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Decision Analysis with Hybrid Uncertain Performance Targets

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***Abstract*—The main focus of this paper is for decision analysis from the target-oriented point of view. Firstly, the target achievement computation method is revised, in which the resulting value function can have four shapes: concave, convex, S-shaped, inverse S-shaped. In addition, it is now more and more widely acknowledged that all facets of uncertainty cannot be captured by a single probability distribution. A fuzzy uncertain target-oriented method is also proposed, in which the proportional approach is selected to transform a possibility distribution into its associated probability distribution, and then based on the random target-oriented model, we can obtain the probability of meeting targets. Three types of fuzzy targets, widely used in Bellman-Zadeh paradigm, are selected to illustrate the fuzzy target-oriented model.**

I. Introduction

Substantial empirical evidence and recent research have shown that it is difficult to build mathematically rigorous utility functions based on attributes [1] and the conventional attribute utility function often does not provide a good description of individual preferences [2]. As a substitute for utility theory, Kahneman and Tversky [2] propose an S-shaped value function, and Heath et al. [3] suggest that the inflection point in this S-shaped value function can be interpreted as a target. To develop this concept further, target-oriented decision analysis involves interpreting an increasing, bounded function, properly scaled, as a cumulative distribution function (cdf) and relating it to the probability of meeting or exceeding a target value.

Berhold [4] notes that "there are advantages to having the utility function represented by a distribution" (p. 825), arguing that it permits the use of the known properties of distribution functions to find analytic results. Manski [5] calls this the "utility mass model". Castagnoli and LiCalzi [6] prove that expected utility can be expressed in terms of "expected probability", with the utility function for performance interpreted as a cdf in the case of a single attribute (see also Bordley and LiCalzi [7]). Interestingly, the Savage's utility function [8] can always be interpreted as the probability of achieving a target [7], [6]. In maximizing expected utility, a DM behaves as if maximizing the probability that performance is greater than or equal to a target, whether the target is real or just a convenient interpretation. Taking a different tack, instead of random uncertainty, Huynh et al. [9] propose a target-oriented approach to decision making under uncertainty with fuzzy targets. More details on target-oriented decision analysis could be referred to [10], [11], [12].

In general, target-oriented decision analysis lies in the philosophical root of bounded rationality [13] as well as represents the S-shaped value function [2]. In most studies on target-oriented decision analysis, monotonic assumption of attribute is given in advance to simplify the decision problems. However, in the context of decision analysis involving targets/goals, usually there are three types of goals: "the more the better" (corresponding to benefit target), "the less the better" (corresponding to cost target), and goal values are fairly fixed and not subject to much change, i.e., too much or too little is not acceptable (we shall call this type of targets as interval targets). Thus it is important to consider these three types of targets. Furthermore, our another motivation comes from the uncertainty representation of targets. It is now more and more widely acknowledged that all facets of uncertainty cannot be captured by a single probability distribution. And usually it is not so easy to find the probability distribution of the uncertain target. In many applications, fuzzy subsets [14] provide a very convenient object for the representation of uncertain information. Thus is is necessary to consider fuzzy target-oriented decision analysis. Although Huynh et al. [9] consider fuzzy uncertainty in target-oriented decision model, they only consider the payoff variables as well as do not analysis the differences between Bellman-Zadeh paradigm and target-oriented decision model.

Based on the above observations, we summarize our primary contributions as follows. Firstly, we propose an approach for computing the probability of meeting random target with respect to different types of target preferences. Although simple and straightforward, the resulting value function can have four shapes: concave, convex, S-shaped, inverse S-shaped. The relationship between target-oriented model and traditional MADM normalization method is also discussed. Furthermore, the fuzzy uncertainty is also considered in target-oriented decision analysis. The proportional conversion method is chosen to transform a possibility distribution into its associated probability distribution, and then based on the random target-oriented model we can obtain the probability of meeting targets. Some fuzzy targets widely used in Bellman-Zadeh paradigm [15] are selected to illustrate the fuzzy target-oriented decision model. The rest of this paper is organized as follows. Section II revises random uncertain target-oriented decision analysis, where three types of target preferences are considered. In Section III, we consider fuzzy target-oriented

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decision analysis via possibility-probability transformation and illustrate our approach by means of three types of fuzzy targets widely used in Bellman-Zadeh paradigm. Finally, some concluding remarks and future work are presented in Section IV.

II. Random Uncertain Target-Oriented Decision Model: Single Attribute Case

For notational convenience, let us designate an evaluation attribute by X, and an arbitrary specific level of that evaluation attribute by x. We also restrict the variable x to a bounded domain $D = [X_{\min}, X_{\max}]$. Suppose that a DM has to rank several possible decisions. Assume for simplicity that the set $\mathcal{A}$ of consequences is finite and completely ordered by a preference relation $\succeq$. Denote by p_d his probability distribution for the random consequence X_d associated with a decision d. The expected utility model suggests that the ranking be obtained by using the following value function

$$V(d) = \mathrm{EU}(X_d) = \sum_x U(x) \cdot p_d(x) \tag{1}$$

where $U(x)$ is a Von Neumann and Morgenstern (NM-)utility function over consequences.

As pointed out by Bordley and Kirkwood [1], an expected utility DM is defined to be target oriented for a single attribute decision if the DM's utility for an outcome depends only on whether a target is achieved with respect to x. Thus a target-oriented DM has only two different utility levels, and because a utility function is only specified to within a positive affine transformation, these two utility levels can be set to one (if the target is achieved) and zero (if the target is not achieved). Then a target-oriented DM's expected utility for alternative a is

$$\begin{aligned} v(a) &= \Pr(X_d \succeq T) \\ &= \sum_x \left[\Pr(x \succeq T) * 1 + (1 - \Pr(x \succeq T)) * 0\right] p_d(x) \\ &= \sum_x \Pr(x \succeq T) p_d(x) \end{aligned} \tag{2}$$

where $\Pr(x \succeq T)$ is the probability of meeting the uncertain target T and T is stochastically independent of X_d. The idea that the NM-utility function U should be interpreted as a probability distribution may appear unusual but, in fact, NM-utilities are probabilities [16], [7]. With the assumption that the attribute is monotonically increasing, x and t are mutually independent, Bordley and Kirkwood [1] suggest the following function

$$\Pr(x \succeq T) = \int_{X_{\min}}^{x} p(t)dt, \tag{3}$$

where $p(t)$ is the probability density function of uncertain target T.

A. Three Target Preferences

In most studies on target-oriented decision making, monotonic assumptions of attributes (e.g., wealth) are given to simplify the problems. In many decision problems involving goals/targets, usually there are three types of goal preferences [17].

- Goal values are adjustable: "more is better" (we shall call benefit targets);
- Goal values are adjustable: or "less is better" (with respect to cost targets);
- Goal values are fairly fixed and not subject to much change, i.e., too much or too little is not acceptable (we shall call this type of targets as interval targets).

The target-oriented decision model assumes that the probability distribution with respect to the uncertain target is unimodal as well as views the mode value of probability distribution of the uncertain target as the reference point, denoted as T_m [7]. To model the three types of goal preference, similar with Bordely and Kirkwood [1], we define

$$\Pr(x \succeq T) = \int_{X_{\min}}^{X_{\max}} u(x,t)p(t)dt. \tag{4}$$

As target-oriented decision model has only two different utility levels, we can define $u(x,t)$ as follows.

1) Benefit target: In this case, the DM has a monotonically increasing preference, i.e. "the more the better". As target-oriented model assumes that there are only two levels of utility (1 or 0), thus, we define as follows:

$$u(x,t) = \begin{cases} 1, & x \geq t; \\ 0, & \text{otherwise.} \end{cases}$$

Then we can obtain the probability of meeting uncertain target as the following function

$$\Pr(x \succeq T) = \Pr(x \geq T) = \int_{X_{\min}}^{x} p(t)dt. \tag{5}$$

This is consistent with the target-oriented model in the literature [7], [6], i.e. the target-oriented model views the cdf as the probability of meeting uncertain targets.

2) Cost target: Similar with the benefit target, for cost target we define

$$u(x,t) = \begin{cases} 1, & x \leq t; \\ 0, & \text{otherwise.} \end{cases}$$

Then the probability of meeting cost target is as follows

$$\Pr(x \succeq T) = \Pr(x \leq T) = \int_{x}^{X_{\max}} p(t)dt \tag{6}$$

3) Interval target: In this case, the mode value T_m is the reference point. There will be added loss of value for missing the reference point on the low side, or added loss for exceeding the reference point. When $x = T_m$ the probability of meeting target should be equivalent to one. Based on this observation, we define the target achievement function as follows:

$$\begin{aligned} &\Pr(x \succeq T) = \Pr(x \in T) \\ &\qquad = \begin{cases} \frac{\int_{X_{\min}}^{x} p(t)dt}{\int_{X_{\min}}^{T_m} p(t)dt}, & \text{if } x < T_m; \\ 1, & \text{else if } x = T_m; \\ \frac{\int_{x}^{X_{\max}} p(t)dt}{\int_{T_m}^{X_{\max}} p(t)dt}, & \text{otherwise.} \end{cases} \end{aligned} \tag{7}$$

It should be note that in case of benefit and cost attribute, we can also use this relative probability of meeting uncertain targets. When the DM prefers monotonically increasing preference, then we can define

$$\Pr(x \geq T) = \frac{\int_{X_{\min}}^{x} p(t)dt}{\int_{X_{\min}}^{X_{\max}} p(t)dt} = \int_{X_{\min}}^{x} p(t)dt.$$

Generally speaking, when the DM has an interval target preference, the value domain below the reference point T_m can be viewed as a pseudo benefit attribute; the value domain upper than the reference point T_m can be viewed as pseudo cost attribute. By means of this relative probability, we can model three types target preferences. As a generation of this type of target preference, the reference point value T_m may have a interval range, such that $T_m \equiv [T_{ml}, T_{mu}]$. In this case, Eq. (7) becomes

$$\Pr(x \succeq T) = \Pr(x \in T) = \begin{cases} \frac{\int_{X_{\min}}^{x} p(t)dt}{\int_{X_{\min}}^{T_{ml}} p(t)dt}, & \text{if } x < T_{ml}; \\ 1, & \text{else if } x \in [T_{ml}, T_{mu}]; \\ \frac{\int_{x}^{X_{\max}} p(t)dt}{\int_{T_{mu}}^{X_{\max}} p(t)dt}, & \text{otherwise.} \end{cases} \quad (8)$$

B. Illustrative Examples

Choosing a suitable probability distribution for uncertain targets is due to specific problems. As the normal distribution is widely used as a model of quantitative phenomena in the natural and behavioral sciences, we shall assume that the uncertain target is normally distributed in the bounded domain D and with mode value T_m. We assume a DM has three types of monotonic preferences: benefit, cost, and interval target. By means of Eq. (5), Eq. (6), Eq. (7), we can obtain the target achievements with respect to these three target preference types, as shown in Fig. 1.

Looking at the target achievement of benefit attribute, $\Pr(x \geq T)$,as shown in Fig. 1. This corresponds to the S-shaped function, which is equivalent to the S-shaped utility function of prospect theory [2] as well as is consistent with "Goals as reference point" by Heath et al. [3]. This induced value function has the following two properties:

1) **Gain and loss**
The target divides the space of outcomes into regions of gain and loss (or success and failure). Thus, the value function assumes that people evaluate outcomes as gains or losses relative to the reference point T_m.
2) **Diminishing sensitivity**
The value function draws an analogy to psychophysical process and predicts that outcomes have a smaller marginal impact when they are more distant from the reference point T_m.

It should be noted that Kahneman and Tversky [2] assume another principle: outcomes that are encoded as losses are more painful than the similar sized gains are pleasurable. In their words, "losses loom larger than gains". The induced value function by target-oriented model does not entirely satisfy the prospect theory. The main reasons for this observation are twofold. The first reason is the distribution type of target. In case of benefit attribute with target having a normal distribution, the target is symmetrically distributed. Another reason is the bounded domain. In fact, when the attribute value has a bounded domain, and the reference point in the prospect theory is the middle value of the domain, the value function induced by prospect theory will also not satisfy this principle.

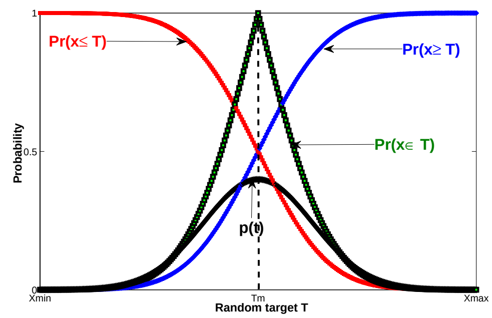


Fig. 1. Target achievements under normal distribution

In case of monotonically decreasing preference, we will obtain the inverse S-shaped function, as shown in $\Pr(x \leq T)$ of Fig. 1. When the DM has an interval target preference, DM will evaluate outcomes as losses relative to the reference point T_m. The attribute value below or exceeding the reference point is viewed as a loss, in which the value function is convex, as shown in $\Pr(x \in T)$ of Fig. 1.

Furthermore, let us consider a special case. Without additional information about the target distribution, we can assume that the random target T has a uniform distribution on D with the probability density function $p(t)$ defined by

$$p(t) = \begin{cases} \frac{1}{X_{\max} - X_{\min}}, & X_{\min} \leq t \leq X_{\max}; \\ 0, & \text{otherwise.} \end{cases} \quad (9)$$

Under the assumption that the random target T is stochastically independent of any alternative, we obtain the probability of meeting uncertain target for benefit and cost attributes as follows

$$\Pr(x \succeq T) = \begin{cases} \Pr(x \geq T) = \frac{x - X_{\min}}{X_{\max} - X_{\min}}, & \text{for benifit target;} \\ \Pr(x \leq T) = \frac{X_{\max} - x}{X_{\max} - X_{\min}}, & \text{for cost target.} \end{cases} \quad (10)$$

From Eq. (10) it is easily seen that, for benefit and cost attribute there is no way to tell whether the DM selects an alternative by traditional normalization method or by target-oriented model. In other words, in this case the target-based decision model with the decision function is equivalent to the traditional normalization function.

C. Comparison and Relationship with Related Research

Prospect theory [2] deals with decision making under risk, where probability distributions of the lotteries are known to agent. Prospect theory assumes that the ranking procedure is linear in the distorted probabilities. In other words, the ranking procedure is generated by the value function

$$v(X_d) = \sum_x U(x)\phi\left[p_d(x)\right], \tag{11}$$

which is linear in ϕ but not in p_d. The weighting function does not obey the axioms of probability theory and it measures the impact of probabilities on choices rather than the likelihood of the underlying events. Therefore, prospect theory postulates a model which in general is not linear in the known probabilities. It is apparent how little prospect theory tries to part away from the expected utility model.

Target-oriented decision model focuses on whether the value function meets a random variable, T having a probability distribution. In addition, target-oriented model views the mode value of probability distribution as reference point, this point was illustrated by Heath et al. [3]. Finally, target-oriented decision model satisfies NM-utility axiomatization [7].

III. Fuzzy Uncertain Target-Oriented Decision Model

It is now more and more widely acknowledged that all facets of uncertainty cannot be captured by a single probability distribution. Furthermore, usually it is not so easy to find the probability distribution of the uncertain target. Possibility theory, introduced by [18] appears as a mathematical counterpart of probability theory, that deals with uncertainty by means of fuzzy sets. Formally, the soft constraint imposed on a variable V is the statement "V is F", where F is a fuzzy set, can be considered as inducing a possibility distribution π on the domain of V such that $\mu(x) = \pi(x)$, for each x. In this paper, we shall use the possibility distribution $\pi(x)$ and membership degree $\mu(x)$ interchangeably.

A. Transformation from Possibility to Probability

The conversion problem between possibility and probability has its roots in the possibility/probability consistency principle of Zadeh [14], that he propose in the paper founding possibility theory in 1978 [18]. The possibility-probability consistency principle is a heuristic relationship between possibilities and probabilities. This principle can be summarized as: "the possibility of an event is always greater than or equal to the probability of the event". This is based on the consideration that possibility representation and probability representation are not just two equivalent representations of uncertainty, but the representation is weaker because it explicitly handles imprecision.

Yager [19] investigates the problem of instantiating a possibility variable over a discrete domain by converting its possibility distribution into a probability distribution, via a simple normalization. This conversion has been extended into a continuous domain [20] as follows:

$$p(x) = \frac{\pi(x)}{\int_x \pi(x)dx}. \tag{12}$$

When applying this proportional probability density distribution to convert the fuzzy number, it is noted that the range of the membership grade of the resultant proportional distribution is greatly reduced when the fuzzy number has a wide domain. Consequently, the ability of the membership function to discriminate precisely among the members of the fuzzy set is impaired. Fortunately however, the domain of the fuzzy number is always sufficiently narrow to avoid this becoming a problem. From the analytical point of view, the proportional proportional transformation approach can deal with different types of possibility distributions while following the possibility/probability consistency principle of Zadeh [18]. From the computational point of view, the proportional approach is convenient and simple in real applications. Thus, in this study, the proportional probability density function will be used to transform the possibility distribution to probability distribution.

B. Fuzzy Target-Oriented Decision Analysis

In many applications, the subjective assessments provided by DM(s) are usually conceptually vague, with uncertainty that is frequently represented in linguistic forms. To help people easily express their subjective assessments, the linguistic variables [21] are used to linguistically express requirements.

Assume that the fuzzy targets linguistically specified by the DM have the canonical form, and $\pi(t)$ is the membership degree/possibility distribution of uncertain target T. Based on the proportional possibility-probability conversion method Eq. (12) and the random target-oriented model for benefit target Eq. (5), we can obtain

$$\Pr(x \geq T) = \int_{X_{\min}}^{x} p(t)dt = \frac{\int_{X_{\min}}^{x} \pi(t)dt}{\int_{X_{\min}}^{X_{\max}} \pi(t)dt} \tag{13}$$

Similarly, we can obtain the target achievement function for cost target as follows

$$\Pr(x \leq T) = \frac{\int_{x}^{X_{\max}} \pi(t)dt}{\int_{X_{\min}}^{X_{\max}} \pi(t)dt} \tag{14}$$

According to the random target-oriented decision model for interval target Eq. (7) and the proportional possibility-probability conversion method Eq. (12), we obtain the target achievement function for interval target as follows:

$$\Pr(x \in T) = \begin{cases} \frac{\int_{X_{\min}}^{x} \pi(t)dt}{\int_{X_{\min}}^{T_{ml}} \pi(t)dt}, & \text{if } x < T_m; \\ 1, & \text{else if } x \in [T_{ml}, T_{mu}]; \\ \frac{\int_{x}^{X_{\max}} \pi(t)dt}{\int_{T_{mu}}^{X_{\max}} \pi(t)dt}, & \text{otherwise.} \end{cases} \tag{15}$$

In decision making involving fuzzy targets, there are three types of commonly used fuzzy targets: "**fuzzy min** T_m", "**fuzzy max** T_m", "**fuzzy equal** T_m or **fuzzy range/interval**: from T_{ml} to T_{mu}". For computational efficiency, trapezoidal or triangular fuzzy numbers are used to represent the above uncertain targets.

1) Fuzzy min: Assume that the DM assesses his target of at least T_m. In this case the DM has a monotonically increasing preference, the fuzzy number can be represented as

$$\pi(t) = \begin{cases} \frac{t - X_{\min}}{T_m - X_{\min}}, & \text{if } X_{\min} \leq t \leq T_m \\ 1, & \text{otherwise.} \end{cases} \tag{16}$$

We can obtain the induced probability of meeting this target according to Eq. (13).

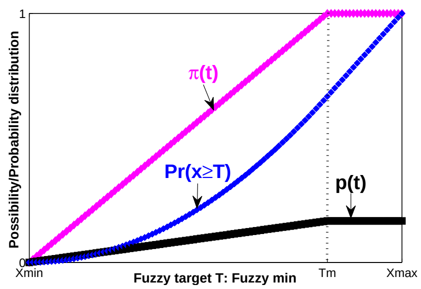


Fig. 2. Target achievements under fuzzy min target

Fig. 2 graphically depicts the membership function of the fuzzy min target, its associated probability distribution and the corresponding probability of meeting the target. As illustrated, the fuzzy min target induces the S-shaped function. When $X_{\max} = T_m$, the fuzzy min target reduces to $(X_{\min}, X_{\max}, X_{\max})$. In this case, $X_{\max}$ is the reference point value, because all the attribute values are below T_m, thus using Eq. (13) we will obtain a convex value function.

2) Fuzzy max: Assume that the DM assesses as the membership function for his target of at most about T_m and we get the membership function for this target as follows:

$$\pi(t) = \begin{cases} 1, & \text{if } X_{\min} \leq t \leq T_m \\ \frac{X_{\max} - t}{X_{\max} - T_m}, & \text{otherwise.} \end{cases} \tag{17}$$

Then we obtain the the induced probability of meeting target according to Eq. (14). The related functions of this target are graphically illustrated in Fig. 3. In this case, we obtain the inverse S-shaped function.

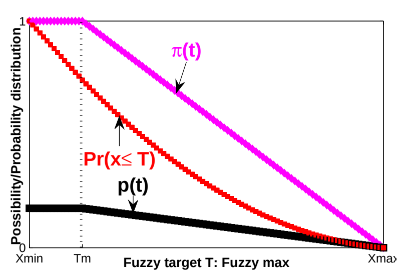


Fig. 3. Target achievements under fuzzy max target

Similarly, there exists a special case of fuzzy max target, such that $T = (X_{\min}, X_{\min}, X_{\max})$. As $X_{\min}$ is the reference point and DM prefers a monotonically decreasing preference, thus this special fuzzy max target induces a convex value function.

3) Fuzzy equal or Fuzzy interval: Another fuzzy target is "fuzzy equal". In this case, the target values are fairly fixed and not subject to much change, i.e., too much or too little is not acceptable. Let us assume that the DM assesses the membership function for his target about T_m as

$$\pi(t) = \begin{cases} \frac{t - X_{\min}}{T_m - X_{\min}}, & \text{if } X_{\min} \leq t < T_m; \\ 1, & \text{if } t = T_m; \\ \frac{X_{\max} - t}{X_{\max} - T_m}, & \text{if } T_m < t \leq X_{\max}. \end{cases} \tag{18}$$

Fig. 4(a) graphically depicts the membership function of the unimodal target, its associated probability density function and the corresponding probability function of meeting this target via Eq. (15). As the DM assesses feels losses with respect to the modal value, the unimodal target induces the convex value function when the possible attribute values are below or upper the mode value.

The "fuzzy equal" target is a special case of "fuzzy interval". In this case, the DM assesses target ranges. The fuzzy target fuzzy from T_{ml} to T_{mu} can be defined as

$$\pi(t) = \begin{cases} \frac{t - X_{\min}}{T_{ml} - X_{\min}}, & \text{if } X_{\min} \leq t < T_m; \\ 1, & \text{if } T_{ml} \leq t \leq T_{mr}; \\ \frac{X_{\max} - t}{X_{\max} - T_{mr}}, & \text{if } T_m < t \leq X_{\max}. \end{cases} \tag{19}$$

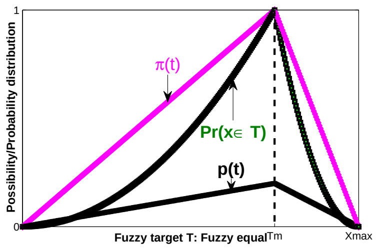
Fig. 4(b) graphically shows the possibility distribution, induced probability distribution, and its associated probability function of meeting targets.

C. Comparison with Bellman-Zadeh's Paradigm

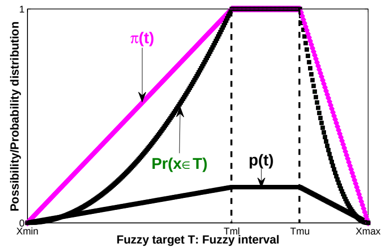
In their pioneering work on MADM, Bellman and Zadeh [15] suggest that a attribute can be represented as a fuzzy subset over the alternatives. In particular, if x is a attribute we can represent this as a fuzzy subset x over A such that $A(x)$ is the degree to which this criterion is satisfied, where $\forall\ A(x) \in [0, 1]$. They use the fuzzy membership function to represent the degree of preference (utility). Both the Bellman-Zadeh's paradigm and our approach use fuzzy subset to model decision making involving targets. The main differences between our approach and Bellman-Zadeh's paradigm are twofold.

The semantics of membership functions of fuzzy sets are different. Bellman and Zadeh view the membership function of fuzzy sets as a kind of utilities, whereas in our approach the membership function of fuzzy sets is viewed as a kind of uncertainty representations, possibility distribution. In fact, according to the context of problems, membership degrees can be interpreted as similarity, preference, or uncertainty [22]. As pointed out by Beliakov and Warren [23]:

> In fuzzy set theory, membership functions of fuzzy sets play the role similar to utility functionsłthe role of degrees of preference. Many authors, including Zadeh himself, refer to membership functions as 'a kind of utility functions'. The equivalence of utility and membership functions extends from semantical to syntactical level. Although this is not the only possible interpretation of membership functions, it allows one to formulate and solve problems



(a) Fuzzy equal target



(b) fuzzy interval target

Fig. 4. Target achievements under fuzzy equal/interval target

of multiple attributes decision making using the formalism of fuzzy set theory.

The semantics of fuzzy numbers are different. In our approach, even the same fuzzy number can have more than one semantic depending on DM's preferences. Whereas, Bellman and Zadeh consider only one semantic. For example, for the fuzzy number $T = (X_{\min}, T_m, X_{\max})$, in the above discussion, we view this fuzzy number as fuzzy equal target. Huynh et al. [9] have also considered this fuzzy target by assuming monotonically increasing target preference. Generally speaking, we suggest that each fuzzy number can have three types of target preference depending on DM's preferences. In this study, we only listed the three types of fuzzy targets commonly used in Bellman-Zadeh paradigm. To be consistent with Bellman-Zadeh paradigm the same semantics of fuzzy numbers is assumed.

IV. Concluding Remarks and Future Work

The importance of behavioral aspects of decision making has grown, and this was recognized by the award of the 2002 Nobel Prize in Economics to Daniel Kahneman [24]. As an emerging area considering behavioral aspects of decision making, target-oriented decision analysis lies in the philosophical root of bounded rationality as well as represents the S-shaped value function.

The contribution of this paper is to propose a hybrid uncertain target-oriented decision analysis model. To do so, firstly we proposed an approach for computing the probability of meeting random target, in which the resulting value function can have four shapes: concave, convex, S-shaped, inverse S-shaped. Furthermore, considering the uncertainty representation of targets, a fuzzy uncertain target-oriented method has been proposed. The proportional approach was used to transform a possibility distribution into the probability distribution. Fuzzy targets widely used in Bellman-Zadeh paradigm [15] were selected to illustrate the fuzzy target-oriented model.

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