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# Using Fuzzy Real Options in a Brownfield Redevelopment Decision Support System

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***Abstract*—A prototype decision support system is developed for private developers to evaluate brownfield redevelopment projects based on fuzzy real options. Redevelopment investments under high uncertainty can be better assessed using fuzzy real options than when employing the traditional Net Present Value (NPV) method. When using NPV, private developers tend to require high risk premiums and regard these projects as unprofitable. Hence, a better valuation method is needed to reveal the real value of a brownfield. The fuzzy real option model seems to be appropriate for this purpose. A Decision Support System (DSS) is proposed based on this model in order to evaluate a brownfield site and provide private developers with better investment advice. This DSS's architecture and user interface are introduced and discussed. Implementation techniques are compared and selected according to their advantages and disadvantages. An illustrative example is used to demonstrate this system's processing capability. It is found that the proposed DSS is flexible and user-friendly. It can generate not only a price, but also an optimized strategy for dealing with uncertainties, which has great potential in applications.**

***Index Terms*—Brownfield Redevelopment, Real Options, Fuzzy Arithmetic, Decision Support System, Net Present Value, System-of-Systems.**

## I. Introduction

A brownfield is an abandoned or underutilized property that possibly contains pollutants or hazardous materials, which are usually left from previous usages, such as industrial activity [1]. Brownfields are common in regions transiting from an industrial to a service-oriented economy, or when industrial enterprises have relocated or restructured themselves [2]. Brownfields are associated with an unsustainable development pattern, as they often arise when property is abandoned in favor of greenfield development.

Brownfield redevelopment is a typical system-of-systems (SoS) problem, as it involves various systems with complex interactions, as shown in Figure 1. Brownfield redevelopment has many of the characteristics of an SoS, such as possessing high uncertainty, exhibiting non-linear behavior, and being interdisciplinary in nature [3]. Due to the complex interactions of soil-groundwater and societal systems, uncertainties in redevelopment costs, knowledge and technologies, and liabilities, are high and play a critical role in preventing redevelopment [4].

Many Canadian cities are encouraging developers to participate in brownfield redevelopment as part of their strategy to enhance local sustainability. However, private developers are usually reluctant to undertake brownfield redevelopment projects as they regard these projects as unprofitable, thereby leading to slow progress in addressing the brownfield problem. However, facts have shown that brownfield redevelopment could be profitable, and, in some cases, even has high investment return [5] [6] [7]. One major reason for this is that the traditional evaluation tool, the Net Present Value method, overlooks the value of uncertainty in brownfields, and hence, often underestimates the value of brownfields [7].

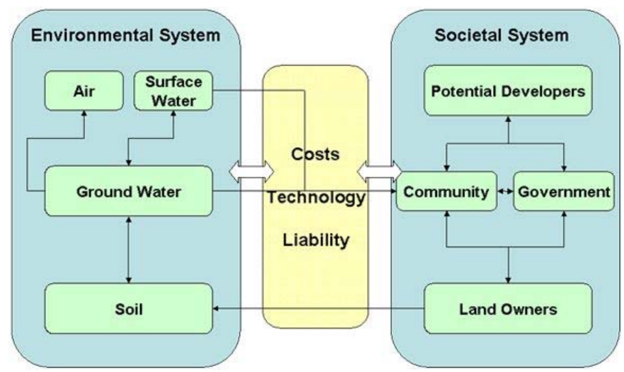


Fig. 1. Systems Diagram of Brownfield Redevelopment

To accelerate brownfield redevelopment, a better evaluation tool, the fuzzy real options approach, demonstrates its capacity in valuing uncertainty and flexibility in dealing with multiple uncertain sources. Fuzzy real options not only reveal higher value of brownfields, but also indicate the amount of financial aid needed to change developer decisions from delaying redevelopment projects to undertaking them immediately.

A prototype DSS is developed to facilitate developers' adoption of the fuzzy real options approach to evaluate brownfield redevelopment projects. Its architecture, user interface, implementation techniques are explained. In an illustrative example, this DSS reveals a higher value of brownfields than NPV and suggests that to wait for redevelopment is the optimal choice for developers, which conforms to the current situation in brownfield redevelopment.



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## II. Background

### A. Real Options

An option is the right, but not the obligation, to buy or sell a certain security at a specified price at some future time [8]. The option pricing model developed by Black and Scholes (1973) [9] has had great success in pricing financial derivatives. Because option pricing quantifies the value of uncertainties, this technique has migrated to broader usage, such as strategy selection [8], risky project valuation [10] [11], and policy assessment [12]. The idea of employing an option pricing model to place value on real assets or investments with uncertainties is usually called the real options approach or real options modeling [13] [8]. In real options, risky projects are modeled as a portfolio of options that can be valued using option pricing equations [10].

Option pricing acknowledges uncertainty explicitly (Table I). As for the assets underlying the derivatives, the option model regards them not as deterministic processes, but as stochastic ones instead. The volatility (denoted as $\sigma$) that reflects uncertainties can be estimated using historical data and has a more solid mathematical basis.

TABLE I
Comparison of Option Models and NPV

| | **NPV** | **Option model** |
|---|---|---|
| **Value of underlying assets** | $\frac{dS}{s} = \mu dt$ | $\frac{dS}{S} = \mu dt + \sigma dz$ |
| **Value of derivative options** | 0 | Black-Scholes |

Note: $S$ denotes the value; $\mu$ is the drift rate; and $z$ is a normal distributed variable with a zero mean and 1 unit standard deviation as $N(0, 1)$.

In a real options model, the value of a risky project includes not only its present value, but also the portfolio of options reflecting the values of uncertainties and associated managerial flexibilities. The following options may exist in different kinds of projects and situations, and can be evaluated using option formulae [8] [11] [13] [14] [15].

- *The option to defer*: The option of waiting for the best time to start a project can be valued as an American call option or a Bermuda call option;
- *The option to expand*: The option of expanding the scale of the project can be valued as an American call option or a barrier option;
- *The option to contract*: The option of shrinking the scale of the project can be valued as an American put option;
- *The option to abandon*: The ability to quit the project can be valued as an American put option or a European put option;
- *The option of staging*: The ability to divide projects into several serial stages, with the option of abandoning the project at the end of each stage ("option on option"), can be valued as a compound option, also called the learning option in some articles;
- *The option to switch*: The flexibility to change the project to another use can be valued as a switch option.
- *The option with multiple uncertainties*: When the underlying asset of an option has more than one uncertainty, it is called the rainbow option and requires more complex pricing formula.

The output of option valuation models usually include the value of the project, critical values, and strategy spaces for different optimal decisions [8]. This important information is explained in the following:

- *Valuations*: The most important output is the value of the risky project. This is the original goal of using a real options model;
- *Critical values*: In project valuation, there are usually formulae for calculating critical values, which are used in deciding whether to undertake the project. Critical values play the same role as Net Present Value (NPV) zero;
- *Strategy space*: The multi-dimensional strategy space is divided into regions, corresponding to which option is best to implement. This is an optional output.

### B. Uncertainties in Brownfield Redevelopment

Various uncertainties involved in the brownfield redevelopment can be classified into three categories based on their originalities. These three main reasons, namely, limited knowledge, complex environmental systems, and sophisticated societal systems, are explained in the following:

- *Uncertainties due to limited knowledge of brownfields*: At present, knowledge and data about brownfields are limited. Identifying appropriate models, characteristics, and parameters are costly and time-consuming. In Canada, the phenomenon of lacking data and knowledge is worse since there is no well-designed and coordinated data collection mechanism [7] [16];
- *Uncertainties originating from environmental systems*: Environmental systems have complex interactions in different systems, especially between groundwater and soil. Brownfields expose environmental risks mainly in the vadose zone, which is the unsaturated layer between ground surface and ground water table [16] [17]. Complex site-specific characteristics, such as unknown distribution of source contaminations and heterogeneous porous media, prevent remediation and redevelopment processes because they usually lead to highly uncertain remediation costs [17];
- *Uncertainties originating from societal systems*: There are various kinds of stakeholders in brownfield redevelopment with complex conflicts and interactions. In some situations, they even hold interchangeable roles. These complex interactions and ambitious boundaries create high levels of uncertainty in liabilities and cost-sharing policies.

Unfortunately, these uncertainties mentioned above are hard to find counterparts for in the market. Hence, the basic assumptions of option pricing model is invalid in brownfield redevelopment [18] [10]:

- *Complete Market*: All risks can be hedged by a portfolio of options [19]. In other words, all risks that have been completely reflected in the market price can be replicated as options. In some literature, this approach is also called Market Asset Disclaimer (MAD) [11];

- *Arbitrage-free Market*: There is no profit opportunity unless players in the market are willing to take some risk [10]. In other words, there is no risk-free way of making money;
- *Frictionless Market*: There is no barrier preventing people from trading, borrowing, or shorting. And there are no transaction costs for doing so. Furthermore, the underlying assets are infinitely divisible [18];
- *Constant Rate*: Parameters like risk-free interest rates and volatility, are constant. For the convenience of computation and formula deduction, these kinds of parameters are assumed to be constant during the entire project lifetime.

These uncertainties are usually called private risks [8]. The process of real options modeling should be customized in order to make the valuation framework flexible enough to fit different situations. Some techniques should be introduced in order to expand real options which are capable of processing private risks.

### C. A Fuzzy Real Options Model of Brownfield Evaluation

Soft-computing techniques have demonstrated their advantages in intelligent behavior. Fuzzy theory is especially suitable for situations in which expert knowledge is required. Hence, fuzzy real options are proposed for dealing with private risks that are hard to objectively estimate based on possibility theory as claimed by Carlsson and Fuller [20] [21]. If private risks are represented as fuzzy variables, possibility theory can be used. In this case, both subjective and objective uncertainties are integrated into the fuzzy real options model.

However, Carlsson's fuzzy real options are limited to the exercise price and current value in the options model [20]. Hence, the transformation method is employed to generalize fuzzy variable representation to any parameter, which overcomes the multiple outputs problem [22]. The idea underlying the transformation method follows three steps: firstly, decompose fuzzy numbers into discrete form; then use an $\alpha$-cut for calculation purposes as a traditional function; and finally, search the coordinates of the points in the hypersurfaces of the cube [22].

Among a couple of available real options models for brownfield redevelopment, the model proposed by Lentz and Tse [23] is chosen to be extended to fuzzy function. Valuation formula of the contaminated property are derived using the option pricing approach. Their model begins with definitions in Table II, which are all assumed to be Geometric Brownie Motions with the drift rates $\mu$, the volatilities $\sigma$, and random process $dz$. The private risks are originally represented as the coefficient between the underlying assets ($x$ and $R$) and their derived options ($P$ and $K$) (Formula (2) and (3)). With these definitions, the formulae are deducted with the ITO lemma and boundary conditions. All necessary parameters are in Equation (1).

$$\begin{aligned}
&g = \mu_x - (\mu_P - r)\beta_x \\
&\beta_x = \rho_{xP}\tfrac{\sigma_x}{\sigma_P} \\
&\beta_R = \rho_{RK}\tfrac{\sigma_R}{\sigma_K} \\
&\omega_K = r + (\mu_K - r)\beta_R \\
&\sigma^2 = \sigma_x^2 + \sigma_R^2 - 2\sigma_{xR} \\
&\gamma = \omega_k - \mu_R \\
&\delta = g - (\mu_R - \mu_K + r) \\
&q = 0.5(\tfrac{\sigma^2 - 2\delta}{\sigma^2} + \sqrt{\tfrac{(2\delta-\sigma^2)^2 + 8\gamma\sigma^2}{\sigma^4}})
\end{aligned} \tag{1}$$

TABLE II
DEFINITIONS OF CONTAMINATED PROPERTY REDEVELOPMENT MODEL

| | Underlying asset | Derived option |
|---|---|---|
| **Uncertain cash flow** | $\frac{dx}{x} = \mu_x dt + \sigma_x dz_x$ | $\frac{dP}{P} = \mu_P dt + \sigma_P dz_P$ |
| **Uncertain cost** | $\frac{dR}{R} = \mu_R dt + \sigma_R dz_R$ | $\frac{dK}{K} = \mu_K dt + \sigma_K dz_K$ |

$$dz_x = \rho_{xp} dz_p + (1 - \rho_{xp}^2)^{1/2} dz_{pe} \tag{2}$$

$$dz_R = \rho_{Rk} dz_p + (1 - \rho_{Rk}^2)^{1/2} dz_{ke} \tag{3}$$

where $\rho_{xp}$ and $\rho_{rk}$ are the correlation coefficients; $dz_{pe}$ and $dz_{ke}$ are random variables conforming to the normal distribution with zero mean and $dt$ variance.

This model includes four coefficient parameters. The parameters $\varphi_1$ and $\phi$ focus on cash flows. As cash flows from all states are proportional to the cash flow from the clean state, the cash flow generated under contamination is $\varphi_1 x$; the cash flow after removal to the clean flow $x$; and cash flow after redevelopment is $\phi x$. The coefficients $\alpha_1$ and $\alpha_2$ denote the removal and restoration costs $\alpha_1 R$ and $\alpha_2 R$, which are assumed to be proportional to the total redevelopment cost $R$. Therefore, the cleanup cost $C$ equals $(\alpha_1 + \alpha_2)R$.

Three critical values are involved in deciding among three strategies, denoted $Z^*$, $Y^*$, and $W^*$ in Formula 5 in [23]: do nothing, remove pollutants and redevelop sequentially, or remove pollutants and redevelop simultaneously. Values to be compared with these critical values are, respectively, $Z = \frac{x}{R}$, the ratio of the clean cash flow ($x$) to the redevelopment cost ($R$), $Y = \frac{x}{C}$, the ratio of the clean cash flow ($x$) to the cleanup cost ($C$), and $W = \frac{x}{(1+\alpha_1)R}$, the ratio of the clean cash flow ($x$) to the combined cost of removal and redevelopment as a joint action.

If the contaminated properties were to be cleaned and redeveloped sequentially, their values, expressed as Formula 4 in [23], depend on the critical value $Y^*$ as Formula 5 in [23]. On the other hand, if the brownfield sites were to be cleaned and redeveloped simultaneously, their values can be expressed in Formula (6) in [23], depending on the critical value of $W^*$ in Formula (5). All decision rules are summarized in Table III:

$$V1 = \begin{cases} \frac{\varphi_1 x}{r-g} + (\frac{(q-1)^{q-1}}{q^q})((\frac{1-\varphi_1}{r-g})^q(\frac{x}{C})^{q-1} + (\frac{\phi-1}{r-g})^q(\frac{x}{R})^{q-1})x \\ \text{if } Y \le Y^*; \\ \frac{x}{r-g} + (\frac{(q-1)^{q-1}}{q^q})(\frac{\phi-1}{r-g})^q(\frac{x}{R})^{q-1}x - C \\ \text{if } Y > Y^*; \end{cases} \tag{4}$$

$$\begin{array}{l} Y^* = \frac{r-g}{1-\varphi_1}\frac{q}{q-1} \\ W^* = \frac{r-g}{\phi-\varphi_1}\frac{q}{q-1} \\ Z^* = \frac{r-g}{\phi-1}\frac{q}{q-1} \end{array} \quad (5)$$

$$V2 = \begin{cases} \frac{\varphi_1 x}{r-g} + (\frac{\phi-\varphi_1}{r-g})^q (\frac{(q-1)^{q-1}}{q^q}) \frac{x^q}{(\alpha_1 R + R)^{q-1}}, & \text{if } W \le W^*; \\ \frac{\phi x}{r-g} - (\alpha_1 R + R), & \text{if } W > W^*; \end{cases} \quad (6)$$

TABLE III
DECISION RULES ON BROWNFIELD REDEVELOPMENT [23]

| Decision rules | | | Output Value | Suggestion |
|---|---|---|---|---|
| $Y \le Y^*$ | $W \le W^*$ | $V_1 \le V_2$ | $V_2$ | Wait until $W \ge W^*$ to remove and redevelop simultaneously |
| $Y \le Y^*$ | $W \le W^*$ | $V_1 > V_2$ | $V_1$ | Wait until $Y \ge Y^*$ to remove and redevelop sequentially |
| $Y \le Y^*$ | $W > W^*$ | $V_1 \le V_2$ | $V_2$ | Remove and redevelop immediately |
| $Y \le Y^*$ | $W > W^*$ | $V_1 > V_2$ | $V_1$ | Wait until $Y \ge Y^*$ to remove and redevelop sequentially |
| $Y > Y^*$ | $W \le W^*$ | $V_1 \le V_2$ | $V_2$ | Wait until $W \ge W^*$ to remove and redevelop simultaneously |
| $Y > Y^*$ | $W \le W^*$ | $V_1 > V_2$ | $V_1$ | Remove now and wait until $Z > Z^*$ to redevelop |
| $Y > Y^*$ | $W > W^*$ | $V_1 \le V_2$ | $V_2$ | Remove and redevelop immediately |
| $Y > Y^*$ | $W > W^*$ | $V_1 > V_2$ | $V_1$ | Remove now and wait until $Z > Z^*$ to redevelop |

## III. THE PROTOTYPE DECISION SUPPORT SYSTEM

### A. System Design

Input all required information, and obtain results, decision regions, and suggestions. The purpose of building a DSS is to permit private developers using fuzzy real options to conveniently evaluate brownfield projects. Hence, the design goals of the DSS should include:

- *Easy to use*: All input parameters need to be easily input, such as riskless rate, volatility, and fuzzy variables. Users can quickly understand the DSS;
- *Provide optimal operation suggestions*: Since the fuzzy real options approach has not been widely adopted by developers, giving value with associated operation suggestions help to convince decision makers that the higher estimated brownfield value generated from fuzzy real options than the result from NPV is achievable;
- *Show strategy spaces graphically*: Plotting the strategy spaces and boundaries can intuitively clarify the project situation and make decision makers aware of how to choose between options in order to attain the maximum value of a brownfield;

Based on the above design requirements, the system architecture of the prototype DSS developed to address this problem is shown in Fig. 2. Experts input parameters via the Windows Presentation Foundation (WPF) layer shown in Fig. 3. After that, an event and process management module controls the work flow to convert all information to the MatLab format and feeds it to MatLab for computation using the proposed fuzzy real options algorithm. Basically, fuzzy data are first converted (fuzzified and then defuzzifed) into the crisp values needed for the real options model. Then, output is obtained by utilizing the real options formula. Finally, the output is presented graphically via WPF.

Although the prototype developed in this paper is primitive, the system architecture is generic and extendable. This DSS can be gradually expanded in scale and complexity with increased functionality. This prototype constitutes a feasibility study of both the algorithm and the technical approach to building the DSS.

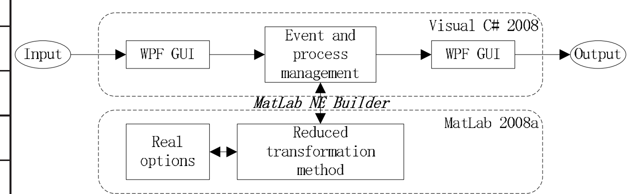


Fig. 2. System architecture of the DSS

### B. Implementation Technologies

Major available implementation technologies are compared and shown in Table IV. In this prototype of DSS, the .NET framework, WPF, and Matlab are selected as optimized techniques, given that they are easy to realize and integrate to make a holistic DSS.

TABLE IV
IMPLEMENTATION TECHNOLOGIES COMPARISON

| Components | Technology | Advantages | Disadvantages |
|---|---|---|---|
| UI | WPF | Vectorized graphic engine, animated scene, and core compatible with Web | Unknown bugs and limited operating system (OS) support |
| | Windows Form | Mature technology and wide adoption | Limited controls and incompatible with Web |
| | Java Swing | Almost all OS support | Limited controls and low speed |
| Algorithm | MatLab | Easy to use, wide adoption, and flexible architecture | Slow speed and high license fee |
| | C++ | High flexibility and quick running speed | Hard to use |
| | C#//F# | High compatibility | Not as easy as MatLab |
| | Java | Almost all OS support | Poor performance |
| Framework | Component Object Model | Mature technology | Prone to produce bugs and hard to use |
| | .NET | Advance and flexible framework and better performance through concurrent computing | Limited adoption |
| | Java | Almost all OS support, wide adoption, and mature technology | Low speed |

Because of the system architecture specified in Fig. 2, the prototype of this DSS has the following advantages and limitations:

- *Easy to be converted into an Internet-enabled DSS*: WPF is built on Extensible Application Markup Language (XAML) and .NET framework, which are both used by Silverlight 2 technology [24]. Hence, all WPF applications except 3D modules can be easily migrated to Silverlight. Furthermore, because Silverlight is developed for facilitating rich media internet application development, this DSS can be easily converted into an Internet-enabled system;
- *Easy to be deployed on mobile devices*: As long as the mobile device runs on the Windows Mobile system, it can use applications on the .NET compact framework, which is a subset of .NET framework without 3D animation. Since 3D is not used in this DSS, over 90% codes of this DSS can be conserved. This DSS can become a mobile application with recompilation for .NET compact framework;
- *Easy to deal with typographic relationships*: Because WPF is superior in vector graphics, interactive graph interface is easy to realize. Hence, typographic relationships can be represented as diagrams. Users can modify them interactively to express complex relationships intuitively;
- *Hard to accommodate neural network right now*: Presently, the MatLab NE builder can only be utilized with existing neural networks [25]. Customizing neural network interactively via WPF will cause exceptions in this application. For this reason, the neuro-fuzzy system is only tested under Matlab manually in this project. There is one potential solution that uses other open-source neural network components written in C#. Because of lack of time, this approach has not yet been tested.

## IV. Conclusions

When using sample data from Lentz and Tse's work to test the DSS [23], which is shown in Figure 3 as the input interface, it is found that the private risk of redevelopment volatility has an effect on the value of brownfield properties. And because the output indicators (Y and W) are less than their corresponding critical values ($Y^*$ and $W^*$), this site is not worth developing now, as demonstrated in Figure 4. This result suggests that developers would be reluctant to undertake this redevelopment task.

Moreover, the critical values are fuzzy outputs as well. The fuzzy boundaries differentiating optimal strategies are illustrated in Fig. 5. Also, these critical values can be converted into units of $x/R$ and shown in a figure as different decision regions in strategy space. Fuzzy areas are calculated based on their fuzzy means and standard deviations. This DSS provides decision makers intuitive decision suggestions with the aid of the decision region chart.

As shown in the illustrative example using the prototype DSS, the fuzzy real options approach can effectively deal with private risk, generate satisfactory evaluations and useful decision suggestions. Hence, a DSS based on the fuzzy real options

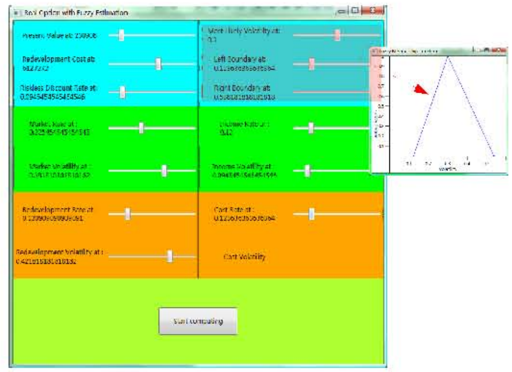

Fig. 3. GUI for input of the DSS

Fig. 4. GUI for output of the DSS

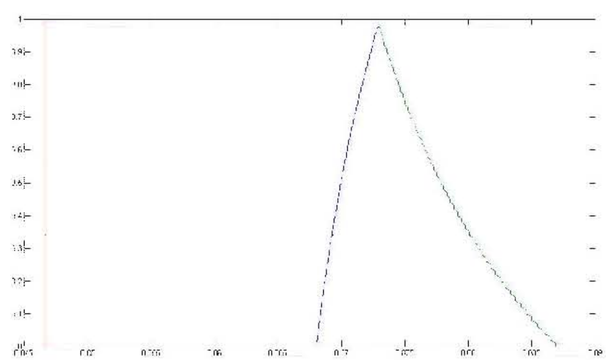

Fig. 5. W on the left and $W^*$ with Fuzzy Boundary (Membership Function) on the right

would constitute a useful tool in risky project evaluations, and therefore have great potential in applications.

At last, from the user-friendly perspective, WPF provides a powerful tool for vector graphics, which are easy for users to understand. A more intuitive input module based geographic information system will be introduced later. With this module, users are allowed to present their expert knowledge as fuzzy boundaries of a brownfield. This DSS will be further developed to be easier to use while to provide more decision suggestions.

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