outputs for the next production period.

Next we consider a numerical example. Table I presents the data of 6 DMUs with two inputs and two outputs. The market demands for output 1 and 2 are forecasted to increase by 2.5 and to drop by 1.5, respectively. Table II reports all DMUs' ideal production plans for output 1 and 2 in the next production period in rows 4 and 5, respectively, and reports the new CCR efficiency scores by adopting ideal production plans in the last row. Note that all new efficiency scores remain undecreasing, among which the new efficiency scores of all inefficient DMUs (DMUs 1, 2, 4) improve significantly.

V. APPLICATION

In this section, we apply our output planning approach to a set of 16 fast-food restaurants located in the city of Hefei, Anhui Province, China. Man-hour and shop size are used as two inputs. Man-hour here refers to the labor force used within a certain period. Shop size refers to total rental floor space of the restaurant that can be used for the purposes of serving customers. The outputs are the sales of dish and beverage. The data are collected for one business month in April 2008, and given in Table III with the original CCR efficiency scores of all restaurants in column 6. According to market investigation, the unit prices to dish and beverage in the whole fast-food industry are 8 (RMB per serving) and 3 (RMB per serving).

Since May is a golden time for tourism, the number of people eating out will normally go up. Meanwhile, it will be increasingly hot in May, thus cool beverages will be in a greater demand. Through investigation into the market and analysis of questionnaires done by potential customers, the demand for dish and beverage in the next business month May 2008 can be forecasted to increase by 3.717×10^3 (servings) and 3.04×10^3 (servings), respectively. The ideal sales plans of all 16 restaurants are showed in columns 7 and 8 in Table III, with the new efficiency scores after planning in the last column. Then all fast food restaurants could arrange their respective raw material stock according to the corresponding ideal sales plans. By comparing the original efficiency scores with the new ones in Table III, we note that all new efficiency scores remain undecreasing, among which DMUs 1, 2, 3, 4, 5, 6, 8, 9, 11, 13, 14, 15, 16 have improved their respective new efficiency scores significantly, and inefficient DMUs 11 and 14 even become efficient ones.

TABLE I. FICTIONAL DATA

DMU	1	2	3	4	5	6
Input 1	4	6	1	2	3	3
Input 2	3	2	3	6	1	2
Output 1	2	1	1	1	1	2
Output 2	1	2	2	1	2	1
Original efficiency	0.73684	0.50000	1.00000	0.50000	1.00000	1.00000

TABLE II. IDEAL OUTPUT PLANS AND NEW EFFICIENCY (INCREASING AND DECREASING DEMANDS)

DMU	1	2	3	4	5	6
Input 1	4.000	6.000	1.000	2.000	3.000	3.000
Input 2	3.000	2.000	3.000	6.000	1.000	2.000
Ideal output 1	2.625	1.625	1.000	1.625	1.000	2.000
Ideal output 2	1.000	1.125	1.125	1.000	1.125	1.000
New efficiency	0.96711	0.81250	1.00000	0.81250	1.00000	1.00000

TABLE III. DATA FOR 16 FAST FOOD RESTAURANTS AND IDEAL SALES PLANS

DMU j	Inputs		Outputs		Original efficiency	Ideal dish sale (10^3 servings)	Ideal beverage sale (10^3 servings)	New efficiency
	Man-hour (10^3 *h*)	*Shop size* (10^2 m^2)	*Dish sale* (10^3 *servings*)	*Beverage sale* (10^3 *servings*)				
1	1.80	0.65	3.20	1.85	0.8875	3.500	2.063	0.9711
2	2.52	1.10	4.50	2.00	0.8911	4.800	2.213	0.9505
3	2.16	0.83	4.15	2.25	0.9587	4.320	2.463	0.9982
4	1.50	0.48	2.65	1.35	0.8817	2.950	1.563	0.9815
5	2.88	1.25	5.20	2.30	0.9010	5.500	2.513	0.9530
6	1.44	0.36	2.50	0.95	0.8677	2.800	1.163	0.9719
7	1.20	0.28	2.40	1.50	1.0000	2.400	1.500	1.0000
8	2.10	0.85	3.80	1.55	0.9030	4.100	1.763	0.9743
9	2.16	0.71	3.70	1.30	0.8548	4.000	1.513	0.9241
10	2.52	0.82	5.05	2.85	1.0000	5.050	2.850	1.0000
11	1.80	0.55	3.35	2.05	0.9302	3.600	2.250	1.0000
12	2.40	1.04	4.80	1.90	0.9980	4.800	2.113	0.9980
13	2.88	1.32	5.40	2.35	0.9356	5.700	2.563	0.9876
14	1.50	0.46	3.00	1.80	0.9992	3.000	1.875	1.0000
15	2.52	0.84	4.40	2.15	0.8713	4.700	2.363	0.9307
16	2.16	0.69	3.85	2.25	0.8900	4.150	2.463	0.9597

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VI. CONCLUSIONS

From a noncooperative view, this paper proposes an effective approach based on DEA and extended Cournot model to make future output plans among a group of individual units according to current input levels and market demand changes. Both numerical example and real world application to fast-food industry have showed that the proposed method is applicable and leads to reasonable results, which could be regarded as a feasible reference for decision makers facing output planning problem.

One possible extension for this paper is to consider changing input levels rather than constant input consumptions before and after planning. In that case, decision making units could adjust their respective input levels according to production need.

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