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A Novel Grey Model to Short-Term Electricity Price Forecasting for NordPool Power Market

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***Abstract*—In order to improve the forecasting precision of traditional grey model for Short-term price in competitive electricity market, a novel grey model is presented in this paper based on period-decoupled price sequence. According to the interval time of market cleaning, the historical price data are divided into 24 sequences or 48 sequences. In the proposed grey model, two kinds of price sequences, called the main sequence (MS) and the reference sequence (RS), are defined. The correlation coefficient between price sequences of adjacent time intervals is analyzed, which is more than 0.9522 obtained from the Nordpool data in 2007. Therefore, it is determined that the MS is composed of prediction-period price data, and the RS is composed of hour-before-period price data. Furthermore, considering the limitation of the least square method (LSM) used in the traditional grey model for identification the developing coefficient *a* and the grey input *b*, the Particle Swarm Optimization algorithm (PSO) is adopted instead of LSM. Thus the PSOGM (1,2) forecasting model to short-term price is founded. The historical data from the Nordpool power market is used for computing, and the numerical results demonstrate that the MAPE of PSOGM (1,2) model for short-term price rolling prediction is 5.0626% and 7.5491% for continuous forecasting, raising 3%~20% compared with traditional grey model.**

***Keywords*—power market, electrcity price forecasting, grey model, particle swarm optimization**

I. Introduction

In the competitive daily power markets, accurate prediction to electricity price is critical important for all market participants. Having reliable daily price forecast information, producers or energy service companies are able to delineate good bilateral contracts and make better financial decision. Meanwhile, the factors impacting electricity price, such as load factors, weather factors, and bidding strategy factors are undulating and undetermined, so the price prediction in power markets has becoming the commonly problem all over the world [1]-[5].

In 1982[6], professor Deng pioneered the grey system theory, abbreviated to GST or GS. Up to now, it has been used successfully to medicine, image processing, robot, industry technology and so on. Relative to the definitions of black system and white system, only part of information known is called grey system. Grey system theory considers random processes as grey processes varying in time-related, and it regards all sorts of random variables as grey variables, so it introduces the method of Grey Data Generation to trim disorder original data. Electricity price is a dynamic process of random and uncertainty. Not all the influence factors are known to us, but the price data of past and actuality can be obtained, thus power markets can be seen as grey systems. so it can be analyzed by grey system theory. Actually, as the mainly grey model, GM (1,1) forecasting model is very important and has been used in power market to predict electricity price [7]-[11].

However, GM (1,1) forecasting model is suitable to predict for those sequences with exponential change law, and always has bad precious when used for fluctuation sequences [12]. However, electricity price is undulating and non- monotonic [13]. So it is unreasonable to predict electricity price with GM (1,1) forecasting model.

The work described in this paper focuses on hour-ahead and day-ahead electricity price forecasts in the Nordpool market with GM (1,2) forecasting model based on period-decoupled price sequence and Particle Swarm Optimization algorithm (PSO). The correlation coefficient between price sequences of adjacent time intervals are analyzed with data obtained from the Nordpool market in 2007, and the results indicate that the adjacent period price sequence have strong correlation, then the prediction period price sequence is chosen as the main sequence, and the hour-before period price sequence is chosen as the reference sequence. Thus the GM (1,2) model for short-term price forecasting is founded.

Normally, the least square method (LSM) is used in traditional GM (1,2) model to identify the developing coefficient *a* and the grey input *b*, where the generalized inverse matrix of grey data matrix **B** must be exist. But in fact, the matrix **B** can't always has the generalized inverse matrix. So the traditional GM (1,2) model will can't be used to those problems, in which the Matrix B doesn't have the generalized inverse matrix. Otherwise, the forecasting result should be unacceptable (As shown in Figure 1).

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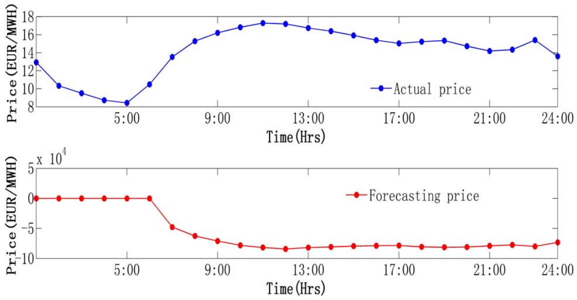

Figure 1 Actual and prediction Price of 27 July, 2007 in Nordpool market

In order to expand the application range of GM (1,2) model, the Particle Swarm Optimization algorithm was adopted to identify the developing coefficient *a* and the grey input *b* instead of LSM, and then the PSOGM (1,2) model for short-term price forecasting is proposed. Posterior error(*C*), micro-error probability (*P*) and mean-absolute-percent error (MAPE) values obtained from the forecasting results demonstrate that the proposed PSOGM (1,2) model functions reasonably well to forecast electricity price for the next-hour and next-day in power market.

This paper is organized as follows. In Section Ⅱ ,we discusses the correlation of price sequence; The method &theory of GM(1,2) and PSOGM(1,2) model is presented in Section Ⅲ; we also evaluates the fitting and forecasting results in Section Ⅳ; Conclusions are made in the final Section.

II. CORRELATION OF PRICE SEQUENCE

Generally, it is important and difficult to choose the reference sequence in the GM (1,2) model for short-term price forecasting. The reference sequence should satisfy following conditions:

(1) The determined reference sequence must has largely correlation degree with the main sequence;

(2) The value of the reference sequence should be known at forecasting hour-spot.

The definition of correlation coefficient (*r*) in the grey system theory is formulated as follow [12].

$$r=\frac{n\sum_{i=1}^{n}MS(i)RS(i)-(\sum_{i=1}^{n}MS(i))(\sum_{i=1}^{n}RS(i))}{\sqrt{\left[n\sum_{i=1}^{n}MS^2(i)-(\sum_{i=1}^{n}MS(i))^2\right]\left[n\sum_{i=1}^{n}RS^2(i)-(\sum_{i=1}^{n}RS(i))^2\right]}} \quad (1)$$

Where *MS(i)* represents the main sequence value at time *i*th; *RS(i)* represent the reference sequence value at time *i*th; *n* represents the length of each sequence.

Referencing to equation (1), the correlation coefficient for adjacent time period price sequences of Nordpool in 2007 can be calculated. It is shown as Fig.2. Obviously, any of two adjacent time period price sequences have high correlation degree. Therefore it is reasonable to choose the hour- before price sequence as the reference price sequence.

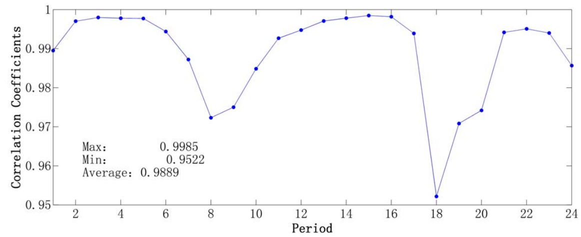

Figure 2 Correlation coefficient for adjacent time period price sequences of Nordpool in 2007

III. METHOD AND THEORY

A. *Principle of Traditional GM (1,2) model*

Given the main raw price sequence **P**(0) and the reference raw price sequence R(0) as follows:

$$\left.\begin{aligned}\mathbf{P}^{(0)}&=\left\{P^{(0)}(1),P^{(0)}(2),\cdots\cdots P^{(0)}(N)\right\}\\ \mathbf{R}^{(0)}&=\left\{R^{(0)}(1),R^{(0)}(2),\cdots\cdots R^{(0)}(N+1)\right\}\end{aligned}\right\} \quad (2)$$

By defining

$$\left.\begin{aligned}P^{(1)}(k)&=\sum_{j=1}^{k}P^{(0)}(j) \qquad k=1,2,\cdots\cdots N\\ R^{(1)}(k)&=\sum_{j=1}^{k}R^{(0)}(j) \qquad k=1,2,\cdots\cdots N+1\end{aligned}\right\} \quad (3)$$

We get new series

$$\left.\begin{aligned}\mathbf{P}^{(1)}&=\left\{P^{(1)}(1),P^{(1)}(2),\cdots\cdots P^{(1)}(N)\right\}\\ \mathbf{R}^{(1)}&=\left\{R^{(1)}(1),R^{(1)}(2),\cdots\cdots R^{(1)}(N+1)\right\}\end{aligned}\right\} \quad (4)$$

To some processes, $\mathbf{P}^{(1)}$ is the solution of the following grey ordinary differential equation.

$$\frac{d\mathbf{P}^{(1)}(t)}{dt}+a\mathbf{P}^{(1)}(t)=b\mathbf{R}^{(1)}(t) \quad (5)$$

Where *a* is called developing coefficient; *b* is called grey input. The equation (5) is called GM(1,2). Here, parameters *a* and *b* usually be identified by the least squares method (LSM) as follows:

$$\left.\begin{aligned}&\begin{bmatrix}a & b\end{bmatrix}^T=(\mathbf{B}^T\mathbf{B})^{-1}\mathbf{B}^T\mathbf{Y}\\ &\mathbf{B}=\begin{bmatrix}-Z^{(1)}(2) & R^{(1)}(2)\\ -Z^{(1)}(3) & R^{(1)}(3)\\ \cdots & \cdots\\ -Z^{(1)}(N) & R^{(1)}(N)\end{bmatrix}\\ &\mathbf{Y}=\left[P^{(0)}(2),P^{(0)}(3),\cdots\cdots,P^{(0)}(N)\right]\\ &Z^{(1)}(k)=0.5P^{(0)}(k-1)+0.5P^{(0)}(k))\end{aligned}\right\}$$

If rank (B)=2, the equation (5) has a unique solution:

$$\left.\begin{aligned}&\hat{P}^{(1)}(k+1)=(P^{(0)}(1)-\frac{b}{a}R^{(1)}(k+1))e^{-ak}+\frac{b}{a}R^{(1)}(k+1)\\&k=0,1,2,\cdots N\end{aligned}\right\}\quad(6)$$

From equation (6), the value of $\hat{P}^{(1)}(k)$ can be computed. Thus we can compute the main price sequence values

$$\hat{P}^{(0)}(k+1)=\hat{P}^{(1)}(k+1)-\hat{P}^{(1)}(k)\quad(7)$$

B. PSOGM (1,2) forecasting model

Obviously, the GM (1,2) model has a unique solution based on LSM only when the rank (**B**)=2. But for some process, this condition can't be satisfied.

In this paper, PSO is adopted to identify the parameters *a* and *b*. PSO is an evolutionary computation technique developed by Eberhart and Kennedy in 1995[14,15], which was inspired by the social behavior of bird flocking and fish schooling. PSO has its roots in artificial life and social psychology, as well as in engineering and computer science. It is a computational intelligence-based technique that is not largely affected by the size and nonlinearity of the problem, and can converge to the optimal solution in many problems where most analytical methods fail to converge. It can, therefore, be effectively applied to identify parameters for GM (1,2) model in electricity power markets. The procedure for identifying can be explained as follows.

Step 1 Initializing the particle swarm parameters.

M particles are produced in N-dimension space randomly. The position vector and velocity vector of particles are signed as

$$\mathbf{X}_i=\{x_{i1},x_{i2},\cdots,x_{iN}\}$$

and

$$\mathbf{V}_i=\{v_{i1},v_{i2},\cdots,v_{iN}\};$$

Where i=1,2…M；N=DW-1；$0\leqslant x_{ik}\leqslant 1$，$-1\leqslant v_{ik}\leqslant 1$;

Step 2 Calculating the adaptive value of particles.

The adaptive value function is defined as

$$f(x_i)=\frac{1}{1+MAPE}\quad(8)$$

Where MAPE is the abbreviation for mean absolute percentage error.

Step 3 Comparison of $f(x_i)$ and $p_i best$.

if $f(x_i)>p_i best$ then $p_i best=f(g_i)$

if $f(x_i)>gbest$ then $gbest=f(g_i)$

Where $p_i best$ and *gbest* represent the best value of $f(x_i)$ for *i*th particle and the best value of $f(x_i)$ for all particles up to now respectively.

Step 4 Updating velocity and position of particles.

The position and velocity of particles are updated according to equation (9).

$$\left.\begin{aligned}&v_{i+1d}=\omega v_{id}+c_1r_1(p_{id}-x_{id})+c_2r_2(p_{gd}-x_{id})\\&x_{i+1d}=x_{id}+v_{id}\end{aligned}\right\}\quad(9)$$

Where v_{i+1d} and x_{i+1d} represent the renewed velocity and position of the *d*th particle respectively; v_{id} and x_{id} represent the present velocity and position of the *d*th particle respectively; ω is called inertia weight coefficient; *c1* and *c2* are called accelerate coefficient with *c1*=*c2*=2 in general; r_1 and r_2 usually have random value range from 0 to 1.

As often, the inertia weight is initialized as 0.9, and end as 0.1. Moreover, the inertia weight of PSO is reduced linearly from maximum to minimum.

$$\omega(k)=\omega_{ini}-\frac{G}{G_{\max}}(\omega_{ini}-\omega_{end})\quad(10)$$

Where ω_{ini} and ω_{end} represent the initial value and end value of the inertia weight respectively. *G* is the evolution generations at present.

Step 5 Rules of stopping.

If the number of iteration reaches at the maximum preset value *Gmax*, then the procedure goes back to step2.

C. Checking methods for the model

Aiming at checking the forecasting precision of the proposed model, after-test residue checking method is applied in this paper.

Posterior error(C) is defined as follow.

$$C=\frac{S_2}{S_1}=\frac{\sqrt{\frac{1}{N}\sum_{j=1}^{N}(q(j)-\bar{q})^2}}{\sqrt{\frac{1}{N}\sum_{j=1}^{N}(P(j)-\bar{p})^2}}\quad(11)$$

Where *q(k)* represents the residual error of raw sequence at time *k*th; $\bar{q}$ represents the mean of sequence *q(j)*; $\bar{p}$ represents the mean of raw sequence P(j); S_1^2 represents the variance of raw sequence *P(j)*; S_2^2 represents the variance of residual sequence *q(j)*.

The micro-error probability (P) is defined as follow.

TABLE I
REFERENCE LIST FOR FORECASTING PRECISION DEGREE

Parameters	Prediction Precision Grade			
	Good	Qualified	Just	Unqualified
P	>0.95	0.8~0.95	0.7~0.8	≤0.7
C	<0.35	0.35~0.5	0.5~0.65	≥0.65

$$P=P\left\{\left|q(k)-\bar{q}\right|<0.6745S_1\right\}\quad(12)$$

The detailed relation between forecasting precision and C, P is shown as TABLE I.

TABLE Ⅱ
TESTING OF FITTING PRECISION FOR FORECASTING MODELS

Period	MAPE (%)			C			P		
	GM(1,1)	*GM(1,2)*	*PSOGM(1,2)*	*GM(1,1)*	*GM(1,2)*	*PSOGM(1,2)*	*GM(1,1)*	*GM(1,2)*	*PSOGM(1,2)*
1	3.5541	3.1691	1.5724	0.7852	0.8950	0.3515	0.5000	0.7857	1.0000
2	3.4127	2.5540	1.1115	0.7146	0.6865	0.2455	0.5714	0.7857	0.9286
3	3.3606	2.2312	0.4961	0.6973	0.6488	0.1099	0.7143	0.8571	1.0000
4	3.5861	2.6002	0.6905	0.7290	0.7450	0.1640	0.7857	0.7857	1.0000
5	3.5400	2.2239	1.2013	0.8209	0.8184	0.3378	0.7143	0.8571	0.9286
6	4.9966	3.6851	1.8869	0.9434	0.8500	0.3519	0.5714	0.7857	1.0000
7	7.0258	4.9697	2.4468	0.9271	0.8048	0.3383	0.5000	0.7857	0.9286
8	12.3667	10.1518	5.1711	0.9750	0.7250	0.6182	0.6429	0.7857	0.9286
9	23.6939	21.6199	7.2222	0.9887	0.5903	0.6144	0.6429	0.8571	0.8571
10	13.7198	12.5091	8.0498	0.9777	1.2480	1.1164	0.5714	0.7857	0.8571
11	9.8652	5.7698	3.7550	0.9588	0.6321	0.5824	0.5000	0.7857	0.8571
12	8.3431	3.1164	1.6364	0.9417	0.4531	0.3033	0.5000	0.8571	0.9286
13	7.8296	2.5114	0.8294	0.9389	0.4091	0.1488	0.4286	0.8571	1.0000
14	7.6006	2.3866	0.8558	0.9412	0.4362	0.1172	0.4286	0.8571	1.0000
15	8.2384	2.8374	0.7891	0.9528	0.4522	0.1358	0.4286	0.8571	1.0000
16	8.5104	2.7705	0.8332	0.9642	0.3898	0.0998	0.5000	0.8571	1.0000
17	13.0590	11.1147	4.3598	0.9823	0.6689	0.6028	0.6428	0.7857	0.9286
18	21.8075	12.9874	7.0563	0.9786	0.4160	0.4401	0.5000	0.9286	0.7857
19	6.7971	5.8711	15.4825	0.9215	0.7630	3.3427	0.3571	0.7857	0.7140
20	4.8829	3.5211	2.0948	0.8786	0.7689	0.4482	0.3571	0.8571	0.8571
21	4.6414	2.4773	0.9326	0.8785	0.7137	0.1842	0.5000	0.8571	1.0000
22	4.6045	2.3470	0.5528	0.8690	0.6633	0.1234	0.5000	0.8571	1.0000
23	3.5656	2.0540	1.3283	0.8928	0.7312	0.3674	0.5000	0.8571	0.9286
24	2.7970	2.2899	1.3260	0.8816	1.0784	0.5040	0.5000	0.7143	0.7857
Average	7.9916	5.3237	2.9867	0.8975	0.6912	0.4853	0.5357	0.8244	0.9256

IV. RESULT ANALYSIS AND DISCUSSION

A. Checking of fitting precision for the model

We take the price data from January 18, to January 31,2007 in Nordpool market for analysis (As shown in Fig.3). From Fig.3 we can see that the electricity price is undulating and non- monotonic. So the proposed PSOGM (1,2) model was trained and checked using this period price data. The checking results for the proposed model are shown in Table.Ⅱ.

It can be seen from TABLE.Ⅱ that the fitting precision of PSOGM (1,2) model is higher than GM (1,1) and GM (1,2) model, and the precision grade of PSOGM (1,2) model is GOOD mostly. Fig.4 shows the electricity price-fitting curve of high-load period at 8:00(from January 18 to 31,2007). The fitting results obtained from the PSOGM (1,2) model are quite close to the actual price value.

B. Analysis of forecasting precision for the model

The proposed PSOGM (1,2) model is used for electricity price forecasting in Nordpool market. According to 24 period-decoupled price sequences from 18 May to 31 May 2007, each period price from 1 June 2007 to 31 September 2007 can be rolling-predicted. Fig.5~Fig.8 show the rolling-prediction

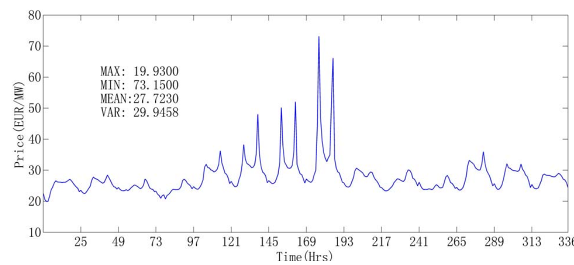

Figure3 Actual price from January 18 to 31,2007 in Nordpool market

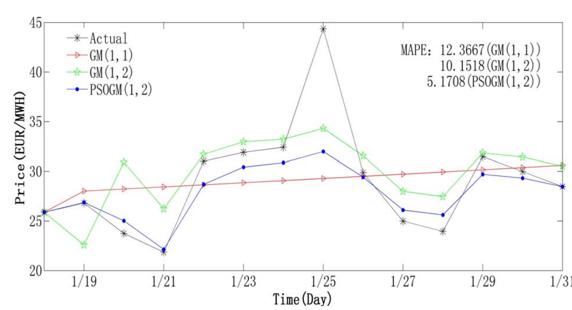

Figure4 8:00 price from January 18 to 31,2007 in Nordpool market results of proposed model, and TABLE.Ⅲ give the comparison between the proposed model and other traditional grey models.

Fig.5 shows the actual price and the forecasts with proposed model at lowest load period 2:00; Fig.6 shows the actual price and the forecasts with proposed model at peak load period 8:00; Fig.7 shows the actual price and the forecasts with proposed model at normal load period 14:00; Fig.8 shows the actual price and the forecasts with proposed model at peak load period 20:00. TABLE. Ⅲ compares the prediction MAPE applying proposed model and other traditional grey models. Obviously, the proposed PSOGM (1,2) model has higher forecasting precision than GM (1,1) and GM (1,2) model.

The proposed PSOGM (1,2) model can also be used for day-ahead electricity price forecasting. Table Ⅳ shows the day-ahead price continuous forecasting results from March to December 2007 in Nordpool market.

In TABLE Ⅳ, the MAPE value of July and August are invalidation with traditional GM (1,2) model, and the results illustrate that the traditional GM (1,2) model has invalidation forecasting value for some grey process.

Moreover, we can see from TABLE Ⅳ that the proposed model has higher precision than GM(1,1) and GM(1,2) model. The MAPE value of PSOGM (1,2) model for each month is less than 10% except for July and August. The July MAPE value is 12.022%, and the August MAPE value is 15.0447%.

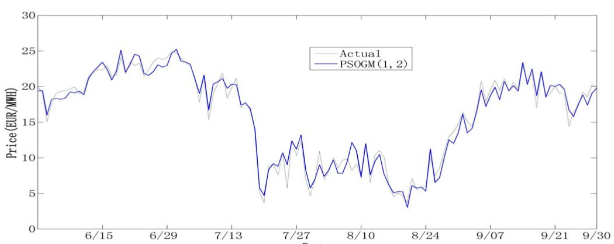

Fig.5 2:00 price from June 1 to September 31,2007 in Nordpool market

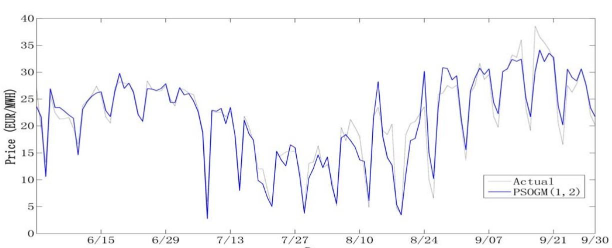

Fig.6 8:00 price from June 1 to September 31,2007 in Nordpool market

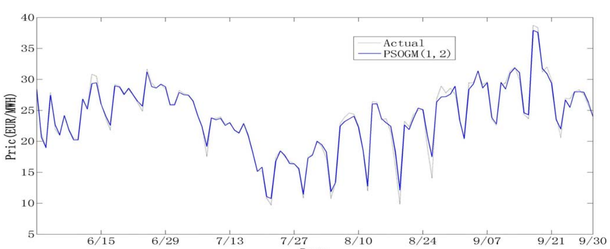

Fig.7 14:00 price from June 1 to September 31,2007 in Nordpool market

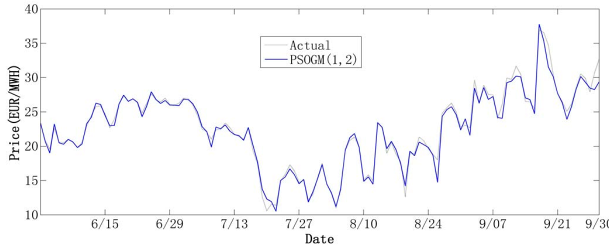

Fig.8 20:00 price from June 1 to September 31,2007 in Nordpool market

TABLE Ⅲ

FORECASTING RESULTS FROM JUNE 1 TO SEPTEMBER 31,2007 IN NORDPOOL MARKET

Forecasting Period	MAPE(%)		
	GM(1,1)	GM(1,2)	PSOGM(1,2)
Low Load Period (2:00)	18.0869	6.1231	6.3446
Peak Load Period (8:00)	46.3883	62.9107	9.2115
Normal Load Period (14:00)	22.2868	3.1064	2.4373
High Load Period (20:00)	14.595	2.2844	2.2571
Average	25.3393	18.6062	5.0626

TABLE Ⅳ

FORECASTING RESULTS FROM MARCH TO DECEMBER 2007 IN NORDPOOL MARKET

Forecasting Month	MAPE(%)		
	GM(1,1)	GM(1,2)	PSOGM(1,2)
March	7.6713	5.3314	4.603
April	8.6339	6.0253	5.6841
May	11.8552	8.7981	9.2769
June	11.9764	9.8081	8.4512
July	17.8107	1.20×10^{14}	12.022
August	19.8366	2.96×10^{17}	15.0447
September	13.1544	11.1603	9.2785
October	4.4532	4.0166	4.0174
November	5.5357	3.7253	3.4212
December	5.3191	3.5133	3.6922
Average	10.6247	2.9621×10^{16}	7.5491

V. CONCLUSION

In this paper, a novel grey model has been proposed. The PSO algorithm is adopted to identify the developing coefficient *a* and grey input *b* instead of LSM in traditional grey model. Thus the PSOGM (1,2) model for short-term electricity price forecasting is founded. Appropriate examples based on data obtained from Nordpool power market are demonstrate the functioning of the proposed PSOGM (1,2) model. All the examples in this paper illustrate that the proposed PSOGM (1,2) model has higher forecasting precision than traditional grey model, and has more widely used range than GM (1,2) model. The average MAPE of proposed model is 5.0626% for short-term electricity price rolling prediction, and 7.5491% for continuous forecasting.

REFERENCES

[1] Zeng Ming, Feng Yi, et al. Electricity Price Forecasting Based on Multi-models Combined by Evidential Theory [J]. Proceedings of the CSEE，2008，28 (16) ：84-89．

[2] Yang Jun, Ai Xin, et al. Electricity Price Forecasting Based on Nonparametric GARCH[J]． Transactions of China Electrotechnical Society，2008，23(10)： 135-142．

[3] Zhang Xian， Wang Xifan， et al． Short term electricity price forecasting based on period decoupled price sequence[J]． Proceedings of the CSEE，2005，25(15)：2-6．

[4] Zhang L， Luh P B． Neural network-based market clearing price prediction and confidence interval estimation with an improved extended Kalman filter method[J]． IEEE Transactions on Power Systems，2005，20(1)：59-66．

[5] Garcia R C，Contreras J，Akkeren M，et al． A GARCH forecasting model to predict day-ahead electricity prices[J]． IEEE Transactions on Power Systems，2005，20(2)：867-874．

[6] JL Deng． The control problem of grey systems[J]． Journal Systems & control Letters, 1982(5)：288-294．

[7] DU Song-huai, HOU Zhi-jian, JIANG Chuan-wen. A New Short-term Grey Forecasting Procedure of Spot Price[J]． Journal of Grey System, 2002, 14(4)：351-358.

[8] MA Xin, HOU Zhi-jian, JIANG Chuan-wen. Grey Forecasting Electricity Forward Price[J]． Journal of Grey System, 2003, 15(3)：263-266．

[9] Cheng Xiaoxin, Zhou Yuhui. Research on electricity price prediction based on improved grey mode[J]． Journal of North China Electric Power University, 2006, 33(1)： 47-50．

[10] WU Xing-hua， ZHOU Hui. A Grey Model of Electricity Price Forecasting Based on Period Residual Modification[J]． Power System Technology, 2008, 32(8)：67~70．

[11] Ma Xin, Hou Zhijian， et al. Study on grey dynamic model for electricity forward price forecasting[J]． Relay. 2003, 33(6)：23-25．

[12] Julong Deng. Grey Prediction and Decision[M]． Huazhong University of Science and Technology Press,1986.

[13] BeniniM, MarracciM, Pelacchi P, et a1.Day-ahead Market Price Volatility Analysis in Deregulated Electricity Markets [A]. Proc of IEEE PES Summer Meeting. 2002, 1354-1359.

[14] Kennedy J, Eberhart R. C. Particle swarm optimization [C]. Proc. IEEE Int. Conf. Neural Networks. Piscataway, NJ：IEEE press, 1995:1942-1948.

[15] R. Eberhart and J. Kennedy, “A new optimizer using particle swarm theory,” in Proc. 6th Int. Symp. Micro Machine and Human Science *(MHS)*, Oct. 1995:39–43.