Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

A Hamilton-Jacobi Formulation for Cooperative Control of Multi-Agent Systems

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***Abstract*—Coordination of autonomous non-point agents is studied in this work. Interactions between vehicles together with their internal dynamics have been modeled as a continuous, unconstrained differential game. In particular, collision prediction is shown to be achievable in a cooperative scheme through solving the boundary value equation associated with the dual problem. Optimal control formulation has been further applied in order to solve the coordination problem while coping with both time constraints and energy aspects of the system. The developed control scheme is further contrasted with the traditional negated gradient laws, which on the other hand can be shown to correspond to Nash Equilibria of non-cooperative strategies, in terms of its complexity and resultant trajectories. Furthermore, state feedback formation control is combined with the proposed cooperative scheme to alleviate its computational concerns. In addition, conflict-free navigation and convergence properties of the proposed scheme are verified in simulation.**

***Index Terms*—Optimal Control, Multi-Vehicle Systems, Collision Avoidance, Cooperative Coordination.**

I. INTRODUCTION

The line of research on cooperative control is motivated by various applications in sensor networks, flight management systems, and automated highways. Cooperative control considers the problem of manipulating the internal states of a multi-variable system such that the output of the system demonstrates desirable properties. In other words, the problem can be stated as that of assigning values to local variables such that a collective objective is attained while satisfying state and/or input constraints.

In this work, we are interested to develop cooperative strategies to resolve conflicts among autonomous vehicles in crossings. It was suggested in [1] that the proposed navigation function be combined with optimal control schemes in order to achieve different navigation patterns in intersections. Toward this goal, we bind the differential game premises with potential functions and demonstrate that some of the well-known shortcomings of potential functions such as their deficiency in dealing with non-convex obstacles can be circumvented in optimal control schemes. The major difference is that in the proposed scheme the trajectories are generated backwards in time whereas in [1] the negated gradient field is the only clue on how to approach the destination. By imposing the final configuration through boundary equations and introducing the collision relation of the navigation function together with an energy term as the cost-to-go in the optimization problem, a different approach has been developed in this work. It should be mentioned that the two-boundary-value problem that is faced by the optimal planner requires solving a cooperative optimization problem.

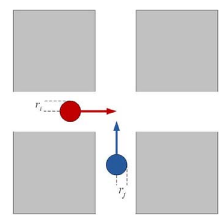


Fig. 1. A conflict scenario with two agents

Optimization problems in cooperative control frameworks are known to be computationally expensive (see [2] for details). Hence, we provide leader based formation control in order to reduce the complexity of the proposed approach. By clustering into lines, not only do the agents free some space, but also they allow the planner to resolve the conflicts amongst the leaders rather than individual agents which can lead to a significant decrease in the calculation cost.

The rest of the paper is organized as follows: The problem formulation is presented in Section II. Section III provides an elementary introduction to optimal control premises used in this work. In the same section cooperative navigation in multi-vehicle systems will be studied. Section IV takes a rather non-cooperative approach to the same problem and analyzes navigation from the point of view of single agents. Nash equilibrium solutions are presented and proven theoretically. We extend the same concept toward state-feedback formation in multi-vehicle systems in section V and conclude in section VI.

II. PROBLEM FORMULATION

We consider a conflict scenario involving a set of N agents passing through the same intersection $I \subset \mathbb{R}^2$. $q_i \in I$ represents the position of the center of agent i which is assumed to occupy a disc of radius r_i. We further assume that the planner has access to global position and desired destination of each agent $q_{d_i} \in D$ where $D \subset \mathbb{R}^2$ stands for the set of desired destinations. Fig. 1 shows a conflict scenario involving two independent agents. It is further assumed that each vehicle is able to communicate with a set of other vehicles. A graph $G = (V, E)$ represents the feasible communications where the nodes V represent the individual vehicles and edges E exists for each communication channel. The set of the neighbors for each vehicle is given by $N^i(G)$. Furthermore, we assume that

978-1-4244-2794-9/09/$25.00

the nearest neighbor for each vehicle is the one among its neighbors that is closest to its destination:

$$N_N^i(G) = \{v \in N^i(G) \mid \ \|q_v - q_{d_i}\| \leq \|q_j - q_{d_i}\| \\ for\ all\ j \in N^i(G)\}$$

Moreover, the motion of each agent is described by a first order dynamics:

$$\dot{q}_i = u_i \tag{1}$$

for each i where $q_i = (q_{x_i}, q_{y_i})^T$ and $u_i = (u_{x_i}, u_{y_i})^T$ are respectively the state and control vectors. The problem is to find the proper control inputs for each agent i such that the controlled agent, undergoing the dynamics given in (1), can reach its destination while avoiding collisions with other agents $j \neq i$. In addition, it is assumed that the desired destinations are located somewhere outside of the working space, i.e, $(D \cap I) = \emptyset$ so that the agents disappear from the environment when they reach their destination. Therefore, the concept of convergence to a final configuration is not critical in this work.

III. Cooperative Control

In this work, we consider the navigation of multi-vehicle systems in intersections. Hence, we map the task of crossing the intersection and avoiding potential collisions into a global cost-to-go. Toward this goal, we state and formulate the underlying optimal control problem and provide a clear notion of some terms and notations that will be used later.

A. Optimal Control Preliminaries

The problem considered as unconstrained optimal control in the literature [5] can be stated in the following form:

Problem 1: Find a measurable control $u(.) : [0,T] \rightarrow R^m$ on the fixed interval $t \in [0,T]$ so as to minimize a scalar function $\phi[x(T)]$ of the terminal state $x(T)$ such that the state trajectory $x(.) : [0,T] \rightarrow R^n$ continues to satisfy the following constraint:

$$\dot{x}(t) = f(x,u)\ ,\ x(0) = x_0$$

and perhaps satisfies an equality constraint of the following form:

$$\psi[x(T)] = 0$$

which corresponds to a terminal state.

In order to satisfy the terminal state constraint while maintaining the dynamics, it is required to assume that the variational equation of the system is controllable. Otherwise, it should be assumed that the initial and final state constraints are imposed on a controllable subset of the system. In some cases, it is possible to perform input-state linearization and transform the system into its controllable canonical form in order to identify the controllable subset of the system.

Having these assumptions in mind, we construct an augmented cost for *Problem 1* by penalizing the constraints through the use of Lagrange multipliers and further adding them to the original cost. Since the state equation holds along the trajectories of the system, one can write:

$$\int_0^T \lambda^T(-\dot{x} + f)dt = 0$$

$$J = \phi[x(T)] + v^T\psi[x(T)] + \int_0^T \lambda^T(-\dot{x} + f)dt \tag{2}$$

Applying the integration in part, one gets:

$$J = \phi[x(T)] + v^T\psi[x(T)] - \lambda^T(T)x(T) + \int_0^T (\dot{\lambda}^T x + \lambda^T f)dt$$

We define the following Hamiltonian for the above system:

$$H = \lambda^T f \tag{3}$$

We further let the pair x^*, u^* be a point and the corresponding control path which locally minimize J. If such a point exists, it will behave as a local attractor in the system. Hence, if we locally perturb the control path:

$$u(.) = u^*(.) + \epsilon p(t)$$

we will get trajectories which stay in a close neighborhood of x^* and will eventually converge to it. Traditional theorems in optimization state that the following holds at x^*:

$$\frac{\partial J(x^*, u^*)}{\partial x} = 0, \frac{\partial J(x^*, u^*)}{\partial u} = 0$$

Applying the above condition to (2) and (3), one will get the following two boundary value problem:

$$\dot{\lambda}^*(t) + \frac{\partial H(x^*, u^*, \lambda^*, v^*, t)}{\partial x} = 0, \lambda^*(T) = \frac{\partial \phi}{\partial x}[x(T)] \tag{4}$$

$$\frac{\partial H(x^*, u^*, \lambda^*, v^*, t)}{\partial u} = 0 \tag{5}$$

$$\dot{x}^*(t) = f(t, x^*, u^*), \quad x^*(0) = x_0 \tag{6}$$

The above three equations are often known as Euler-Lagrange equations [3] which correspond to a minimization problem with no state inequality. It should be noted that the collision avoidance can also be obtained through introducing state inequalities. In the presence of state inequalities, solving the boundary value problem requires additional certificates than (4) on the dual variable λ^*. In this work, we do not deal with the state inequalities and focus on attaining collision avoidance by assigning a proper navigation function as the cost of the optimization problem.

B. Navigation Function

A navigation function can be viewed as a smooth mapping which is analytic on the working space and whose negated gradient is attractive toward the goal and repulsive from moving and static obstacles. Therefore, such a function can be combined with proper control laws in order to derive agents toward their destination on collision free trajectories. The following decentralized navigation function has been proposed in [1]:

$$\begin{aligned}\phi_i &= \gamma_1 \left\|q_i - q_{d_i}\right\|^2 + \gamma_2 \sum_{j\neq i} \tfrac{1}{\alpha + \|q_i - q_j\|^2 - (r_i + r_j)^2} \\ &+ \gamma_3 \sum_{k=1}^{m} c_k(q_i)\end{aligned} \tag{7}$$

The above function is composed of three terms. The first term is the squared distance of agent i from its destination and attains small values as the agent approaches the goal. The second term is a collision relation between agents i and all other agents and its negated gradient is repulsive from them. It is worth mentioning that no agents can enter the physical boundary of other agents, therefore, the denominator of the second term is always positive provided that α is a positive constant. Furthermore, it is interesting to note that the closer two agents are the higher is the value of the second term. The collision relation of agent i is composed of a constant γ_2 which determines the importance of the second term with respect to other terms, and a rational function proportional to the inverse of its relative distance from other agents. It should be emphasized that the negated gradient is the most repulsive from the nearest agent. The third term is to avoid static obstacles and walls, i.e., $c_k(q_i)$ is a positive definite function which indicates the k-th nonlinear constraint of the environment. r_i and r_j are radius of the discs occupied by agents i and j respectively. α,γ_1,γ_2 and γ_3 are positive constants. It is worth mentioning that the constant α determines the magnitude of the repulsive force generated by agent i which is experienced by agent j when it has come to a close vicinity of agent i.

C. Multi-Vehicle Systems

In order to tackle the problem stated in section II, one can define a performance function which maps the accomplishment of the task into a cost function for *Problem 1*:

$$J_i = \phi[x(T)] + \int_0^T g(t, x(t), u(t))dt \tag{8}$$

The cost function defined above is comprising of two terms. The first term is the same as the one defined in *Problem 1* and penalizes the final state of the system. The integral can be used to penalize the energy consumption or to define an ideal path for the system. Note that the necessary conditions for optimality that were obtained early in this section were independent of the structure of the cost. Hence, the additional terms in the integral can be simply added to the Hamiltonian,i.e., the equation (3) can be modified as:

$$H = g + \lambda^T f \tag{9}$$

and the equations (4)-(6) still hold. A cooperative strategy is an assignment of control inputs to vehicles such that the cost in (8) is minimized. It remains to find a cost which, when minimized, puts the agents in their desired locations and does so, ideally, while respecting the time and energy constraints of the system. Artificial potential functions are good candidates to accomplish the task defined above and can be combined with energy terms to obtain the desired characteristics of the system. In particular, the cooperative control problem defined in (9) is considered in combination with the navigation function in (7). The following definition is adopted from [3]:

Definition 1: Consider a problem with $i = 1, ..., N$ players and a set of strategies $\gamma \in \Gamma$. Each players controls a local strategy γ_i and likes to minimize a local cost function $J_i(\gamma)$. A set of strategies is called Pareto Efficient if the set of strategies which allow for

$$J_i(\gamma) \leq J_i(\tilde{\gamma}), i = 1, ..., N$$

with at least one strict inequality is empty.
The following problem is of particular interest:

Problem 2: Consider the navigation of multiple vehicles $i = 1, ..., N$. Find a Pareto optimal path which minimizes the following cost function:

$$J = \sum_{i \in N} J_i$$

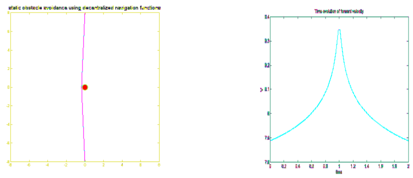
where

$$J_i = (q_i[T] - q_{d_i})^T Q(q_i[T] - q_{d_i}) +$$
$$\int_0^T \gamma_2 \sum_{j\neq i} \frac{1}{\alpha + \|q_i - q_j\|^2 - (r_i + r_j)^2} dt$$

subject to:

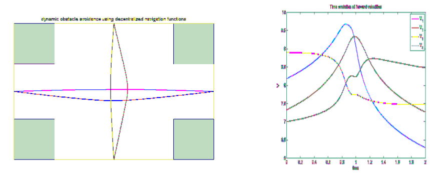
$$\dot{q_i} = u_i \ , \ \ q_i[0] = q_{i_0} \ \ \forall i \in N$$

It is worth mentioning that the cost function defined in *Problem 1* is a multi-variable non-convex function. Hence, the problem is implicitly seeking for a local optimal control. However, a Pareto optimal path shall be provided. In our framework, this gives a notion of efficiency as no vehicle can improve its status through local changes on its control input without incuring damage to other vehicles. As will be seen in the formation section, the optimality may require that some agents wait more than others. Note that we are relying on a numerical solver to find the optimal inputs. Hence, all claims on optimality are both approximate and local. It should be further noted that the constraints provided to the solver must be consistent and feasible. Hence, the inclusion of numerous vehicles may lead to complicated scenarios in which some vehicles will not be able to meet their assigned deadlines.



(a) Trajectories (b) Velocities

Fig. 2. Simulation in MATLAB: *a) trajectory:* the agent is able to avoid the obstacle using the control law given in *Proposition 1*. *b) velocity:* The agent accelerates in advance in order to compensate for the delay caused by the obstacle.



(a) Trajectories (b) Velocities

Fig. 3. Simulation in MATLAB: *a) trajectory:* the agents are able to avoid each other through the cooperative control law given in *Proposition 1*. The boundary equation is solved in MATLAB. *b) velocity:* Some agents accelerate in advance in order to compensate for the delay caused by their neighbors. They decelerate after the conflict is resolved. The pattern is reversed for the other agents.

Proposition 1: Define the Hamiltonian as follows:

$$H = \sum_{i=1}^{N} u_i^T R_i u_i + \sum_{i=1}^{N} \lambda_i^T u_i + \sum_{j \neq i} \frac{1}{\alpha + \|q_i - q_j\|^2 - (r_i + r_j)^2}$$

The optimal control satisfies the following system of boundary equations:

$$2Ru_i^* + \lambda_i^* = 0 \quad \forall i \in N$$

$$\dot{\lambda}^* = \sum_{i \neq j} \frac{2(q_i^* - q_j)}{(\alpha + (q_i^* - q_j)^T (q_i^* - q_j))^2} \quad \forall i \in N$$

$$\dot{q}_i^* = u_i^* \quad \forall i \in N$$

subject to:

$$q_i(0) = q_{i_0} \ , \ \ q_i(T) = q_{i_d} \quad \forall i \in N$$

The tradeoff between energy consumption and security is of vital importance. A too heavy weight on the control input will risk the vehicles in adopting shortest path strategies and even incurring conflicts in order to avoid the energy expenses. A protocol is provided in MATLAB (see [8]) which scans the motions of vehicles and detects the potential conflicts in advance. This can be done by taking the inner product of the relative positions and velocities. More precisely, for any two vehicles that navigate within the range of the intersection if

$$c(q_i, v_i, q_j, v_j) = (q_i - q_j)^T (v_i - v_j) + 1$$

is close to zero, a conflict might exist. If a conflict exists, all vehicles in the range of the intersection will be urged to adopt the centralized policy. This helps sustain the optimality of the trajectories found by the solver. It is also possible to resolve the conflicts for a subset of the agents in order to increase the speed of the solver and assign priorities to centralized trajectories. Other vehicles can rely on local controllers which will be explained in the non-cooperative section of this report. The results for a cooperative scenario are shown in Fig. 3.

IV. Non-Cooperative Control

The same control law as the one proposed in [4] can be applied in order to generate trajectories which lie on the negated gradient of the above navigation field:

$$u_{q_i} = -K_i \frac{\partial \phi_i}{\partial q_i} \tag{10}$$

Note that u_{q_i} denotes an arbitrary component of the control vector. Intuitively, the above control law attempts to minimize the navigation function in (7) by deriving the controlled agent i on the negated gradient of the function. (7) when combined with (10) will generate trajectories which are directed toward the goal and avoid both static and moving obstacles.

Definition 2: A set of control inputs $u_1, ..., u_N$ for the agents in the scenario given in Problem 7 is said to define a Nash equilibrium for the game if:

$$J_i(u_1^*, ..., u_i^*, ...u_N^*) \leq J_i(u_1^*, ..., u_i, ...u_N^*) \quad \forall i, \forall u_i \in U$$

where U is the set of admissible control.
The following is proposed:
Theorem 1: The set of inputs $(u_1, ..., u_N)$ *where* $u_i = -K_i \frac{\partial \phi}{\partial q_i}$ *defines a Nash equilibrium for the game described in Problem 7 with the following cost function:*

$$J_i = \int_0^T (uu^T + K_i \nabla\phi_i^T \nabla\phi_i) dt, \ \ i = 1, ..., N$$

where ϕ_i *is given by (7).*

Proof: The claim is that the control law given by (10) is a weakly dominant strategy for each agent. Hence, it is also a Nash equilibrium. Since any local optimal control is weakly dominant, it suffices to show that the given law satisfies the local optimality axioms. Define the Hamiltonian as follows:

$$H = u^T u + \lambda^T u + K_i \nabla\phi_i^T \nabla\phi_i$$

optimality requires that:

$$\lambda^* = -2u^*$$

which is:

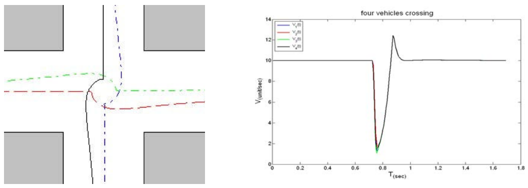
$$\lambda^* = -2K_i \nabla\phi_i$$

it remains to show that:

$$\dot{\lambda} + \frac{\partial H}{\partial q} = 0$$

We start from $\lambda^* = 2K_i \nabla\phi_i$ and derivate both sides w.r.t time to obtain:

$$\dot{\lambda}^* = 2K_i \nabla\dot{\phi}_i$$



(a) Trajectories (b) Velocities

Fig. 4. Simulation in MATLAB: *a) trajectories:* from a scenario involving 4 agents passing through the same crossing in a symmetric situation. *b) velocities:* They all decelerate at the same time and accelerate after resolving the conflict. This might imply some lose of efficiency

that is:

$$\dot{\lambda}^* = 2K_i\nabla^2\phi_i\dot{q}_i = -2K_i\nabla^2\phi_i\nabla\phi_i$$

Having $\dot{\lambda}^*$ from the above, one can write:

$$-2K_i\nabla^2\phi_i\nabla\phi_i + 2K_i\nabla^2\phi_i^T\nabla\phi_i = 0 \qquad \text{Q.E.D}$$

Note that the cost function given in *Theorem 1* has interesting properties. Near a local minimum for a smooth function such as navigation functions the size of the gradient is known to be small. It is also known that at any point of the function, the negative gradient points to direction along which the magnitude of the gradient decreases the most. Intuitively, the above control law attempts to minimize the navigation function in (7) by deriving the controlled agent i on the negated gradient of the function. (7) when combined with (10) will generate trajectories which are directed toward the goal and avoid both static and moving obstacles. This is a familiar results viewed from the point of view of multi-player games. The equilibrium of *Theorem 1* has also desirable energy characteristics. It can be perceived from the cost function that the control policy minimizes a reasonable energy cost in the system.

V. Formation Control

Feedback Formation Protocol: Each agent i, except for the leader, follows its nearest neighbor $j = N_N^i$ of the its same type through the following formation dynamics:

$$\dot{q}_i = q_j - q_i + d_{ij}$$

which is the consensus equation except for d_{ij} which is distributed as additional error over the consensus. One component of d_{ij} is always zero, depending on the type of the agents. The other components must be defined in such a way that prevents collisions and imposes a desirable security zone for the platoon: $r_i + r_j < |d_{ij}| < r_i + r_j + s$. For instance, s can be defined as the minimum length of the vehicles, i.e., $s = 2r_{min}$ where r_{min} is the radius of the smallest disc in the working arena. The leader of each type is the closest to leave the intersection.

The above protocol is shown in Fig. 5. The claim is that the above formation graph is controllable in input-state feedback, i.e., using inputs on the leader, we can derive the whole platoon from to a desirable state.

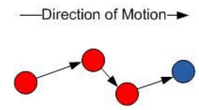


Fig. 5. Leader-follower network. The leader is the right most node, in the direction of motion.

Theorem 3: The feedback formation protocol is leader-follower stable, i.e., the following system can be controlled using the input on the leader.

$$\dot{q} = \begin{bmatrix} 0 & 0 & \cdots & & 0 \\ 1 & -1 & \cdots & & 0 \\ 0 & 1 & -1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & -1 \end{bmatrix}_{n\times n} q + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{n\times 1} u$$

Note that the leader is assumed to be agent 1, without loss of generality.

Proof: It suffices to prove that the system is controllable in the open loop form [7]:

$$\dot{q} = \begin{bmatrix} 1 & 0 & \cdots & & 0 \\ 1 & 0 & \cdots & & 0 \\ 0 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \ddots & \vdots \\ 0 & 0 & \cdots & -1 & 0 \end{bmatrix}_{n\times n} q + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}_{n\times 1} u$$

which can be transformed to the feedback form using the feedback gain:

$$F = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$$

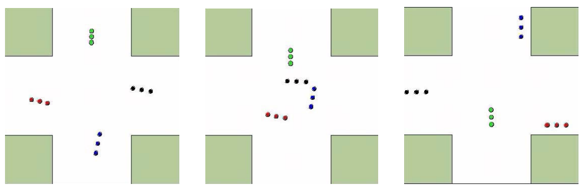
Now, for any n, the controllability matrix is the upper triangular unity matrix:

$$C = \begin{bmatrix} 1 & 1 & \cdots & & 1 \\ 0 & 1 & \cdots & & 1 \\ 0 & 0 & 1 & \cdots & 1 \\ \vdots & \vdots & \cdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

Thus, the leader-follower network is controllable. Q.E.D

A. Cooperative Multi-Vehicle Formation

The feedback formation protocol introduced in the previous section can be combined with the cooperative control protocols of section 2.4, in order to achieve conflict-free trajectories for a group of vehicles. Local changes can be made to the centralized solution in order to guarantee collision avoidance.



(a) Conflict (b) Avoidance (c) Resolved

Fig. 6. Each leader avoids its adjacent platoons through the cooperative scheme of *Proposition 2*. Planning ahead of time allows for collision prediction as shown in the figures above. See [8] for animated movies.

For instance, if the approximate trajectories do not respect the security zones, the agents can rely on their own decentralized controller to avoid possible collisions.
We need an additional definition for conflict-free platooning.

Definition 4: For every leader l, the set of collision relations will be defined w.r.t members of every neighbor platoon. The set of such members is defined as:

$$N_O^l = \{j \notin \Omega^l\}$$

where Ω^l is the set of the agents in the formation graph of leader l.
The following navigation scheme is further proposed:

Proposition 2: For the leaders in each of the M leader-follower networks, define the Hamiltonian as follows:

$$H = \sum_{l=1}^{M} u_l^T R_l u_l + \sum_{l=1}^{M} \lambda_l^T u_l$$

$$+ \sum_{l=1}^{M} \sum_{j \in N_O^l} \frac{1}{\alpha + \|q_l - q_j\|^2 - (r_l + r_j)^2}$$

The optimal control satisfies the following system of boundary equations:

$$2Ru_l^* + \lambda_l^* = 0 \quad \forall l$$

$$\dot{\lambda}_l^* = \sum_{j \in N_O^l} \frac{2(q_l^* - q_j)}{(\alpha + (q_l^* - q_j)^T (q_l^* - q_j))^2} \quad \forall l$$

$$\dot{q}_l^* = u_l^* \quad \forall l$$

subject to:

$$q_l(0) = q_{l_0}\ ,\ \ q_l(T) = q_{l_d} \quad \forall l$$

Conflict free trajectories for the leaders can be obtained by solving the above boundary value equations. Feedback formation protocol can be combined with the centralized inputs in order to obtain a semi-decentralized navigation scheme in our framework.
Fig. 6 shows a view of the platform in operation using the above navigation scheme. Animated simulation of the presented scenario is available at [8].

VI. Conclusion

Cooperative control schemes were developed for navigation of multi-vehicle systems. Optimality of the proposed schemes was demonstrated analytically in both cooperative and non-cooperative cases. In non-cooperative frameworks, it was further demonstrated that the negated gradient law commonly used for potential functions corresponds to an efficient Nash Equilibrium of the underlying multi-player game. Stability in this local sense, preventing local changes from further minimizing the cost objective, invokes a sense of cooperation between the vehicles in their non-cooperative environment.
Furthermore, a future research might aim at analyzing the convergence of the formation under potential connection failures. Robustness measures can be provided to approve the collision-free navigation under communication failures. A more practically concerned research may aim at investigating the feasibility of providing approximate solutions to the boundary value problem in order to improve the computational efficiency of the proposed scheme.

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