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A New Method of Fuzzy Risk in Studying Landfall Typhoon

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***Abstract*—We can obtain a conservative risk value, a venture risk value and a maximum probability risk value. Under such an α level, three risk values can be calculated. As α adopts all values throughout the set [0, 1], it is possible to obtain a series of risk values. Therefore, the fuzzy risk can be a multi-valued risk or set-valued risk. Calculation of the fuzzy expected value of landfall typhoon risk in Zhejiang province has been performed based on the interior-outer set model. Selection of an α value depends on the confidence in different groups of people, while selection of a conservative risk value or venture risk value depends on the risk preference of these people.**

***Keywords*— α level; fuzzy risk; fuzzy expected value; landfall typhoon**

I. INTRODUCTION

The lives of people and property in Zhejiang province, China are threatened by landfall typhoons, which have been listed as one of the gravest natural disasters to affect social and economic development in Zhejiang [1]. Typhoon Winnie landed in Wenling, Zhejiang on the 18 August, 1997, causing direct economic losses as high as 18.6 billion RMB. Typhoon Saomai landed in Cang'nan, Zhejiang on the 10 August, 2006, leaving a death toll of 104 with 190 missing and direct economic losses of 11.3 billion RMB. Landfall typhoon affects vast areas, is difficult to resist and causes the greatest destruction, representing a huge threat to lives and property, and is a risk facing mankind [2].

Risk is always accompanied by uncertainty [3]. If there were no uncertainty, there would be no risk. Due to many factors, human beings cannot accurately predict many incidents. Uncertainty always exists and, therefore, risk is inevitable [4]. Landfall typhoon variations in the future are an issue of great uncertainty, and risk analysis of landfall typhoons is absolutely necessary.

Fuzzy risk differs from probability risk in terms of the probability distribution introduced for exceeding probability, indicating the possibility of occurrence of some probabilities [5]. This not only reveals the inaccuracy of the exceeding probability estimation, but also provides a way for the model to accommodate fuzzy information. Huang established the interior-outer set model by using the information distribution method to calculate the fuzzy risk as an expression of the fuzzy nature of probability estimation [6]. The interior-outer set model, due to the complexity of the combined calculation involved in its traditional expression, is not easy to popularize in practical applications. However, the matrix arithmetic method can be used for this purpose. Moraga provided another method of presentation for the interior-outer set model that has proven to be quite easy to compute [7]. Since no landfall typhoon analysis has been performed using the theory of fuzzy risk [8], studying future landfall typhoon variation using the interior-outer set model is recommended.

II. METHODS

A. Interior-outer Set Model

Let $X = \{x_i \mid i = 1,2,\cdots,n\}$ represent the sample of the incident where the sample point is $x_i \in R^1$ (real number set). For instance, $X = \{x_i \mid i = 1,2,\cdots,n\}$ consists of precipitation records in a certain historic period with a field of *U*. When *X* represents a precipitation sample, the field is [0, 1800]. Let $u_1, u_2, \cdots, u_m$ represent discrete points with a given step length Δ . U is used to express this discrete field, e.g. $U = \{u_i \mid i = 1,2,\cdots,m\}$. From the point of view of information distribution, a sample point x_i can allocate its information with a value of q_{ij} to point u_j , as expressed by the following equation:

$$q_{ij} = \begin{cases} 1 - |x_i - u_j| / \Delta, & \text{for } |x_i - u_j| \le \Delta \\ 0, & \text{for } |x_i - u_j| > \Delta \end{cases} \tag{1}$$

where x_i is the observation value, u_j is a controlled point and Δ is the step length of the controlled point. The interior-outer set model can be used to estimate the fuzzy probability of an incident occurring during the following interval:

$$I_j = [u_j - \Delta/2, u_j + \Delta/2), \quad j = 1,2,\cdots,m \tag{2}$$

Intervals are selected so that all of the sample points x_i lie within a certain interval I_j only. When a sample point is subjected to random disturbance, it may depart from the interval I_j, while a point outside may also enter the interval. Changes in the relationship between the sample point and the interval due to random disturbance are referred to as leaving in the former case, and joining in the latter case. The possibility of

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sample point x_i leaving or joining the interval I_j is expressed as q_{ij}^- and q_{ij}^+, respectively. For the purpose of computation, the definition of the interior set and outer set of I_j is as follows:

Interior set: $X_{in-j} \overset{\Delta}{=} X \cap I_j$, i.e. a set composed of all elements contained within I_j.

Outer set: $X_{out-j} \overset{\Delta}{=} X \setminus X_{in-j}$, i.e. a set composed of all elements not contained within I_j.

Let $S_j = \{s_j \mid \{x_{s_j}\}_{s_j} = X_{in-j}\}$ represent the interior index set. Similarly, let $T_j = \{t_j \mid \{x_{t_j}\}_{t_j} = X_{out-j}\}$ represent the outer index set.

Let $|S_j| = n_j$, then $|T_j| = n - n_j$ where n_j represents the volume of S_j or X_{in-j}. Let

$$q_{ij}^- = \begin{cases} 1 - q_{ij}, & x_i \in X_{in-j} \\ 0 \end{cases} \quad (3)$$

$$q_{ij}^+ = \begin{cases} q_{ij}, & x_i \in X_{out-j} \\ 0 \end{cases} \quad (4)$$

Consequently, a possibility-probability risk distribution of random incident in interval I_j can be calculated corresponding to the field of interval $I = \{I_j \mid j = 1,2,\cdots,m\}$ and the field of discrete probability $P = \{p_k \mid k = 0,1,2,\cdots,n\} = \{k/n \mid k = 0,1,2,\cdots,n\}$:

$$\Pi_{I,P} = \{\pi_{I_j}(p) \mid I_j \in I, p \in P\} \quad (5)$$

where, $\pi_{I_j}(p)$ represents the possibility of an incident in the interval I_j with a probability *p*.

A simplified calculation method for equation (5) was given by Huang using the following technique [9]:

First, calculate the leaving value set $Q_j^- = \{q_{ij}^-\}$ for interval I_j by taking its interior set X_{in-j}, and then calculating the joining value set $Q_j^+ = \{q_{ij}^+\}$ for the same interval using its outer set X_{out-j}. Define:

$\uparrow Q_j^- = [q_{j0,j}^-, q_{j1,j}^-, \cdots, q_{jn_j-1,j}^-]$, where $q_{js,j}^- \le q_{jt,j}^- (\forall s < t)$, i.e. the elements of Q_j^- are arranged in order of ascending magnitude.

$\downarrow Q_j^+ = [q_{jn_j+1,j}^+, q_{jn_j+2,j}^+, \cdots, q_{jn,j}^+]$, where $q_{js,j}^+ \ge q_{jt,j}^+ (\forall s < t)$, i.e. the elements of Q_j^+ are arranged in order of descending magnitude.

Finally, let $p_k = k/n \quad (k = 0,1,2,\cdots,n)$, and equation (5) can be simplified as follows:

$$\pi_{I_j}(p) = \begin{cases} q_{j0,j}^- & p = p_0 \\ q_{j1,j}^- & p = p_1 \\ \cdots & \\ q_{jn_j-1,j}^- & p = p_{n_j-1} \\ 1, & p = p_{n_j} \\ q_{jn_j+1,j}^+ & p = p_{n_j+1} \\ q_{jn_j+2,j}^+ & p = p_{n_j+2} \\ \cdots & \\ q_{jn,j}^+ & p = p_n \end{cases} \quad (6)$$

where $q_{j0,j}^-$ is the 1st element of $\uparrow Q_j^-$, $q_{j1,j}^-$ the 2nd element of $\uparrow Q_j^-$, $q_{jn_j-1,j}^-$ the last element of $\uparrow Q_j^-$, $q_{jn_j+1,j}^+$ the 1st element of $\downarrow Q_j^+$, $q_{jn_j+2,j}^+$ the 2nd element of $\downarrow Q_j^+$, $q_{jn,j}^+$ the last element of $\downarrow Q_j^+$.

Equation (6) is a form of the interior-outer set model suitable for simplified computation.

B. *Fuzzy Expected Value Calculation Based on Possibility-Probability Distribution*

The possibility-probability risk calculated using the interior-outer set model is referred to as fuzzy risk. The simplest way to perform fuzzy risk assessment is to calculate the fuzzy expected value and convert fuzzy risk into non-fuzzy risk, making it a clear number. In doing so, there must be a transition from a fuzzy set to a clear set. In order to obtain a clear set from a fuzzy set, the first thing that must be done is to define a standard α (level or threshold value), and then select the elements *x* with subordinate degree $A(x) \ge \alpha \ (0 \le \alpha \le 1)$. Therefore, the level value α is a key concept in this transition.

Definition: Let Ω represent the space of incident *x*, and *P* the probability field,

$$\underset{\sim}{p}(x) \overset{\Delta}{=} \{\mu_x(p) \mid x \in \Omega, p \in P\} \quad (7)$$

This equation represents a fuzzy probability distribution with. $\forall \alpha \le [0,1]$, and then select the elements p_i of the subordinate degree $\mu_x(p) \ge \alpha$. Let

$$\begin{aligned} \underline{p}_\alpha(x) &= \min\{p \mid p \in P, \mu_x(p) \ge \alpha\} \\ \overline{p}_\alpha(x) &= \max\{p \mid p \in P, \mu_x(p) \ge \alpha\} \end{aligned} \quad (8)$$

where, $\underline{p}_\alpha(x)$ is referred to as the minimum probability with regard to *x* in the cut set α-level; and $\overline{p}_\alpha(x)$ is referred to as the maximum probability accordingly. For example in Figure 1 (Omitted), the minimum probability with regard to *x* in the cut set α-level is p_2; the maximum probability is p_3 accordingly. A sketch of the fuzzy cut set of the possibility-probability distribution is shown in Figure 1. In this paper, a triangular function is adopted as the subordinate function.

The finite closed interval

$$p_\alpha(x) \overset{\Delta}{=} [\underline{p}_\alpha(x), \overline{p}_\alpha(x)] \quad (9)$$

is referred to as the α level cut set of the fuzzy set $p(x)$ with regard to *x*. The transition from fuzzy set $p(x)$ to clear set $p_\alpha(x)$ is achieved with α as the smallest value of the subordinate degree, i.e.,

$$\pi_x(p) = \{p \mid p \in P, \mu_x(p) \geq \alpha\} \quad (10)$$

Here, the level value α is referred to as the possibility level of probability. $p_\alpha(x)$ is an interval derived from the possibility distribution $\pi_x(p)$ with given values of *x* and α, and not a function with π as a variable. $p_\alpha(x)$ has a alterable boundary $0 \leq \alpha \leq 1$, and is a set with an elastic boundary. That is to say, for any given possibility level α, there is a corresponding $p_\alpha(x)$. Thus, the higher the value of α, the greater the corresponding possibility of probability. It can be seen from Figure 1 that the triangular curve is just the subordinate function $\mu_x(p)$. The conversion from $p(x)$ to $p_\alpha(x)$ begins at α. This means that the higher the possibility level α is, the less the element in the set $p_\alpha(x)$; the lower the level α is, the more the element in the set $p_\alpha(x)$. Hence, the higher the value of α, the lower the uncertainty, and the closer to the true value the probability; the lower the value of α, the higher the uncertainty, and the less practical use the value of probability will have. The possibility level α is dependant on technical conditions and knowledge.

Let

$$\underline{p}'_\alpha(x) = \underline{p}_\alpha(x) / \int_\Omega \underline{p}_\alpha(x)dx, \quad \overline{p}'_\alpha(x) = \overline{p}_\alpha(x) / \int_\Omega \overline{p}_\alpha(x)dx, \quad \text{or}$$

$$\underline{p}'_\alpha(x) = \underline{p}_\alpha(x) / \sum_\Omega \underline{p}_\alpha(x)dx \quad \overline{p}'_\alpha(x) = \overline{p}_\alpha(x) / \sum_\Omega \overline{p}_\alpha(x)dx \quad (11)$$

$\underline{p}'_\alpha(x)$ and $\overline{p}'_\alpha(x)$ represent the normalization of $\underline{p}_\alpha(x)$ and $\overline{p}_\alpha(x)$, respectively.

Let

$$\underline{E}_\alpha(x) = \int_\Omega x\underline{p}'_\alpha(x)dx, \quad \overline{E}_\alpha(x) = \int_\Omega x\overline{p}'_\alpha(x)dx, \quad \text{or}$$

$$\underline{E}_\alpha(x) = \sum_\Omega x\underline{p}'_\alpha(x)dx \quad \overline{E}_\alpha(x) = \sum_\Omega x\overline{p}'_\alpha(x)dx \quad (12)$$

Then

$$E_\alpha(x) \overset{\Delta}{=} [\underline{E}_\alpha(x), \overline{E}_\alpha(x)] \quad (13)$$

represent the expected interval of the α level cut set of $p(x)$ with respect to *x*. Where, $\underline{E}_\alpha(x)$ and $\overline{E}_\alpha(x)$ are referred to as the fuzzy expected value of minimum probability and the fuzzy expected value of maximum probability for $p_\alpha(x)$, respectively. Especially when α=1, the occurrence of incident I_j is the only possibility. Hence, $E_1(x)$ for α=1 is called as the fuzzy expected value of maximum possibility probability for $p_\alpha(x)$.

A fuzzy expected value of the possibility-probability distribution is a set with $\underline{E}_\alpha(x)$ and $\overline{E}_\alpha(x)$ as its boundaries. Here, to simplify calculation, $\underline{E}_\alpha(x)$ and $\overline{E}_\alpha(x)$ are selected as fuzzy expected values of possibility-probability distribution. In fact ,the number of an expected value of possibility-probability distribution can be 3 or more. For any possibility-probability distribution with a given non-empty α cut set, there must be corresponding $\underline{E}_\alpha(x)$ and $\overline{E}_\alpha(x)$. Therefore, under the α level, the fuzzy expected values are $\underline{E}_\alpha(x)$ and $\overline{E}_\alpha(x)$. When the cut set technique for fuzzy sets is applied using α and adopting all values in the range [0, 1], it is possible to obtain the whole hierarchical structures of $\underline{E}_\alpha(x)$ and $\overline{E}_\alpha(x)$. The fuzzy expected values $\underline{E}_\alpha(x)$ and $\overline{E}_\alpha(x)$ of the possibility-probability distribution are really the fuzzy risk values. Generally speaking, incidents with high probability values occur with less intensity while incidents with low probability values occur with stronger intensity. Therefore, we refer to the fuzzy expected value $\underline{E}_\alpha(x)$ as conservative risk value (R_C), and the fuzzy expected value $\overline{E}_\alpha(x)$ as venture risk value (R_V). The fuzzy expected value $E_1(x)$ is referred to as maximum probability risk value (R_M). For such an α level, 3 risk values can be obtained. With α taking all values throughout [0, 1], it is possible to obtain a series of risk values. Therefore, the fuzzy risk may be a multi-valued risk or set-valued risk.

III. RESULTS

A. *Landfall Typhoon Risk Analysis*

The magnitudes of typhoon hazards mainly depend on their intensity when they land. Maximum wind force, maximum wind velocity and minimum central pressure are the three indicators used to describe the intensity of landfall typhoons. The central pressure *P*, due to its relatively high observation precision, is used to describe the intensity of a landfall typhoon.

According to statistics, since 1949 a total of 160 typhoons have affected or landed in Zhejiang province. Most of these are influential typhoons and landfall typhoons of lower intensity. These typhoons alleviate the ravages of droughts between July and September. Only a few landfall typhoons with high intensities have caused disasters in Zhejiang. Therefore, a year when typhoons land with a central pressure ⩽960hPa is referred to as a strong typhoon landing year. Typhoon sampling is performed according to this criterion (Table I). In recent years, typhoon disasters have become increasingly serious as fast social and economic development occurs. Therefore, it is necessary to analyze landfall typhoon risks to ensure healthy and sustainable economic development.

TABLE I. LANDFALL TYPHOONS OF MINIMUM CENTRAL PRESSURE $P\leqslant$ 960HPA SINCE 1981 IN ZHEJIANG

Year	*P* (hPa)	*x*	Year	*P* (hPa)	*x*
1981	960	x_1	2005	950	x_5
1994	960	x_2	2005	945	x_6
1997	955	x_3	2006	920	x_7
2004	950	x_4			

The type of probability pattern (for instance normal distribution or exponential distribution) that landfall typhoon variations in Zhejiang follow is still unknown, and the sample volume actually measured is relatively small, with the characteristics of fuzzy information. Since probability estimation based on such samples is not accurate, it is advisable to determine landfall typhoon risk in the future using the possibility-probability distribution.

Taking into consideration the intervals $I_1=[915,935)$, $I_2=[935,955)$, $I_3=[955,975)$ with corresponding discrete fields $U=\{u_j \mid j=1,2,3\}$ = {925, 945, 965} and a controlling point step length of $\Delta=20$, calculations are performed based on table calculation methods as follows:

Step 1: Calculate the distributed information q_{ij} using equation (1) for the individual sampling points. The results are shown in Table (Omitted).

Step 2: Calculate the leaving information q_{ij}^- (Table omitted) using equation (3). Arrange q_{ij}^- in ascending order over the given interval I_j to obtain the leaving information $\uparrow Q_j^-$ of ascending magnitude, as shown in Table (Omitted).

Step 3: Calculate the joining information q_{ij}^+ (Table omitted) using equation (4). Arrange q_{ij}^+ in descending order over the given interval I_j to obtain the joining information $\downarrow Q_j^+$ of descending magnitude, as shown in Table (Omitted).

Step 4: Let

$$P=\{p_k=k/n\} \tag{14}$$

Here, *k*=1, 2, $\cdots$, 7, where *n* is the total number of samples. Substitute the figures in Table (Omitted) into equation (6) and get the fuzzy risk calculated by the possibility-probability distribution (Table omitted).

B. Fuzzy Expected Value of Landfall Typhoon Risk

The α cut set technique can be used to obtain the fuzzy expected value of landfall typhoon risk. Firstly, take a level cut set with α=0.1. We can obtain the following from Table (Omitted) based on equations (8) and (9):

$$p_{0.1}(I_1)=[0,0.14],\quad p_{0.1}(I_2)=[0.14,0.86] \\ p_{0.1}(I_3)=[0,0.71] \tag{15}$$

Then:

$$\begin{aligned} \underline{p}_{0.1}(I) &= \{0,\quad 0.14,\quad 0\} \\ \overline{p}_{0.1}(I) &= \{0.14,\quad 0.86,\quad 0.71\} \end{aligned} \tag{16}$$

Secondly, we can obtain the following sets based on equation (11) by normalizing $\underline{p}_{0.1}(I_j)$ and $\overline{p}_{0.1}(I_j)$:

$$\begin{aligned} \underline{p}'_{0.1}(I) &= \{0,\quad 1,\quad 0\} \\ \overline{p}'_{0.1}(I) &= \{0.08,\quad 0.50,\quad 0.42\} \end{aligned} \tag{17}$$

Finally, we calculate the fuzzy expected values $\underline{E}_{0.1}(I)$ and $\overline{E}_{0.1}(I)$ based on equation (12). Where I_j is replaced by the interval center point U_j, so:

$$\begin{aligned} \underline{E}_{0.1}(I) &= 0\times925+1\times945 \\ &\quad +0\times965=945 \\ \overline{E}_{0.1}(I) &= 0.08\times925+0.50\times945 \\ &\quad +0.42\times965=952 \end{aligned} \tag{18}$$

From the fuzzy expected value of the landfall typhoon risk in Zhejiang province listed in Table II, we can get a conservative risk value (R_C) 945hPa for landfall typhoon risk in Zhejiang province, a venture risk value (R_V) 952hPa, and a maximum probability risk value (R_M) 951hPa for the α=0.1 level cut set.

TABLE II. FUZZY EXPECTED VALUE OF THE LANDFALL TYPHOON RISK IN ZHEJIANG PROVINCE

α	0.1	0.3	0.6	1
R_C (hPa)	945	948	951	951
R_V (hPa)	952	950	951	951

IV. DISCUSSION AND CONCLUSION

It can be seen from the above that the fuzzy risk of an incident calculated based on the fuzzy cut set technique is a multi-valued risk, allocating many risk values to a given unit. These risk values are organized by some hierarchy for different values of α. For each value of α, the expected values of possibility-probability risk are $\underline{E}_\alpha(x)$, $\overline{E}_\alpha(x)$, and $E_1(x)$.

The simplest way to perform fuzzy risk analysis is to calculate the fuzzy expected value, and convert fuzzy risk into non-fuzzy risk, so as to obtain a clear number. There exists a problem concerning the transformation from a fuzzy set to a clear set. Therefore, it is necessary to locate the α level value in advance, and then select the element *x* with subordinate degree $A(x)\geq\alpha$. The fuzzy expected value of a possibility-probability distribution is a set with $\underline{E}_\alpha(x)$ and $\overline{E}_\alpha(x)$ as its boundaries. The fuzzy expected values $\underline{E}_\alpha(x)$ and $\overline{E}_\alpha(x)$ of a possibility-probability distribution represent the fuzzy risk value. Therefore, we can obtain a conservative risk value, a venture risk value and a maximum probability risk value. With different values of α, the results fall into different conceptual categories such as "low-risk area", "high-risk area" and "acceptable risk area" in the same geographical area.

This case study has been performed separately using α=0.1, 0.3, 0.6 and 1.0. The selection of the value α depends on the extent of confidence in different groups of people. The lower the confidence in people the bigger the difference between the conservative risk value and the venture risk value. In the case of secured confidence, both results may be reduced to one, i.e. the maximum probability risk value (R_M=951hPa).

Selection of a conservative risk value or venture risk value depends on the risk preference of different groups of people. For instance, an investment activity with small investment and significant benefits (regardless of high risk probability) in an area where landfall typhoon risk is high may appeal to a tourism project investor who might be interested in the venture risk value; In the building of a nuclear power station, an investor with a fairly large amount of capital may give up an activity with "possibly" high benefits, for an activity with low risk probability and select a conservative risk value. This result, which has a very clear and practical significance, can not only facilitate the study and application of the regional risk planning theory, but also helps scientists and decision-makers, and is in the public interest as it makes full use of information about the risk being studied.

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