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Direct Adaptive Controller for Nonaffine Discrete-Time Systems Based on Fuzzy Rules Emulated Networks

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***Abstract*—A direct adaptive control system for a class of unknown nonaffine discrete-time plants is introduced in this article. The proposed control law is constructed by the estimated system linearization with adjustable networks called Muti-input Fuzzy Rules Emulated Networks or MIFRENs. Only on-line learning phase, the bounded parameters inside MIFRENs and the boundary of control error are given by the proposed theorem. The validation of the main theorem is demonstrated by computer simulation system.**

***Index Terms*—Nonlinear discrete-time; Fuzzy logic; Neural networks; Adaptive control.**

I. INTRODUCTION

An adaptive controller based on system linearization with artificial intelligence techniques such as neural networks, fuzzy logic systems and neurofuzzy networks of a class of unknown discrete-time dynamic systems has been an active research field recently. The closed-loop system stability and tracking error have been analyzed in the case of neural network adaptive control [4], [5] but during the learning phase the stability and convergence cannot be ensured because of the special conditions. The system stability or bounded signals analysis has been verified [1], [7] and references therein. The discrete-time projection has been introduced for adaptive control systems in [9]. In [8], the unknown nonlinear part has been compensated by neural networks and the closed-loop system stability has been also guaranteed for a class on discrete-time systems. The dead-zone function has been applied for feedback linearization systems [6] in the case of robust system but this control algorithm is only limited for the system with slow trajectory tracking.

In this work, we introduce the controller for a class of nonlinear discrete-time systems with estimated unknown nonlinear functions by Muti-input Fuzzy Rules Emulated Networks (MIFRENs). These nonlinear functions are occurred when the control law is constructed and they are completely unknown *a priori*. All adjustable parameters inside MIFRENs are automatically tuned by the proposed leaning algorithm. By the theoretical analysis, these parameters are all bounded during the system operation with out any request of off-line learning phase. The closed-loop tracking error is also bounded by the universal function approximation of MIFREN.

II. STATEMENT OF PROBLEM AND PRELIMINARIES

A. Formulation of Nonlinear discrete-time systems

In this work, we devote our interest in to the discrete-time systems which can be described by

$$y(k+1) = f(p(k), u(k)), \tag{1}$$

where $f(\cdot,\cdot)$ is an unknown nonlinear function, k is time index, $y(k) \in R$ denotes the measurable output, $u(k) \in R$ is the control effort and $p(k) = [y(k), y(k-1), \dots, y(k-n+1), u(k-1), u(k-2), \dots, u(k-m+1)]$ when $m \leq n$. For system design in the next section, these following assumptions are still needed

- Let define two compact sets Ω_y and Ω_u for the system output y and the control effort u, respectively. The derivative of $f(\cdot,\cdot)$ in (1) with respect to the control effort $u(k)$ is always existed $\forall k = 1, 2, \cdots$ and $0 < |\frac{\partial f(\cdot,u)}{\partial u}| \leq \bar{y_u}$ when $y(\cdot) \in \Omega_y$ and $u(\cdot) \in \Omega_u$ where $\bar{y_u}$ is a finite positive value.
- For any desired trajectories $r(k)$, let the ideal control effort of the system (1) $u^*(k)$ be existed by

$$u^*(k) = g_u(p(k), r(k+1)), \tag{2}$$

when $g_u(\cdot,\cdot)$ is a smooth function.

With the ideal control effort obtained by (2), the controlled system can provide the output to be the desired trajectory as

$$r(k+1) = f(p(k), u^*(k)). \tag{3}$$

Let $u^*(k) \in \Omega_{u^*}$ and $r(k) \in \Omega_r$, for the output $y(k) \in \Omega_y$ such that $\Omega_r \subset \Omega_y$. The function $g_u(-)$ is a one-to-one mapping function of Ω_r into Ω_{u^*}, that is $\Omega_{u^*} \subset \Omega_u$. With the last assumption, $g_u(-)$ is smooth and Ω_r is a compact set, then Ω_{u^*} is a compact set.

B. Function Approximation with MIFREN

In [2] and [3], the function approximation MIFREN property had been introduced. An unknown nonlinear function $f_u(.)$ can be estimated by MIFREN as

$$f_u(k) = \beta^T F_\mu(y(k), \cdots, y(k-\hat{n}-1), \cdots, u(k-\hat{m}-1)) + \varepsilon(k), \tag{4}$$

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where β^T is the target linear parameter of MIFREN, $F_\mu(.)$ is the rule vector at MIFREN's rule-layer $\hat{n}$ and $\hat{m}$ are designed delay-order integers for y and u, respectively and $\varepsilon(k)$ stands for the MIFREN function approximation error. Eventually, the using function approximation result of MIFREN can be given as

$$\hat{f}_u(k) = \hat{\beta}^T(k)F_\mu(y(k),\cdots,y(k-\hat{n}-1),\cdots,u(k-\hat{m}-1)), \tag{5}$$

when $\hat{\beta}(k)$ is the actual linear parameter vector of MIFREN. The vector $\hat{\beta}(k)$ can be automatically tuned via the proposed algorithm as will be discussed in the next section. In [2], [3], the property of an universal function approximation has presented by using the Stone-Weierstrass theorem [1], [10].

III. ADAPTIVE CONTROLLER

A. Control law

From the system equation described in (1), let use the second-order Taylor expansion with the mean value theorem, we have

$$\begin{aligned} y(k+1) &= f(p(k),u(k-1)) + f_1(p(k),u(k-1)) \\ &\quad \times\Delta u(k) + f_2(p(k),\bar{u}_k)\Delta u^2(k), \end{aligned} \tag{6}$$

where $\bar{u}_k = \gamma u(k) + (1-\gamma)u(k-1)$ with $0 \le \gamma \le 1$ $\Delta u(k) = u(k) - u(k-1)$, $f_1(p(k),u(k-1)) = \frac{\partial f(p(k),u)}{\partial u}\Big|_{u=u(k-1)}$ and $f_2(p(k),\bar{u}_k) = \frac{1}{2}\frac{\partial^2 f(p(k),u)}{\partial u^2}\Big|_{u=\bar{u}_k}$. By using (2) and the second assumption mentioned in the previous section, the control effort $u(k)$ can be obtained by

$$u(k) = g_u(p(k), y(k+1)). \tag{7}$$

By substituting this controller into (6), we have

$$\begin{aligned} y(k+1) &= f_3(p(k),y(k+1)) + f_1(p(k),u(k-1)), \\ &\quad \times\Delta u(k), \end{aligned} \tag{8}$$

where $f_3(p(k),y(k+1)) = f(p(k),u(k-1)) + \bar{f}_2(p(k),y(k+1))$. Let $r(k)$ be the desired tracking trajectory then the ideal control effort $u^*(k)$ can be obtained by

$$\begin{aligned} u^*(k) &= u(k-1) + f_1^*(p(k))r(k+1) \\ &\quad - f_2^*(p(k),r(k+1)), \end{aligned} \tag{9}$$

when $f_1^*(p(k)) = \frac{1}{f_1(p(k),u(k-1))}$ and $f_2^*(p(k),r(k+1)) = \frac{f_3(p(k),r(k+1))}{f_1(p(k),u(k-1))}$. From the control law given by (9), the singularity problem of $\frac{1}{f_1(p(k),u(k-1))}$ can be avoided by MIFREN approximation which will be discussed later. These nonlinear functions $f_1^*(\cdot,\cdot)$ and $f_2^*(\cdot,\cdot)$ are unknown. Two MIFRENs are constructed to approximate $f_1^*(\cdot,\cdot)$ and $f_2^*(\cdot,\cdot)$ by MIFREN$_1$ and MIFREN$_2$, respectively. We have

$$\begin{aligned} u^*(k) &= u(k-1) + [\beta_1^{*T}F_1(p(k)) + \varepsilon_1(k)]r(k+1) \\ &\quad - \beta_2^{*T}F_2(p(k),r(k+1)) - \varepsilon_2(k), \end{aligned} \tag{10}$$

where $F_1(\cdot)$ and $F_2(\cdot)$ are rule-functions of MIFREN$_1$ and MIFREN$_2$, respectively, $\beta_1^* = [\beta_{1,1}^* \quad \beta_{1,2}^* \quad \cdots \quad \beta_{1,n_1}^*]^T$, $\beta_2^* = [\beta_{2,1}^* \quad \beta_{2,2}^* \quad \cdots \quad \beta_{2,n_2}^*]^T$ are ideal weight vectors, n_1 and n_2 denote number of rules for each MIFREN and $\varepsilon_1(\cdot)$ and $\varepsilon_2(\cdot)$ are approximation errors. Let us neglect these errors and use the actual weight vector as $\beta_1(k)$ and $\beta_2(k)$ thus the proposed control law can be given by

$$\begin{aligned} u(k) &= u(k-1) + [\beta_1^T(k)F_1(p(k))]r(k+1) \\ &\quad - \beta_2^T(k)F_2(p(k),r(k+1)). \end{aligned} \tag{11}$$

Let the control error be defined by

$$e(k+1) = r(k+1) - y(k+1), \tag{12}$$

for time index $k+1$. Substitute $y(k+1)$ from (8) into (12) and use Taylor expression and mean value theorem, the control error can be obtained as

$$\begin{aligned} e(k+1) &= r(k+1) - \Big[f_3(p(k),r(k+1)) \\ &\quad + \frac{\partial f_3(p(k),y)}{\partial y}(y(k+1) - r(k+1))\Big] \\ &\quad - f_1(p(k))\Delta u(k), \end{aligned} \tag{13}$$

where $\bar{y}_{k+1}$ is between $r(k+1)$ and $y(k+1)$. Let us consider the system in (8) with the control effort given by (7), we have

$$\begin{aligned} y(k+1) &= f_3(p(k),y(k+1)) + f_1(p(k)) \\ &\quad \times[g_u(p(k),y(k+1)) - u(k-1], \end{aligned} \tag{14}$$

and

$$\begin{aligned} \frac{\partial f_3(p(k),y(k+1))}{\partial y(k+1)} &= 1 - f_1(p(k),u(k-1)) \\ &\quad \times\frac{\partial g_u(p(k),y(k+1))}{\partial y(k+1)}. \end{aligned} \tag{15}$$

Substitute (15) into (13), we have

$$\begin{aligned} e(k+1) &= r(k+1) - f_3(p(k),r(k+1)) + e(k+1) \\ &\quad - f_1(p(k))\Delta u(k) - f_1(p(k)) \\ &\quad \times\frac{\partial g_u(p(k),y)}{\partial y}\Big|_{y=\bar{y}(k+1)} e(k+1). \end{aligned} \tag{16}$$

For the controllable system in (8), clearly, $f_1(p(k)) \neq 0$ and $u^*(k) = g_u(p(k),r(k+1))$ or $u(k) = g_u(p(k),y(k+1))$ thus the system sensibility $[\frac{\partial y}{\partial u}]^{-1}$ should be obtained as

$$\begin{aligned} \frac{\partial u(k)}{\partial y}\Big|_{y=y(k+1)} &= \frac{\partial g_u(p(k),y)}{\partial y}\Big|_{y=y(k+1)}, \\ &= \frac{1}{\gamma_y(k)}. \end{aligned} \tag{17}$$

With MIFRENs approximation, we have

$$\begin{aligned} e(k+1) &= \gamma_y(k)\Big[\beta_1^{*T}F_1(p(k))r(k+1) \\ &\quad - \beta_2^{*T}F_2(p(k),r(k+1)) - \Delta u(k)\Big] \\ &\quad + \gamma_y(k)\Big[\varepsilon_1(k)r(k+1) - \varepsilon_2(k)\Big]. \end{aligned} \tag{18}$$

Substitute the proposed control law (11) into (18), we obtain

$$\begin{aligned} e(k+1) &= \gamma_y(k)\Big[\widetilde{\beta}_1^T(k)F_1(k)r(k+1) \\ &\quad - \widetilde{\beta}_2^T(k)F_2(k)\Big] + \gamma_y(k)\varepsilon_t(k), \end{aligned} \tag{19}$$

when $\widetilde{\beta}_i^T(k) = \beta_i^{*T} - \beta_i^T(k)$ for $i = 1, 2$ and $\varepsilon_t(k) = \varepsilon_1(k)r(k+1) - \varepsilon_2(k)$.

B. MIFRENs tuning laws

The parameter vectors $\beta_1(k)$ and $\beta_2(k)$ are required to update during the system operation or on-line learning. To simplify, let us rewrite (19) to be

$$e(k+1) = \gamma_y(k)\Big[\widetilde{\beta}_1^T(k) \quad \widetilde{\beta}_2^T(k)\Big]F(k) + \gamma_y(k)\varepsilon_t(k), \quad (20)$$

where $F(k) = \begin{bmatrix} F_1(k)r(k+1) \\ -F_2(k) \end{bmatrix}$. With (20), we can define the update law as the following:

$$\begin{bmatrix} \beta_1(k+1) \\ \beta_2(k+1) \end{bmatrix} = \begin{bmatrix} \beta_1(k) \\ \beta_2(k) \end{bmatrix} + \frac{\eta}{\bar{y_u}||F(k)||^2}F(k)\mathfrak{D}(e(k)), \quad (21)$$

where η is the selected learning rate which will be discussed next and $\mathfrak{D}(\cdot)$ is the dead-zone function which can be defined by

$$\mathfrak{D}(e(k)) = \begin{cases} e(k) - \varepsilon_m & \text{if } e(k) > \varepsilon_m \\ 0 & \text{if } |e(k)| \leq \varepsilon_m \\ e(k) + \varepsilon_m & \text{if } e(k) < -\varepsilon_m, \end{cases} \quad (22)$$

when $|\gamma_y(k)\varepsilon_t(k)| \leq \varepsilon_m$ as a small positive number. In the case of $|e(k-1)| > \varepsilon_m$, with the dead-zone function (22) and the next time-index error (20), we have

$$\mathfrak{D}(e(k+1)) = \alpha_D\gamma_y(k)\Big[\widetilde{\beta}_1^T(k) \quad \widetilde{\beta}_2^T(k)\Big]F(k), \quad (23)$$

where $0 < \alpha_D \leq 1$.

IV. Closed-loop system performance

To analyze the system performance and stability, the bounded weight vectors $\widetilde{\beta}_i^T(k)$ and the bounded tracking error $e(k)$ are both given in this work.

Lemma 1: For the nonlinear discrete-time system given in (1) with the control law defined in (11), the error weight vectors $\widetilde{\beta}_i^T(k)$ for $i = 1, 2$ are bounded by the tuning law in (21) and the selected learning rate η as the followings:

$$0 < \eta < \frac{2\bar{y_u}}{\alpha_D\gamma_y(k)}, \quad (24)$$

when $0 < \bar{y_u}$, and

$$\frac{2\bar{y_u}}{\alpha_D\gamma_y(k)} < \eta < 0, \quad (25)$$

when $\bar{y_u} < 0$.

Proof: Let us define a Lyapunov candidate function as

$$V_{\widetilde{\beta}}(k) = \widetilde{\beta}_1^T(k)\widetilde{\beta}_1(k) + \widetilde{\beta}_2^T(k)\widetilde{\beta}_2(k). \quad (26)$$

The first difference can be obtained by

$$\begin{aligned}\Delta V_{\widetilde{\beta}}(k) &= \widetilde{\beta}_1^T(k+1)\widetilde{\beta}_1(k+1) + \widetilde{\beta}_2^T(k+1)\widetilde{\beta}_2(k+1) \\ &\quad -\widetilde{\beta}_1^T(k)\widetilde{\beta}_1(k) - \widetilde{\beta}_2^T(k)\widetilde{\beta}_2(k). \quad (27)\end{aligned}$$

Substitute (27) and use (23), we obtain

$$\begin{aligned}\Delta V_{\widetilde{\beta}}(k) &= -\frac{2\eta}{\alpha_D\gamma_y(k)\bar{y_u}||F(k)||^2}\mathfrak{D}^2(e(k+1)) \\ &\quad +\frac{\eta^2}{\bar{y_u}^2||F(k)||^2}\mathfrak{D}^2(e(k+1)) \\ &= \Big[\frac{-2}{\alpha_D\gamma_y(k)} + \frac{\eta}{\bar{y_u}}\Big]\frac{\eta}{\bar{y_u}||F(k)||^2} \\ &\quad \times\mathfrak{D}^2(e(k+1)). \quad (28)\end{aligned}$$

With the selected learning rate defined by (24) and (25) and $\gamma_y(k)$ given in (17), the first difference of Lyapunov function is negative, thus $\widetilde{\beta}_i^T(k)$ for $i = 1, 2$ are bounded.

□

Remark: Normally, with out loss of generality, $\bar{y_u}$ is assumed to be positive thus $\gamma_y(k) < \bar{y_u} : \forall k$.

The bounded tracking error for the closed-loop system is introduced by the following theorem.

Theorem 4.1: For the nonlinear discrete-time system given in (1) with the control law defined in (11), let define a compact set $\Omega_\varepsilon = \{e(k)||e(k) \leq 4\varepsilon_m\}$, thus the ultimate boundary on the tracking error is $\lim_{k\to\infty}|e(k)| \leq \varepsilon_m$ or in a compact set Ω_ε.

Proof: Let a Lyapunov candidate function be given by

$$V_e(k) = \frac{\eta}{2\bar{y_u}^2F_o^2}e^2(k) + V_{\widetilde{\beta}}(k), \quad (29)$$

when F_o is defined by $0 < ||F(k)|| \leq F_0, \forall k$. The first difference can be obtained by

$$\begin{aligned}\Delta V_e(k) &= \frac{\eta}{2\bar{y_u}^2F_o^2}[e^2(k+1) - e^2(k)] \\ &\quad +\Delta V_{\widetilde{\beta}}(k). \quad (30)\end{aligned}$$

Substitute (28) into (30), we have

$$\begin{aligned}\Delta V_e(k) &= \frac{\eta}{2\bar{y_u}^2F_o^2}[e^2(k+1) - e^2(k)] \\ &\quad -\frac{2\eta\mathfrak{D}^2(e(k+1))}{\alpha_D\gamma_y(k)\bar{y_u}||F(k)||^2} \\ &\quad +\frac{\eta^2\mathfrak{D}^2(e(k+1))}{\bar{y_u}^2||F(k)||^2}. \quad (31)\end{aligned}$$

From the learning rate given by (24- 24), we can rearrange (31) as

$$\begin{aligned}\Delta V_e(k) &< \frac{\eta}{2\bar{y_u}^2F_o^2}e^2(k+1) - \frac{\eta}{\bar{y_u}^2F_o^2}\mathfrak{D}^2(e(k+1)), \\ &= \frac{\eta}{2\bar{y_u}^2F_o^2}[e^2(k+1) - 2\mathfrak{D}^2(e(k+1))]. \quad (32)\end{aligned}$$

In this proof, we need to provide only the case when $|e(k+1)| > \varepsilon_m$. With $|e(k+1)| > \varepsilon_m$, the dead-zone function in (22) can be obtained as

$$\mathfrak{D}(e(k+1)) = e(k+1) - \varepsilon_m\text{sign}\{e(k+1)\}. \quad (33)$$

Substitute (33) into (32), we have

$$\begin{aligned}\Delta V_e(k) &= \frac{\eta}{2\bar{y_u}^2 F_o^2}\Big[-e^2(k+1) - 2\varepsilon_m^2 \\ &\quad +4|e(k+1)|\varepsilon_m\Big], \\ &< \frac{\eta}{2\bar{y_u}^2 F_o^2}\Big[-e^2(k+1) \\ &\quad +4|e(k+1)|\varepsilon_m\Big]. \qquad (34)\end{aligned}$$

Consider the result in (34), clearly, $\Delta V_e(k)$ is always negative where $|e(k+1)| > 4\varepsilon_m$, thus $\Delta V_e(k) < 0$ when $|e(k+1)|$ is out side a compact set Ω_ε.

□

V. SIMULATION RESULTS

The proposed control algorithm and theorem are verified by the computer simulation. The selected controllable system is described by

$$y(k+1) = \sin(y(k)) + \cos(y(k)u(k))u(k) + 5u(k). \quad (35)$$

All system parameters introduced in previous sections can be designed as $\varepsilon_m = 0.0001$, $\eta = 0.75$ and $\bar{y_u} = 6.25$. All IF-THEN rules for both MIFRENs are given by the followings:

$MIFREN_1$

If $y(k)$ is N	and $u(k-1)$ is N	Then $f_{1,1}(k) = \beta_{1,1}(k)F_{1,1}(k)$,
If $y(k)$ is N	and $u(k-1)$ is Z	Then $f_{1,2}(k) = \beta_{1,2}(k)F_{1,2}(k)$,
If $y(k)$ is N	and $u(k-1)$ is P	Then $f_{1,3}(k) = \beta_{1,3}(k)F_{1,3}(k)$,
If $y(k)$ is Z	and $u(k-1)$ is N	Then $f_{1,4}(k) = \beta_{1,4}(k)F_{1,4}(k)$,
If $y(k)$ is Z	and $u(k-1)$ is Z	Then $f_{1,5}(k) = \beta_{1,5}(k)F_{1,5}(k)$,
If $y(k)$ is Z	and $u(k-1)$ is P	Then $f_{1,6}(k) = \beta_{1,6}(k)F_{1,6}(k)$,
If $y(k)$ is P	and $u(k-1)$ is N	Then $f_{1,7}(k) = \beta_{1,7}(k)F_{1,7}(k)$,
If $y(k)$ is P	and $u(k-1)$ is Z	Then $f_{1,8}(k) = \beta_{1,8}(k)F_{1,8}(k)$,
If $y(k)$ is P	and $u(k-1)$ is P	Then $f_{1,9}(k) = \beta_{1,9}(k)F_{1,9}(k)$,

$MIFREN_2$

If $y(k)$ is N	and $r(k+1)$ is N	Then $f_{2,1}(k) = \beta_{2,1}(k)F_{2,1}(k)$,
If $y(k)$ is N	and $r(k+1)$ is Z	Then $f_{2,2}(k) = \beta_{2,2}(k)F_{2,2}(k)$,
If $y(k)$ is N	and $r(k+1)$ is P	Then $f_{2,3}(k) = \beta_{2,3}(k)F_{2,3}(k)$,
If $y(k)$ is Z	and $r(k+1)$ is N	Then $f_{2,4}(k) = \beta_{2,4}(k)F_{2,4}(k)$,
If $y(k)$ is Z	and $r(k+1)$ is Z	Then $f_{2,5}(k) = \beta_{2,5}(k)F_{2,5}(k)$,
If $y(k)$ is Z	and $r(k+1)$ is P	Then $f_{2,6}(k) = \beta_{2,6}(k)F_{2,6}(k)$,
If $y(k)$ is P	and $r(k+1)$ is N	Then $f_{2,7}(k) = \beta_{2,7}(k)F_{2,7}(k)$,
If $y(k)$ is P	and $r(k+1)$ is Z	Then $f_{2,8}(k) = \beta_{2,8}(k)F_{2,8}(k)$,
If $y(k)$ is P	and $r(k+1)$ is P	Then $f_{2,9}(k) = \beta_{2,9}(k)F_{2,9}(k)$,

when N, Z and P denote negative, zero and positive linguistic levels respectively. The membership functions for these rules are illustrated in Fig. (1) and (2). In this work, we use the same membership functions of $y(k)$ and $r(k+1)$ because these variables have equality linguistic levels in the sense of human. The initial setting $\beta_{i,j}(1)$ for $i = 1, 2$ and $j = 1, 2, \cdots, 9$ can be given as

$\beta_{i,1}(1)$=-1	$\beta_{i,2}(1)$=-0.75	$\beta_{i,3}(1)$=-0.5,
$\beta_{i,4}(1)$=-0.25	$\beta_{i,5}(1)$=0	$\beta_{i,6}(1)$=0.25,
$\beta_{i,7}(1)$=0.5	$\beta_{i,8}(1)$=0.75	$\beta_{i,9}(1)$=1.

In Fig. 3, the tracking performance is quite satisfied with out the off-line learning. The control effort is illustrated in Fig. 4. The convergence of $\beta_i(k)$ is shown by $||\beta_i(k)||$ in Fig. 7 for both MIFRENs.

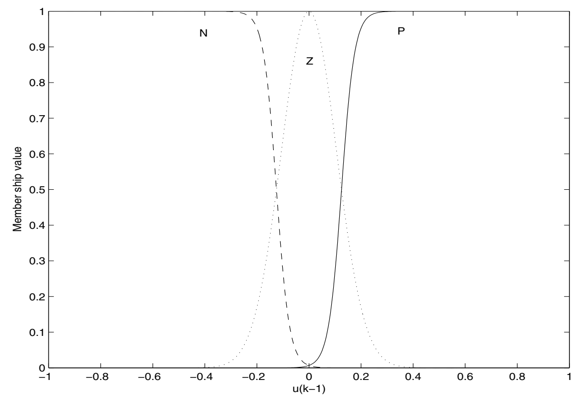

Fig. 1. Membership functions of $u(k-1)$

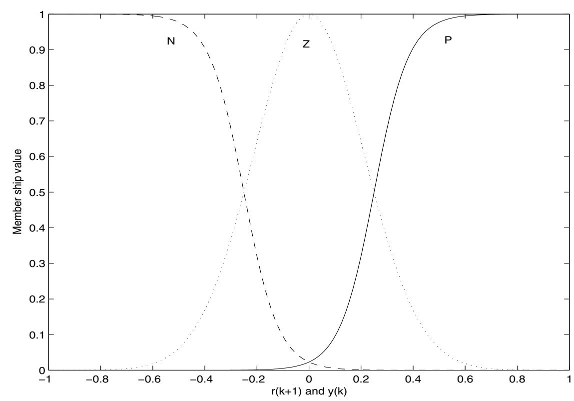

Fig. 2. Membership functions of $r(k+1)$ and $y(k)$

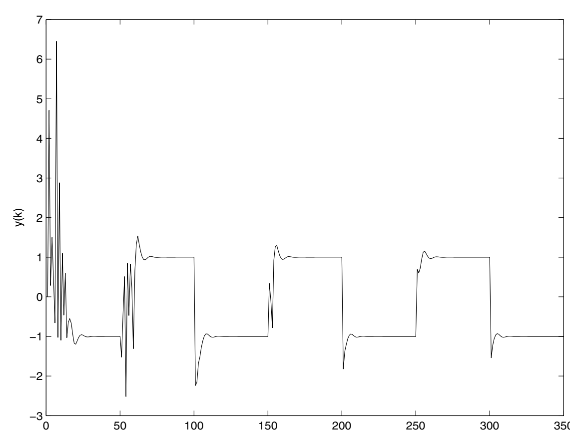

Fig. 3. Tracking performance $y(k)$.

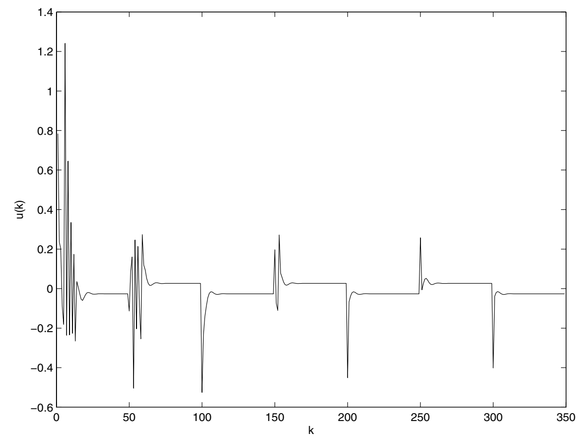

Fig. 4. Control effort $u(k)$.

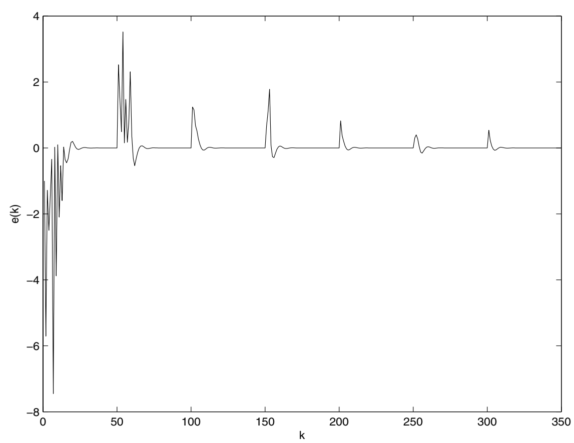

Fig. 5. Closed-loop error $e(k)$.

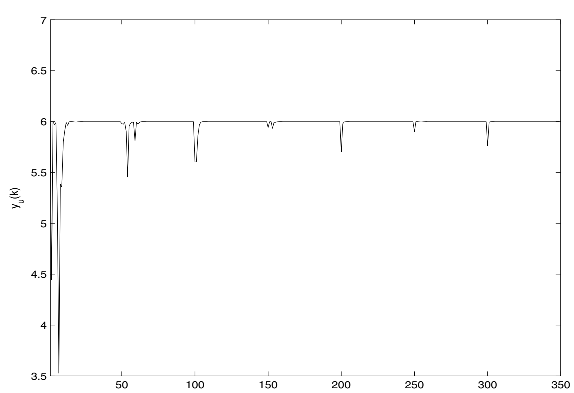

Fig. 6. Time variation of $y_u(k)$.

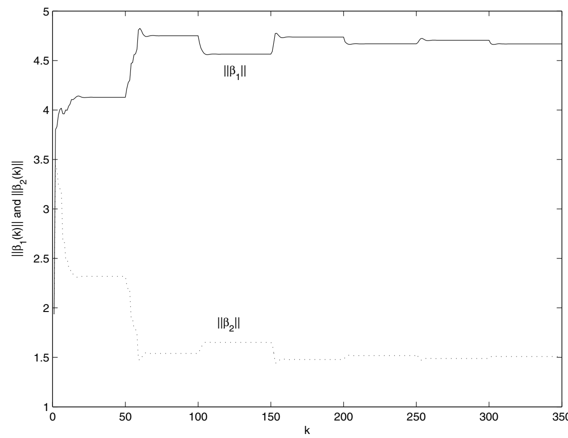

Fig. 7. Time variation of weight parameters $||\beta_i(k)||$.

In the robust system case, the uncertainty terms $\Delta f_1(k)$ and $\Delta f_2(k)$ are included in the system (35) as

$$\begin{aligned} y(k+1) &= \sin(y(k)) + \Delta f_1(k) + \cos(y(k)u(k))u(k) \\ &\quad +5u(k) + \Delta f_2(k)u(k), \end{aligned} \tag{36}$$

when

$$\Delta f_1(k) = \begin{cases} 0.25 & \text{if } 0 < k < 80 \\ 0.75 & \text{if } 80 \leq k < 120 \\ -0.5 & \text{if } 120 \leq k < 175 \\ -0.75 & \text{if } 175 \leq k < 250 \\ 0.5 & \text{if } 250 \leq k, \end{cases} \tag{37}$$

and

$$\Delta f_2(k) = \begin{cases} -0.25 & \text{if } 0 < k < 80 \\ -0.75 & \text{if } 80 \leq k < 120 \\ 0.5 & \text{if } 120 \leq k < 175 \\ 0.75 & \text{if } 175 \leq k < 250 \\ -0.5 & \text{if } 250 \leq k. \end{cases} \tag{38}$$

We use the initial setting IF-THEN rules, membership functions, ε_m, η, $\bar{y_u}$ and parameter vectors β_i, as the same as the previous one. With out any off-line learning for MIFRENs, the tracking performance is represented in Fig. 9. The control effort $u(k)$ is shown in Fig. 10. The time variation of $||\beta_i(k)||$ can be illustrated in Fig. 11. These uncertainty terms $\Delta f_1(k)$ and $\Delta f_2(k)$ are varied with time but the tuning vectors are all bounded.

VI. Conclusion

An adaptive controller for a class of non affine discrete-time systems has been introduced by the approximation based on Taylor and mean value theorem and MIFRENs. Two MIFRENs are implemented to estimate these unknown functions inside the control law. The learning algorithm for parameters inside both MIFRENs is guaranteed the convergence of weight parameters and the closed-loop tracking error are bounded by the main theorem. The computer simulation system demonstrates the accuracy of our mathematic proof and the validation of the proposed algorithm.

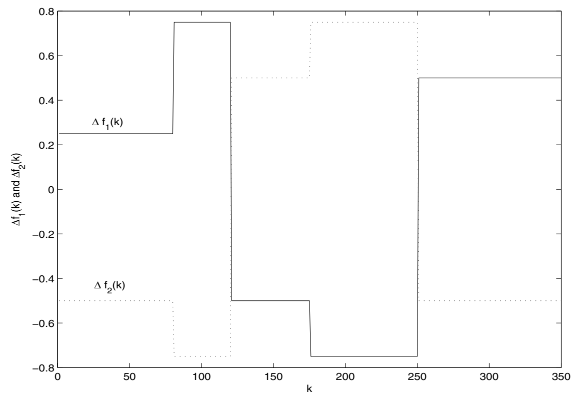

Fig. 8. Illustration of uncertainty $\Delta f_1(k)$ and $\Delta f_2(k)$.

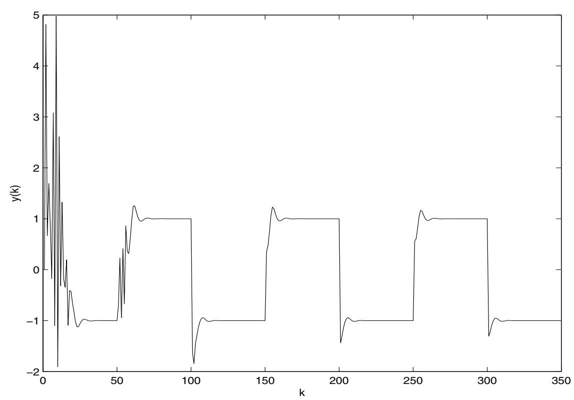

Fig. 9. Tracking performance $y(k)$ for robust system.

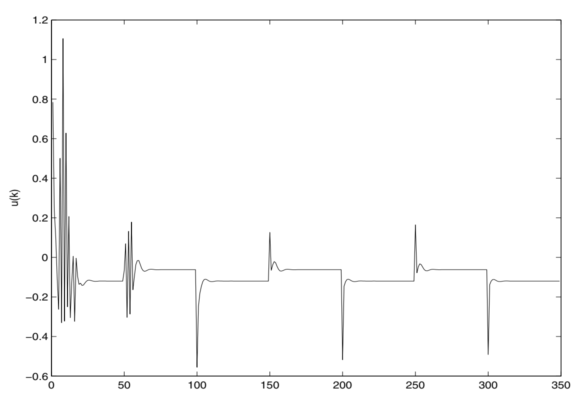

Fig. 10. Control effort $u(k)$ for robust system.

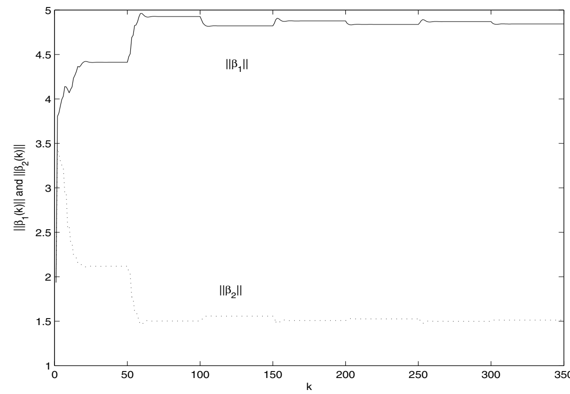

Fig. 11. $||\beta_i(k)||$ for robust system.

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