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A Novel Vector-Field Transform for Image Analysis Inspired by Electrostatics

Xiao-dong Zhuang
Electronics & Engineering Department
Automation & Engineering College, Qingdao University
Qingdao 266071, China
zhuang@qingdaonews.com

Hui Zhu
College of Automation & Electronics Engineering
Qingdao University of Science and Technology
Qingdao 266042, China
x_sys@tom.com

***Abstract*—A novel vector field transform is proposed for gray-scale image based on the electro-static analogy. By introducing the item of gray-scale difference into the virtual repulsive force between image points, the diffusing vector field is obtained by the transform. The primitive areas can be extracted based on the diffusing vector field with a proposed area-expanding method. A novel image segmentation method is then presented by merging primitive areas. The experimental results prove the effectiveness of the proposed image segmentation method.**

***Keywords*—Image transform, diffusing vector field, primitive area, image structure, image segmentation**

I. Introduction

Image transform is fundamental for image processing theory and techniques [1]. The image transform generates another space or field, where some characteristics of the generated space may be exploited for effective and efficient processing of the image. Classical image transform includes mathematical transform such as Fourier Transform, Walsh Transform, etc. A relatively new technique is the Wavelet Transform. In these techniques, the digital image is regarded as a discrete 2-D function and is transformed to the coefficient space. A more general view of image transform may include the transformation to the feature space. The gradient field can be a typical case, which is generated by the convolution of the image and the gradient template. In the gradient field, the edge feature of the digital image can be extracted.

Currently, many image transform methods result in a space or field of scalar coefficients or scalar feature values. Some others can result in a vector field, such as the gradient field. The gradient templates can extract the components of image gradient on the direction of x-coordinate and y-coordinate respectively. A general idea about image transform may include transformation to both scalar space and vector field.

Because the vector field possesses information of both intensity and direction, the vector field transform may give a detailed representation of image structure and features. Some physics-based approaches have been applied in image processing, which take an electro-static analogy in the transformation from the image to the vector field [2,3]. Such methods have got effective and promising results in skeletonization, shape representation, human ear recognition, etc [2-7].

In this paper, a novel vector field is proposed to represent image structure of different areas in the image. The diffusing vector field is defined by extending the vector field of the electro-static analogy to a more generalized form. Based on the diffusing vector field, the source points of diffusing vectors can be extracted and the image can be decomposed to primitive areas, based on which the image segmentation can be implemented by merging the primitive areas. The experimental results indicate the effectiveness of the proposed segmentation method.

II. Diffusing Vector Field Generated by Images

In physics, a charged area with certain distribution of charge generates its electric field within and outside the area [8,9]. In this section, a novel vector transform of gray-scale image is proposed based on an electro-static analogy, in which the image is regarded as a charged area. In the proposed transform, the virtual field force is extended by introducing the gray-scale difference between the related image points. With such definition of the transform, in the generated field the vectors in a homogeneous area diffuse towards the outside of that area, and the generated field is named the diffusing vector field.

A. The electrostatic field force in physics

The force of two charges q_1 and q_2 is given as following [8,9]:

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon} \cdot \frac{q_1 q_2}{r_{12}^2} \cdot \frac{\vec{r}_{12}}{r_{12}} \qquad (1)$$

where $\vec{F}_{12}$ is the force of q_1 on q_2, $\vec{r}_{12}$ is the vector from q_1 to q_2, r_{12} is the length of $\vec{r}_{12}$, $4\pi\varepsilon$ is an item of constant.

The form of static electronic force can be introduced into vector field transform of images. If two image points are regarded as two charged particles, the force vector generated by one point on the other can be defined. Of course, such definition of vector transform between two image points

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must be reasonable and proper in order to reflect certain image features and thus has potential applications.

B. *The virtual repulsive force between image points*

The form of electronic force formula has some characteristics as follows:

(1) The formula has the power of distance r as one of the denominator. The larger the distance between two charged particles, the smaller the force. In images, this causes a kind of local feature extraction. One image point has strong effect on the points nearby, but has little effect on distant points.

(2) The force between two charged particles is related to the electric quantity of both charged particles. In images, the effect of one image point on the other point can also be defined with relation to the intensities (i.e. gray-scale values) of the two image points. Thus certain image features may be extracted by the vector field transform.

In this paper, the vector generated by one image point $g(i, j)$ on another position (x,y) is defined with direct relation to the reciprocal of the intensity difference of the two image points. The definition is proposed to generate virtual repulsive force between neighboring points in homogeneous areas. The virtual repulsive vector is defined as following:

$$\vec{V} = \frac{A}{(|g(i,j) - g(x,y)| + \varepsilon) \cdot r^2_{(i,j)\to(x,y)}} \cdot \frac{\vec{r}_{(i,j)\to(x,y)}}{r_{(i,j)\to(x,y)}} \quad (2)$$

where $\vec{V}$ is the vector generated by image point (i,j) on position (x,y), g represents the intensity of image points, $\vec{r}_{(i,j)\to(x,y)}$ is the vector from (i,j) to (x,y), $r_{(i,j)\to(x,y)}$ is the length of $\vec{r}_{(i,j)\to(x,y)}$, ε is a pre-defined small positive value which guarantees that the above definition is still valid when $g(i,j)$ is equal to $g(x,y)$, A is a pre-defined item of constant. According to the above definition, the two components of $\vec{V}$ are as following:

$$V_x = \frac{A \cdot (x-i)}{(|g(i,j) - g(x,y)| + \varepsilon) \cdot ((x-i)^2 + (y-j)^2)^{3/2}} \quad (3)$$

$$V_y = \frac{A \cdot (y-j)}{(|g(i,j) - g(x,y)| + \varepsilon) \cdot ((x-i)^2 + (y-j)^2)^{3/2}} \quad (4)$$

where V_x and V_y are the components on the direction of x-coordinate and y-coordinate respectively.

C. *The definition of diffusing vector field*

In the above sections, a definition of the repulsive vector is proposed for one image point on another. Based on the repulsive vector, the vector field transform can be defined for the whole image by summing up the vectors produced by all image points on any image points. The vector generated by the whole image on point (x,y) is defined as following:

$$\vec{V}(x,y) = \sum_{\substack{i=1 \\ (i,j)\neq(x,y)}}^{W} \sum_{j=1}^{H} A \cdot \frac{\vec{r}_{(i,j)\to(x,y)}}{(|g(i,j) - g(x,y)| + \varepsilon) \cdot r^3_{(i,j)\to(x,y)}} \quad (5)$$

where $\vec{V}(x,y)$ is the vector produced by the transform on position (x,y), W and H are the width and height of the image respectively. According to the above definition, the two components of $\vec{V}(x,y)$ are as following:

$$V_x(x,y) = \sum_{\substack{i=1 \\ (i,j)\neq(x,y)}}^{W} \sum_{j=1}^{H} \frac{A \cdot (x-i)}{(|g(i,j) - g(x,y)| + \varepsilon) \cdot r^3_{(i,j)\to(x,y)}} \quad (6)$$

$$V_y(x,y) = \sum_{\substack{i=1 \\ (i,j)\neq(x,y)}}^{W} \sum_{j=1}^{H} \frac{A \cdot (y-j)}{(|g(i,j) - g(x,y)| + \varepsilon) \cdot r^3_{(i,j)\to(x,y)}} \quad (7)$$

where $V_x(x,y)$ and $V_y(x,y)$ are the components on the direction of x-coordinate and y-coordinate respectively.

Because the effect of an image point on another decreases quickly with the increase of distance, the vector on any image point is determined by two major factors: the strong effect of a few neighboring points, and the accumulated effect of large amount of distant points. In the definition of the diffusing vector field, the smaller the gray-scale difference the relatively larger the vector magnitude. Therefore, a diffusing vector field will appear in each homogeneous area because the strong "repulsion" between similar image points. On the other hand, at the boundary of two different areas, the vectors field at one side of the boundary will be in opposite directions of those at the other side.

To investigate the property of the proposed transform, several simple test images are transformed to the diffusing vector field. The algorithm is implemented under the Visual C++ 6.0 developing environment. Three of the test images are shown in Fig. 1, Fig. 4 and Fig. 7. These images are of size 32×32. For a clear view, they are also shown 4 times of original size. Fig. 2, Fig. 5 and Fig. 8 show the magnitude of each vector in the transformed field respectively, where larger gray-scale values correspond to larger vector magnitude. The results are also shown 4 times of original size for a clear view. The direction of each vector in the transformed field is digitalized into 8 discrete directions for further processing. Fig. 3, Fig. 6 and Fig. 9 show the direction distribution of the transformed field for each test image.

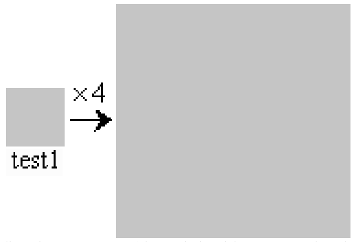

Figure. 1 The first image *test1* (the original image, and 4 times of original size on the right)

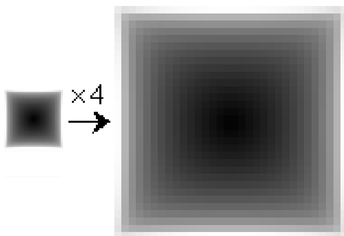

Figure. 2 The vector magnitude in the transformed field of *test1* (the original image; 4 times of original size on the right)

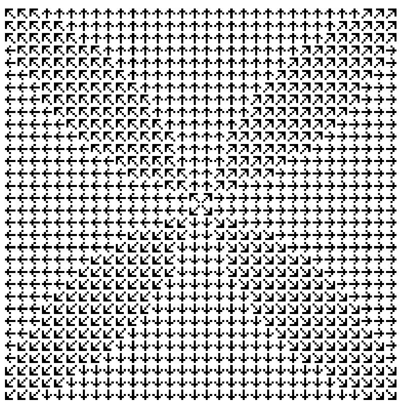

Figure. 3 The direction of each vector in the transformed field of *test1*

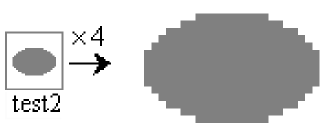

Figure. 4 The second image *test2* (the original image on the left, and 4 times of original size on the right)

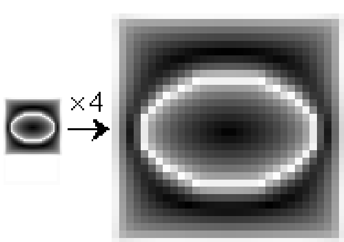

Figure. 5 The vector magnitude in the transformed field of *test2* (the original image; 4 times of original size on the right)

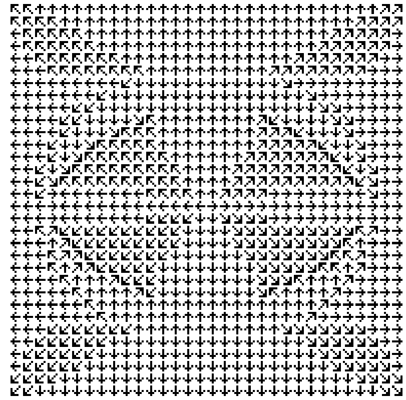

Figure. 6 The direction of each vector in the transformed field of *test2*

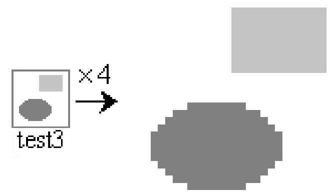

Figure. 7 The third image *test3* (the original image on the left, and 4 times of original size on the right)

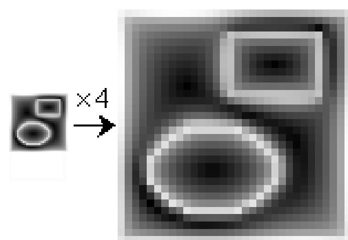

Figure. 8 The vector magnitude in the transformed field of *test3* (the original image; 4 times of original size on the right)

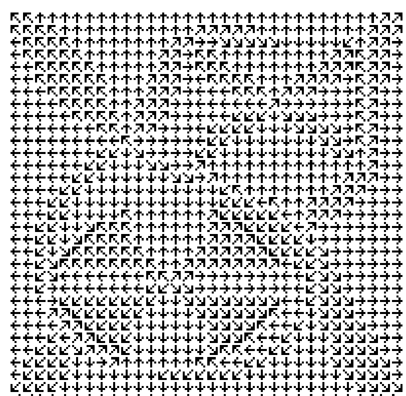

Figure. 9 The direction of each vector in the transformed field of *test3*

The image *test1* is an image of monotonous gray-scale, i.e. the whole image is a homogeneous area. In the transformed field of *test1*, it is obvious that the whole field is diffusing from the center of the image towards the outside. There is an ellipse area in image *test2*. In image *test3*, there are an ellipse area and a rectangle area. In their transformed fields, the fields in the homogeneous areas are diffusing outward from the center of each area. On the boundaries of the areas, it is obvious that the vectors at one side of the boundary line have opposite directions of those on the other side. The experimental results of the test images indicates that the proposed transform produce diffusing vector field within the homogeneous areas, but generates vectors of opposite directions at the two opposite sides along the area boundary.

III. IMAGE SEGMENTATION BASED ON THE DIFFUSING VECTOR FIELD

A. *The primitive areas in images*

The experimental results of the test image indicate that in the homogeneous area a diffusing vector field will be produced, and the diffusing field ends at the boundary of the homogeneous area because the vectors outside have opposite directions of those within the area along the boundary. Therefore, the homogeneous areas in the image can be represented by those with consistent diffusing vectors in the transformed field. Each diffusing vector area corresponds to an area of homogeneous image points. The area of consistent diffusing vectors extracted from the transformed field is defined as a primitive area, which can be regarded as an elementary component of an image because it is regarded as homogeneous in the transform process.

According to the definition, the image *test1* is a whole *primitive* area, while the image *test3* has at least two primitive areas: the ellipse, the rectangle and the background area. All the primitive areas can be extracted from the diffusing vector field, which can be exploited in further image analysis. In this paper, the primitive area forms the basis of the proposed image segmentation method.

B. *Diffusing center for the primitive area*

In each primitive area, the vector field diffuses from the center towards the outside, thus the area center becomes the source of the diffusing field. Therefore, the area centers are the begin points to extract primitive areas. Here the source of the diffusing field is defined as the diffusing center. According to the experimental results of the test images, the definition of the diffusing center is given as following: for a square area consists of four image points, if none of the vectors on these points has component of inward direction into the area, the square area is part of a diffusing center. Fig. 10 shows the allowed vector directions on each point in a diffusing center.

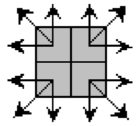

Figure. 10 The allowed vector directions in diffusing center

In Fig. 3, Fig. 6 and Fig. 9, according to the above definition the diffusing centers can be found, which are shown in Fig. 11, Fig. 12 and Fig. 13. The source points in the diffusing centers are indicated in gray.

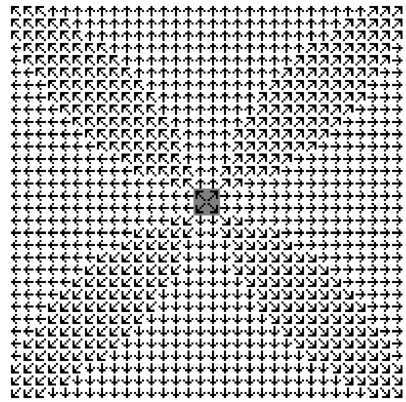

Figure. 11 The diffusing centers in Fig. 3

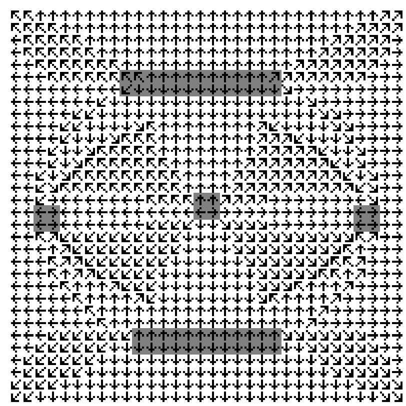

Figure. 12 The diffusing centers in Fig. 6

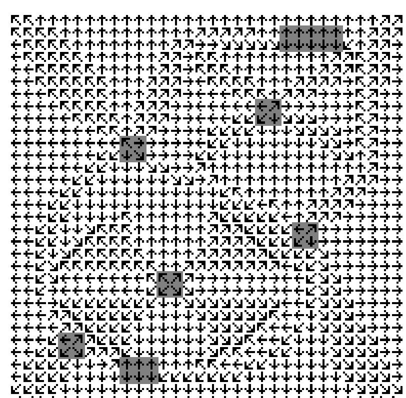

Figure. 13 The diffusing centers in Fig. 9

The image *test1* is a homogeneous area, therefore there is only one diffusing center found in Fig. 11. There is an area of ellipse in the image *test2*, and the diffusing center of the ellipse can be found in Fig. 12. Moreover, there are also four other diffusing centers found in the background area. The image *test3* has an ellipse and a rectangle. Correspondingly, in Fig. 13 there is one diffusing center for the ellipse, one for the rectangle, and five for the background area. It is indicated that in a large and irregular area there may be more than one diffusing center found, such as the background area.

C. Primitive area extraction by the area-diffusing method

The primitive areas are the basic elements in the diffusing vector field, which is a kind of representation of the image structure. From the analysis and experimental results in above sections, in a primitive area the vectors diffusing outwards from the diffusing center (i.e. the area center). Moreover, the diffusing vectors in the primitive area end at the area boundary where opposite vectors at the outside are encountered. Therefore, the primitive area can be extracted by expanding outwards from the diffusing center along the directions of the diffusing vectors. The proposed area-expanding method to extract the primitive area is as follows:

***step*1:**
Get the diffusing vector field of the image by the transform proposed in above sections, and each image point now has a vector on it (the vector is discretized into 8 directions).

***step*2:**
Label the diffusing center points in the diffusing vector field.

***step*3:**
Assign each diffusing center a unique area label (here a unique area number is given to the points in each diffusing center, while the points not in the diffusing center are left unlabeled).

***step*4:**
Then a process of area-expanding in the diffusing vector field is implemented to extract the primitive areas.

For each labeled point in the image, select five of its eight neighboring points that are nearest to its vector's direction. For each of the five selected neighboring points, if it is unlabeled and its vector is not opposite to the labeled point's vector (i.e. the area boundary is not reached), it is labeled the same area number of the labeled point. On the other hand, if the neighboring point has been labeled with another area number, a principle of least gray-scale difference is applied to decide which of the two areas the point should belong to. The difference between its gray-scale and either area's average gray-scale is calculated. The point will belong to the area with less gray-scale difference. By this way, the primitive area can expand by iteration until the area boundary is reached.

The above process is repeated until the areas all stop expanding (i.e. no more unlabeled point can be given a new area number).

***step*5:**
If there are still unlabeled points when the expanding of the areas stops, the principle of least gray-scale difference is applied to assign each unlabeled point an area number.

For each unlabeled point, calculate the difference between its gray-scale and the average gray-scale of its neighboring areas. Then this unlabeled point is merged into the neighboring area that is of the least difference.

The primitive areas are extracted for the three test images according to the proposed area-expanding method based on the diffusing vector fields. The experimental results are shown in Fig.14, Fig. 15 and Fig. 16. In these three figures, the original images and the results of primitive areas extraction are shown. The results are also shown 4 times of original size for a clear view. In these figures, different primitive areas are distinguished from each other by different gray-scale values.

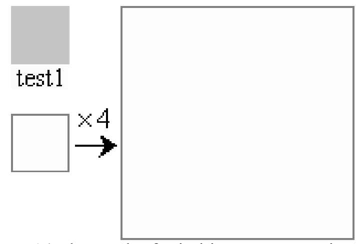

Figure. 14 The result of primitive area extraction for *test*1

The image *test*1 is a homogeneous area. Therefore the primitive area extracted in *test*1 is only one complete area (i.e. the image itself).

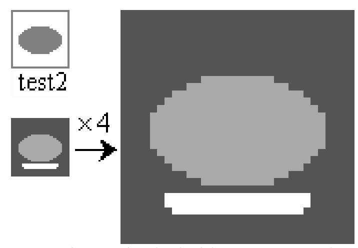

Figure. 15 The result of primitive area extraction for *test*2

The image *test*2 contains an ellipse, and 3 primitive areas are obtained. The ellipse is extracted as one primitive area, and there are 2 other primitive areas extracted in the background area of *test*2.

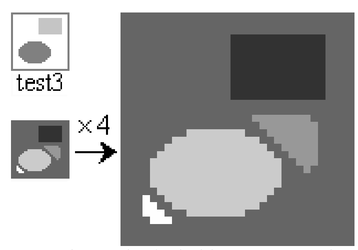

Figure. 16 The result of primitive area extraction for *test*3

The image *test*3 contains an ellipse and a rectangle, and 5 primitive areas are obtained. The ellipse and rectangle in ellipse and rectangle are extracted as two primitive areas, and there are 3 other primitive areas extracted in the background area of *test*3.

The experimental results for the test images show that the object areas can be extracted as primitive areas such as the ellipse in *test*2 and the ellipse and rectangle in *test*3. On the other hand, the number of primitive areas may be less than the number of diffusing center extracted. This is because two or more diffusing center may merge into one area in ***step*4** in the proposed area-expanding method.

D. Image segmentation based on the diffusing vector field and primitive area

Compared with the test images, real world images are more complex. The boundaries between areas in these images are not as clear and distinguishable as in the test images. In the experiments, the primitive areas are also extracted for the pepper image, the cameraman image and the house image. These images are of the size 128×128. The experimental results show that there are a large number of primitive areas extracted from the real world images. There are 341 primitive areas in the pepper image, 305 in the cameraman image and 263 in the house image. This is because the complexity of these images.

The primitive areas are a kind of representation of image structure. To implement meaningful image segmentation, area merging must be done to get more practically useful result. An area merging method is proposed to combine primitive areas based on the least gray-scale difference principle. First an expected number of remaining areas after merging is given. Then the following steps are carried out to merge areas until the expected area number is reached:

***step*1:** For each area in the image, calculate its average gray-scale.

***step*2:** Find the pair of neighboring areas with least average gray-scale difference, and merge them into one area.

***step*3:** If current area number is larger than the final area number, return to ***step*1**; otherwise, end the merging process.

The original image of the pepper image, the cameraman image and the house image are shown in Fig. 17, Fig. 19 and Fig. 21. The result of merging primitive area is shown in Fig. 18, Fig. 20 and Fig. 22 respectively, where different areas are distinguished from each other by different gray-scale values. Fig. 18 shows the result of merging 341 primitive areas into 20 areas for the peppers image. Fig. 20 shows the result of merging 305 primitive areas into 12 areas for the cameraman image. Fig. 22 shows the result of merging 263 primitive areas into 20 areas. The experimental results indicate that the image segmentation can be effectively implemented by the method of merging primitive areas.

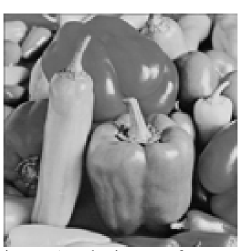

Figure. 17 The image of peppers

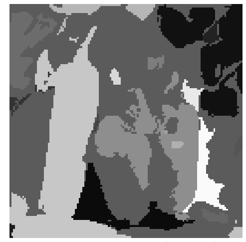

Figure. 18 The primitive area merging result for the peppers image (20 areas remained)

Figure. 19 The image of cameraman

Figure. 20 The primitive area merging result for the cameraman image (12 areas remained)

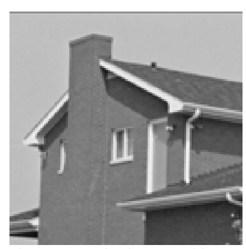

Figure. 21 The image of house

Figure. 22 The primitive area merging result for the house image (20 areas remained)

IV. CONCLUSION

In the research of image transform, vector field transformation is a promising methodology, in which both vector magnitude and vector direction can be exploited for feature extraction and analysis. Electro-static analogy has become a useful way for designing vector field transform of images. In this paper, the diffusing vector field transform is proposed by introducing the factor of gray-scale difference into the electro-static analogy. In the diffusing vector field of images, homogeneous areas are expressed as the areas with a vector group diffusing outwards from the center.

Based on the proposed transform, an effective image segmentation method is presented. By finding the area center and the area-expanding method, primitive areas can be extracted. Then image segmentation is implemented by merging the primitive areas. The experimental results indicate the effectiveness of the segmentation method. Objects can be successfully extracted in real world images in real world with the proposed method. Further research work will investigate more applications of the diffusing vector field transform in other tasks of image processing and analysis.

REFERENCES

[1] YuJin Zhang. Image Engineering: Image Processing and Analysis, TUP Press, Beijing, China, 1999.

[2] D. J. Hurley, M. S. Nixon and J. N. Carter, Force field feature extraction for ear biometrics, Computer Vision and Image Understanding, Vol. 98, No. 3, June 2005, pp. 491-512.

[3] D. J. Hurley, M. S. Nixon and J. N. Carter, Force Field Energy Functionals for Image Feature Extraction, Proceedings of the British Machine Vision Conference, 1999, pp. 604-613.

[4] Luo, B., Cross, A. D. and Hancock, E. R., Corner Detection Via Topographic Analysis of Vector Potential, Pattern Recognition Letters, Vol. 20, No. 6, 1999, pp. 635-650.

[5] N. Ahuja and J. H. Chuang, Shape Representation Using a Generalized Potential Field Model, IEEE Transactions PAMI, Vol. 19, No. 2, 1997, pp. 169-176.

[6] T. Grogorishin, G. Abdel-Hamid and Y.H. Yang, Skeletonization: An Electrostatic Field- Based Approach, Pattern Analysis and Application, Vol. 1, No. 3, 1996, pp. 163-177.

[7] G. Abdel-Hamid and Y.H. Yang, Multiscale Skeletonization: An electrostatic field-based approach, Proc. IEEE Int. Conference on Image Processing, Austin, Texas, November 1994, Vol. 1, pp. 949-953.

[8] I. S. Grant and W. R. Phillips, Electromagnetism, John Wiley & Sons, Second Ed., 1990.

[9] P. Hammond, Electromagnetism for Engineers: An Introductory Course, Oxford University Press, USA, forth edition, 1997.