Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

Feedback-Error-Learning For Stability of Double Inverted Pendulum

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Abstract— **Double Inverted Pendulum is a nonlinear system, unstable and fast reaction system. Double Inverted Pendulum is stable when its two Pendulums allocated in vertically position and have no oscillation and movement and also insertion force should be zero. The main target of this research is design a controller based on Neuro-Fuzzy methods by using feedback-error-learning for controlling double inverted Pendulum.**

Keywords— **Double Inverted Pendulum, Neural Network, Neuro-Fuzzy Controlling, Feedback error learning.**

I. INTRODUCTION

Pendulum system is one of the classical questions of study and research about non-linear control and a suitable criterion for harnessing the mechanical systems. The inverted pendulum, because of its non-linearity and instability is a useful method for testing control algorithms (PID controls, neural network, fuzzy control, genetics algorithm, etc.).In 1972, two researchers from the control domain succeeded, using an analogue computer, in controlling an inverted pendulum in standing position on a cart, which was stabilized by horizontal force [1]. The simple inverted pendulum, the rotary inverted pendulum, double inverted pendulum, and the rotary double inverted [2] pendulum are types of inverted pendulum in control systems. The double inverted pendulum is a non-linear, unstable and fast reaction system. This system consists of two inverted pendulums assembled on each other, as shown in fig. 1, and mounted on a cart that can be controlled and stabilized through applying the force F.

In stabilization of the double inverted pendulum diverse methods have been employed like using fuzzy control systems [1,3-5] and other methods like [6-9]. In this paper a controller for stabilization of a double inverted pendulum will be designed using the feedback error learning method.

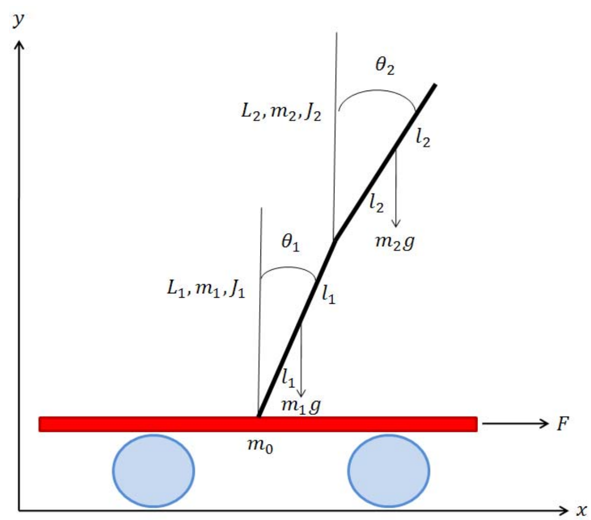

Figure 1. Double inverted pendulum on a cart

II. MODELLING

To control this system, its dynamic behavior must be analyzed first. The dynamic behavior is the changing rate of the status and position of the double inverted pendulum proportionate to the force applied. This relationship can be explained using a series of differential equations called the motion equations ruling over the pendulum response to the applied force. In fig. 1, two pendulums with m_1 and m_2 mass and L_1, L_2 length, and with mass center of l_1 and l_2 and a mass moment of inertia of J_1, J_2 have been assembled on each other. The said two rods are placed on a cart with the mass of m0, subject to the stabilizing force of F. Two angles θ_1 and θ_2 indicate the deviation value from the upright position. Also the friction effect has been ignored in calculations. To derive its equations of motion, one of the possible ways is to use Lagrange equations [10]:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = Q_i \quad i = 1, \dots, n \tag{1}$$

where L = T - V is a Lagrangian, Q is a vector of generalized forces (or moments) acting in the direction of generalized coordinates q and not accounted for in formulation of kinetic energy T and potential energy V.

The kinetic energy of the system, according the fig.1, is composed of the cart, the first pendulum and the second pendulum kinetic energies. The cart kinetic energy is:

$$T_0 = \frac{1}{2} m_0 \dot{x}^2 \tag{2}$$

The first pendulum kinetic energy is equal to the sum of three horizontal, vertical and rotational energy of pendulum.

$$T_1 = \frac{1}{2} m_1 [\dot{x} + l_1 \dot{\theta}_1 \cos\theta_1]^2 + \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 \sin^2\theta_1 + \frac{1}{2} J_1 \dot{\theta}_1^2 \tag{3}$$

The kinetic energy of the second pendulum is also identical to the first pendulum.

$$T_2 = \frac{1}{2} m_2 [\dot{x} + l_1 \dot{\theta}_1 \cos\theta_1 + l_2 \dot{\theta}_2 \cos\theta_2]^2 + \frac{1}{2} m_2 [L_1 \dot{\theta}_1 \sin\theta_1 + l_2 \dot{\theta}_2 \sin\theta_2]^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 \tag{4}$$

The system kinetic energy is gained from the sum of three (2), (3) and (4) equations.

$T=T_0+T_1+T_2$

$$T = \frac{1}{2} m_0 \dot{x}^2 + \frac{1}{2} m_1 [\dot{x} + l_1 \dot{\theta}_1 \cos\theta_1]^2 + \frac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 \sin^2\theta_1 + \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} m_2 [\dot{x} + l_1 \dot{\theta}_1 \cos\theta_1 + l_2 \dot{\theta}_2 \cos\theta_2]^2 + \frac{1}{2} m_2 \left[L_1 \dot{\theta}_1 \sin\theta_1 + l_2 \dot{\theta}_2 \sin\theta_2\right]^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 \tag{5}$$

By simplification of the relationship (5) we have:

$$\boldsymbol{T = \frac{1}{2} (m_0 + m_1 + m_2)\dot{x}^2 + \frac{1}{2}(m_1 l_1^2 + m_2 L_1^2 + J_1)\dot{\theta}_1^2 + \frac{1}{2}(m_2 l_2^2 + J_2)\dot{\theta}_2^2 + (m_1 l_1 + m_2 L_1)\dot{x}\dot{\theta}_1 \cos\theta_1 + m_2 l_2 \dot{x}\dot{\theta}_2 \cos\theta_2 + m_2 L_1 l_2 \cos(\theta_1 - \theta_2)\ \dot{\theta}_1 \dot{\theta}_2} \tag{6}$$

Now we calculate the potential energy of the system separately for three parts of the system. The cart potential energy is zero.

$$V_0 = 0 \tag{7}$$

The potential energy of the two pendulums will be as the following respectively:

$$V_1 = m_1 g_1 l_1 \cos\theta_1 \tag{8}$$

$$V_2 = m_2 g_2 (L_1 \cos\theta_1 + l_2 \cos\theta_2) \tag{9}$$

The system potential energy is gained from the sum of three (7), (8) and (9) equations.

$$V=V_0+V_1+V_2 \tag{10}$$

$$V = (m_1 l_1 + m_2 L_1) g \cos\theta_1 + m_2 l_2 g \cos\theta_2$$

Putting the (6) and the (10) equations in the Lagrange equation we have:

$$L = \tfrac{1}{2}(m_0 + m_1 + m_2)\dot{x}^2 + \tfrac{1}{2}(m_1 l_1^2 + m_2 L_1^2 + J_1)\dot{\theta}_1^2 + \tfrac{1}{2}(m_2 l_2^2 + J_2)\dot{\theta}_2^2 + (m_1 l_1 + m_2 L_1)\dot{x}\dot{\theta}_1 \cos\theta_1 + m_2 l_2 \dot{x}\dot{\theta}_2 \cos\theta_2 + m_2 L_1 l_2 \cos(\theta_1 - \theta_2)\ \dot{\theta}_1 \dot{\theta}_2 - (m_1 l_1 + m_2 L_1) g \cos\theta_1 - m_2 l_2 g \cos\theta_2 \tag{11}$$

Differentiating the Lagrangian by q and $\dot{q}$ yields Lagrange equation (1) as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = u \tag{12}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \frac{\partial L}{\partial \theta_1} = 0 \tag{13}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \frac{\partial L}{\partial \theta_2} = 0 \tag{14}$$

According to the equation (12), there is an external force of u only in X direction. Then we apply the equations (12), (13) and (14) separately on the equation (11).

$$\begin{aligned}(m_0 + m_1 + m_2)\ddot{x} + \; & (m_1 l_1 + m_2 L_1)\ddot{\theta}_1 \cos\theta_1 \\ & + m_2 l_2 \ddot{\theta}_2 \cos\theta_2 \\ & - (m_1 l_1 + m_2 L_1)\dot{\theta}_1^{\,2} \sin\theta_1 \\ & - m_2 l_2 \dot{\theta}_2^{\,2} \sin\theta_2 = u\end{aligned} \tag{15}$$

$$\begin{aligned}(m_1 l_1^2 + m_2 L_1^2 + J_1)\ddot{\theta}_1 + & (m_1 l_1 + m_2 L_1)\ddot{x}\cos\theta_1 \\ & + m_2 L_1 l_2 \dot{\theta}_2^2 \sin(\theta_1 - \theta_2) \\ & + m_2 L_1 l_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) \\ & - (m_1 l_1 + m_2 L_1) g \sin\theta_1 = 0\end{aligned} \tag{16}$$

$$\begin{aligned}(m_2 l_2^2 + J_2)\ddot{\theta}_2 + \; & m_2 l_2 \ddot{x} \cos\theta_2 \\ & + m_2 L_1 l_2 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) \\ & - m_2 L_1 l_2 \dot{\theta}_1^2 \sin(\theta_1 - \theta_2) \\ & - m_2 l_2 g \sin\theta_2 = 0\end{aligned} \tag{17}$$

In the first step, the state variables are selected. Then we assume the x as the cart displacement, x as the displacement velocity, θ_1 the first pendulum angle and $\dot{\theta}_1$ its angular velocity, θ_2 the second pendulum angle and $\dot{\theta}_2$ its angular velocity, all as the state variables of the double inverted pendulum system.

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \\ x_5(t) \\ x_6(t) \end{bmatrix} = \begin{bmatrix} x(t) \\ \dot{x}(t) \\ \theta_1(t) \\ \dot{\theta}_1(t) \\ \theta_2(t) \\ \dot{\theta}_2(t) \end{bmatrix} \tag{18}$$

The selected state variables must be replaced in the equations (15), (16) and (17). Three equations have been represented in matrix form for facilitation:

$$\begin{bmatrix} a_0 & a_1 \cos x_3 & a_3 \cos x_5 \\ a_1 \cos x_3 & a_2 & a_4 \cos(x_3 - x_5) \\ a_3 \cos x_5 & a_4 \cos(x_3 - x_5) & a_5 \end{bmatrix} \begin{bmatrix} \dot{x}_2 \\ \dot{x}_4 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} u + a_3 x_6^{\,2} \sin x_5 + a_1 x_4^2 \sin x_3 \\ a_1 g \sin x_3 - a_4 x_6^2 \sin(x_3 - x_5) \\ a_3 g \sin x_5 + a_4 x_4^2 \sin(x_3 - x_5) \end{bmatrix} \tag{19}$$

That

$a_0 = m_0 + m_1 + m_2$

$a_1 = m_1 l_1 + m_2 L_1$

$a_2 = m_1 l_1^2 + m_2 L_1^2 + J_1$

$a_3 = m_2 l_2$

$a_4 = m_2 L_1 l_2$

$a_5 = m_2 l_2^2 + J_2$

For further facilitation, the determinal of the first matrix of the (19) equation has been represented by the variable $\det E$. By linearization and replacement of the variables, the linear system equations will be as follows:

$$\det E = a_0 * (a_2 * a_5 - a_4^{\,2} \cos^2(x_3 - x_5)) - a_1 \cos x_3 * (a_5 a_1 \cos x_3 - a_3 a_4 \cos(x_3 - x_5) \cos x_5) + a_3 \cos x_5 * (a_1 a_4 \cos x_3 \cos(x_3 - x_5) - a_2 a_3 \cos x_5) \tag{20}$$

Now we write the non-linear system status space equations

$\dot{x}_1(t) = x_2(t)$

$$
\begin{aligned}
\dot{x}_2(t) = \frac{1}{\det E}\Big(&(u + a_3 {x_6}^2 \sin\ x_5 + a_1 x_4^2 \sin x_3) \\
&* (a_2 * a_5 - {a_4}^2\, cos^2(x_3 - x_5)) \\
&- a_1 \cos x_3 \\
&* \big(a_5(a_1 g \sin x_3 \\
&-\ a_4 x_6^2 \sin(x_3 - x_5)) \\
&- a_4 \cos(x_3 - x_5)\ (a_3 g sinx_5 \\
&+\ a_4 x_4^2 \sin(x_3 - x_5))\big) +\ a_3 cosx_5 \\
&* \big(a_4 \cos(x_3 - x_5)\,(a_1 g \sin x_3 \\
&-\ a_4 x_6^2 \sin(x_3 - x_5)) \\
&- a_2(a_3 g sinx_5 \\
&+\ a_4 x_4^2 \sin(x_3 - x_5))\big)\Big)
\end{aligned}
$$

$$\dot{x}_3(t) = x_4(t)$$

$$
\begin{aligned}
\dot{x}_4(t) = \frac{1}{\det E}(a_0 & \\
&* \big(a_5(a_1 g\ \ sin\, x_3 \\
&-\ a_4 x_6^2\, sin(x_3 - x_5)) \\
&- a_4\, cos(x_3 - x_5)\,(a_3 g sinx_5 \\
&+\ a_4 x_4^2 \sin(x_3 - x_5))\big) - (u \\
&+ a_3 {x_6}^2 sin\, x_5 + a_1 x_4^2\, sin\, x_3) \\
&* (a_5 a_1\, cos\, x_3 \\
&- a_3 a_4\, cos(x_3 - x_5)\ \ cosx_5) \\
&+\ a_3 cosx_5 * (a_1\, cos\, x_3\, (a_3 g sinx_5 \\
&+\ a_4 x_4^2\, sin(x_3 - x_5)) \\
&- (a_1 g\ \ sin\, x_3 \\
&-\ a_4 x_6^2\, sin(x_3 - x_5))\, a_3 cosx_5)\ )
\end{aligned}
$$

$$\dot{x}_5(t) = x_6(t)$$

$$
\begin{aligned}
\dot{x}_6(t) = \frac{1}{\det E} a_0 * \big(a_2 & \\
&* (a_3 g sinx_5 +\ a_4 x_4^2\, sin(x_3 - x_5)) \\
&- a_4\, cos(x_3 - x_5)\,(a_1 g\ \ sin\, x_3 \\
&-\ a_4 x_6^2\, sin(x_3 - x_5))\big) - a_1\, cos\, x_3 \\
&* \big((a_3 g sinx_5 \\
&+\ a_4 x_4^2 \sin(x_3 - x_5))a_1\, cos\, x_3 \\
&- a_3\ cosx_5 * (a_1 g\ \ sin\, x_3 \\
&-\ a_4 x_6^2\, sin(x_3 - x_5))\big) +\ (u \\
&+ a_3 {x_6}^2 sin\, x_5\ + a_1 x_4^2\, sin\, x_3) \\
&* (a_1 a_4\, cos\, x_3\, cos(x_3 - x_5) \\
&- a_2 a_3 cosx_5) + a_1 x_4^2\, sin\, x_3) \\
&* (a_1 a_4\, cos\, x_3\, cos(x_3 - x_5) \\
&- a_2 a_3 cosx_5)
\end{aligned}
\tag{21}
$$

III. FEEDBACK ERROR LEARNING(FEL)

The human being movements are formed based on the central neural system called as the motor control in neurophysiology. The motor control in the human mind is highly accurate compared with the human like robots, and acts conveniently. The cause is the deficiencies of the sensors and their inaccuracy and also the delays inherent in the systems or control processes. The brain acts like a two degrees freedom control, with inverse of the under controlled model in the feed forward path and presence of a controller in the feedback path. This method has already proved itself properly in the motor control of the human being's brain [10]. IN this method, the output of the classical controller is used as the learning signal of the inverse model. After complete constructing of the inverted process, the output of the classical controller is automatically off the circuit, for the error will be zero. Now, if any noise or change of the process parameters occurs, the classical controller is automatically activated and takes control of the system, starting constructing of a new inverse of the process.

IV. THE NEURO-FUZZY MAMDANI NETWORK WITH FEL

The neuro-fuzzy mamdani network structure consists of 5 layers. The first layer is considered as the input layer of this structure. The second layer undertakes the task of fuzzification of the inputs. The third layer indicates the fuzzy rules. The forth layer is used for normalization, causing the convergence of the network parameters. Finally, the fifth layer is the defuzzification layer. For the learning of the output layer weight of the radial basic function network the decrease gradient method is used. So the goal function is defined as the equation (22), where

$$E = \frac{1}{2}\sum e_i^2 \tag{22}$$

the e_i will be :

$$e_i = U_i - u_{ffi} \tag{23}$$

In equation (23), the U_i is the system input value and the U_{ffi} is the output value of the neural network in the time *i*. Through employing the decrease gradient algorithm that is the change of the weight coefficients towards the maximum decrease of the gradient, the parameters are adjusted in proportion to the error derivation in relation with the weights with opposite sign. The equation of the output layer weight adjustment is calculated from the equation (24) [11].

$$w_m(k+1) = w_m(k) + (-\eta \frac{\partial E(k)}{\partial w_m(k)}) \tag{24}$$

η is the learning rate of the network and $\frac{\partial E(k)}{\partial w_m(k)}$ is calculated as the following:

$$\frac{\partial E}{\partial w_m} = \frac{\partial E}{\partial e_i} * \frac{\partial e_i}{\partial u_{ffi}} * \frac{\partial u_{ffi}}{\partial w_m} = e_i(k) * (-1) *\ o_l^4 \tag{25}$$

o_l^4 is the output of the fourth layer and $e_i(k)$ is the feedback controller output or u_{fbi} . As a result, the average learning of the centers for the m node of the equation (24) is calculated from the following equation:

$$w_m(k+1) = w_m(k) +\ \eta *\ e_i(k) * o_l^4 \tag{26}$$

V. SIMULATION

The double inverted pendulum system of the neuro-fuzzy network is designed using FEL. The double inverted pendulum includes an input control signal and six outputs .The block diagram of this controller is shown as the fig. (2). The Controller of double inverted pendulum system will have 6 inputs, 18 membership function, and 729 rules (3^6).

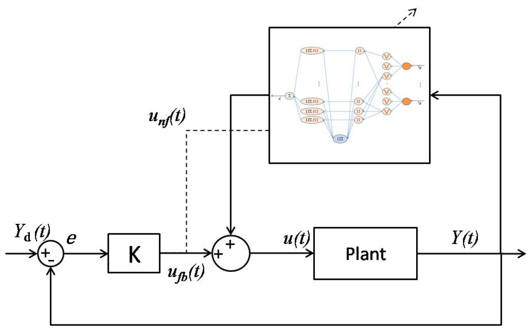

Figure 2 Feedback-error-learning with neuro-fuzzy

The initial conditions are determined as $x = [0; 0; 0.05; 0; -0.04; 0]$. The simulation result is shown as Figure 3 and Figure 4.

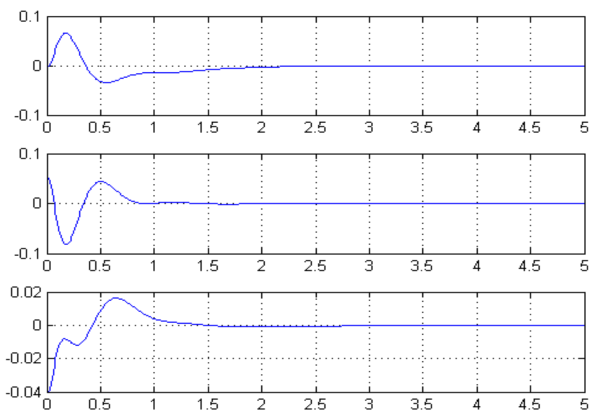

Figure 3. The simulation result for state variables, The initial conditions are θ_1=0.05 rad and θ_2=-0.04 rad

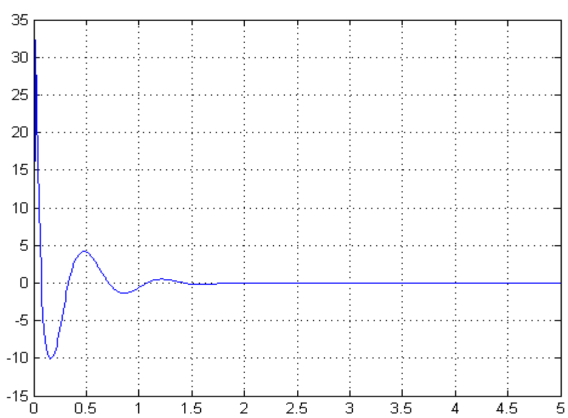

Figure 4. Simulation of controller output signal *U*.

And for another simulate, the initial conditions are determined as $x = [0; 0; 0; 0; 0; 0]$ and the desired conditions are determined as $x = [0.01; 0; 0; 0; 0; 0]$. The simulation result is shown as Figure 5.

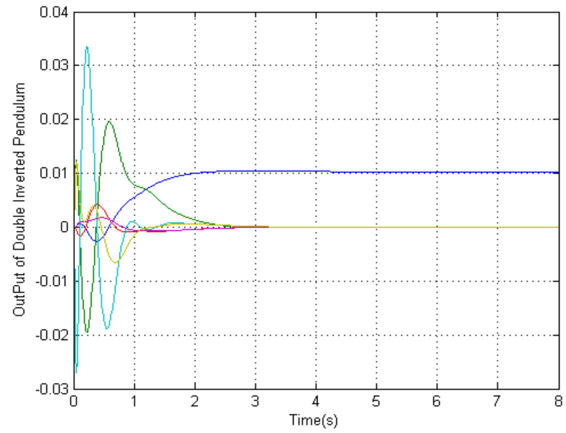

Figure 5. The simulation result for state variables

VI. CONCLUSION

For controlling the double inverted pendulum system, the state feedback and the neuro-fuzzy network together with the feedback error learning has been used. The state feedback in this structure undertakes providing the stability and the output of this controller have been used for learning of the weights of the neuro-fuzzy controller. Finally, by zero approaching of this output, the intelligent controllers take control of the system. Taking into account the learning of the system by the feedback, occurring the error status in the preliminary moments and in the noise cases are lesser than the direct situation and the intelligent controller causes the system to reach faster to the desirable values. In this method, considering the existence of the state feedback, the stability of the system is more reliable.

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