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# Modeling and Optimal Control of Automated Trolleys

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*Abstract*—**Automated trolleys have been developed to meet the needs of material handling in industries. The velocity of automated trolleys is regulated by an S-shaped (or trapezoid-shaped) acceleration and deceleration profile. In consequence of the velocity profile, the control system of automated trolleys is nonlinear and open-looped. In order to linearize the control system, we use a second order dynamic element to replace the acceleration and declaration curve in practice, and design an optimal controller under the quadratic cost function. Performance of the proposed approach is also compared to the conventional method. The simulation shows a better dynamic performance of the developed control system.**

*Keywords*—**optimal control, automated trolleys, LQ problem**

## I. INTRODUCTION

Automated trolleys are typically required to move on straight rails at high speeds. They are widely used in ports, factories, and warehouses ([1][2]), for the carrying of cargo along rails.

Automated guarded vehicles (AGV) have been studied for a long history, which is focused on dispatching, scheduling, and path-planning ([3][4][5]). These problems are not applicable to automated trolleys because their trajectories are limited in straight rails, and the primary focus is the position control of automated trolleys. There are also a few papers about the control of trolleys. Weehong Tan in [6] presented the derivation of an AGV model and its reduction to linear model, and then H∞ robust control methods were used to design the controller. In [7], Corteletti R. presented a technique that generated an uncoupled system which can be controlled by SISO controllers, and an iterative algorithm was applied to SISO controllers. In [8], a fuzzy control scheme was presented to eliminate disturbances in the environment which caused position errors of the automatic guided vehicles. However, these studies were focused on the uncoupling of lateral and longitudinal control of vehicles.

The trajectory of automated trolleys is limited to straight rails, so we don't have to consider the problems of lateral control. The biggest challenge to us is the precision position control with the requirements of frequent starts and stops at high speed.

The conventional control of trolleys is open-looped. A typical point-to-point motion profile can be divided into three phases: accelerating, constant speed, and decelerating. The speed of trolleys is regulated as trapezoidal-curve or s-cure. We cannot use the knowledge of control theory to design controller because of the nonlinearity and open-loop. In our study, we are trying to design a closed-loop control system. Firstly, the response of a second order dynamic element is used as a speed regulator to produce the acceleration and deceleration profile, which ensures that the control system is linear. Secondly, an optimal output tracking controller is designed under the quadratic cost function. Finally, the performance of the proposed approach is also compared to the conventional one. The simulation shows a better dynamic performance of the developed control system.

## II. DYNAMIC MODEL OF AUTOMATED TROLLEYS

A hierarchical structure is commonly used in the control system of automated trolleys. The upper layer is programmable logic controller (PLC), which regulates the automated trolley to a smooth acceleration and deceleration speed profile. In the lower layer, servo drivers receive command signals from PLC, amplify the signals, and transmit electric currents to servo motors in order to produce motion proportional to the command signals. A schematic of this structure is shown in Fig.1.

In a properly configured system, the servo motor rotates at a velocity that very closely approximates the velocity signals being received by the servo drive from the control system. Therefore, the dynamics of servo driver and motor could be described as

$$\dot{s} = v$$

which means the position $s$ of automated trolleys is the integration of motor speed $v$.

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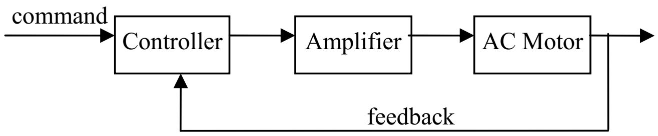


Figure 1. Block diagram of the trolley

The Acceleration and deceleration curve in Fig.2 has been used in practice for a long time. Automated trolleys stop at the destination with the speed zero after a procedure of acceleration, constant movement and deceleration. This brings us two challenges. The first is the control system is nonlinear because of the acceleration and deceleration curve. The second is that this control is open-looped, consequently the position control error is dependent on the precision of the deceleration and the brake, so we cannot design a controller by the methods in control theory.

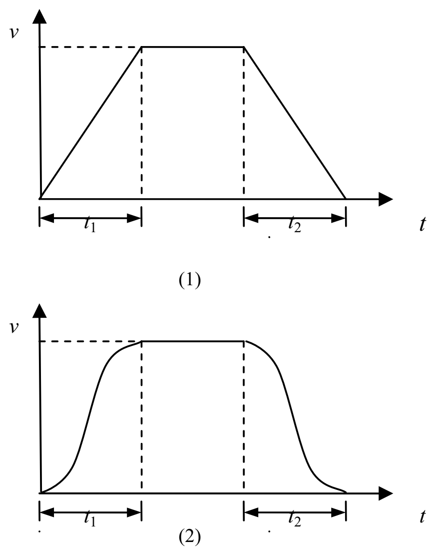


Figure 2. Velocity profile

In order to eliminate the nonlinear element, the following second order dynamic element is used as path planning

$$G(s) = \frac{K}{T^2 s^2 + 2\xi T s + 1} \tag{1}$$

where $\xi > 1$ to avoid oscillation.

In this way, the response of the acceleration and deceleration commands of trolleys is shown in Fig.3. It ensures a smooth speed profile. (1) can also be regarded as a second order low-pass filter, it produce a smooth response to the velocity command.

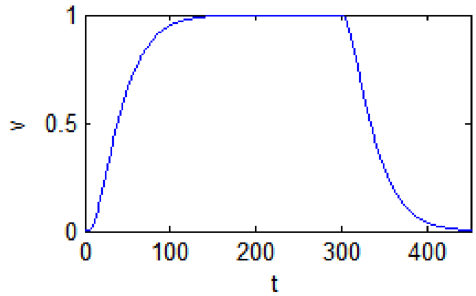


Figure 3. Response of acceleration and deceleration command

Then we can design controller based on the close-looped system. Define state vector by position $x_1$, speed $x_2$, and acceleration $x_3$, the dynamic model of automated trolleys is described by the following linear state equations

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1/T^2 & -2\xi/T^2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ K/T^2 \end{bmatrix}$$

$$C = [1 \quad 1 \quad 1].$$

## III. CONTROLLER DESIGN

The optimal control of linear quadratic (LQ) problem is concerned with linear system and a quadratic cost function defined as

$$\begin{aligned} J = & \frac{1}{2} e^T(t_f) F e(t_f) \\ & + \frac{1}{2}\int_0^{t_f} [\, e^T(t) Q(t) e(t) + u^T(t) R(t) u(t)\,] dt \end{aligned} \tag{3}$$

where $e(t)=y_r(t)-y(t)$, and $y_r(t)$ is the objective trajectory to be tracked.

This cost function includes terminal error, dynamic error, and energy consumption, which exists in most control systems. The optimal control of LQ problem turns out to be obtainable from state feedback, and thus is widely used in practice.

The automated trolleys are required to stop at a specified position. This implies the terminal demands to the position and speed of automated trolleys are specified position and zero. We can write it as

$$y_r(t) = [d \quad 0 \quad 0]^T$$

$$F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{4}$$

where $d$ is a constant which indicates the destination position of the automated trolleys. The values of elements in matrix $F$ are alterable.

In the dynamic process, the speed of automated trolleys is required to satisfy a smooth profile. This is guaranteed by the second order dynamic system with $\zeta>1$ (damp ratio). In order to make the trolleys arrive at the specified position as soon as possible, dynamic error of position is considered, while dynamic errors of speed and acceleration are neglected here, because we are not concerned with them. In addition, energy consumption also needs to be considered in most cases. Therefore, $R$ and $Q$ could be

$$R = 1 \tag{5}$$

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{6}$$

The value and the structure of $F$, $Q$ and $R$ are flexible. When dynamic error is not required to the control system, we can let $Q$ equal 0. The same happens to $R$. If we want trolleys to run more smoothly, dynamic error of speed and acceleration should be considered in $Q$.

According to the results of LQ problem, the solution to optimal control problem defined by (2) and (3) is

$$u^*(t) = -R^{-1}B^T[P(t)x(t) - g(t)] \tag{7}$$

where $P(t)$ is the unique positive-definite solution to the following differential matrix Riccati equation.

$$\begin{aligned} -\dot{P}(t) = P(t)A + A^T P(t) \\ - P(t)BR^{-1}B^T P(t) + C^T QC \end{aligned} \tag{8}$$

with terminal condition

$$P(t_f) = C^T FC$$

and $g(t)$ is found by solving the following differential vector equation

$$-\dot{g}(t) = [A - BR^{-1}B^T P(t)]^T g(t) + C^T Q y_r \tag{9}$$

with terminal condition

$$C(t_f) = C^T F y_r$$

The control system designed by LQR optimal control method is shown in Fig.4.

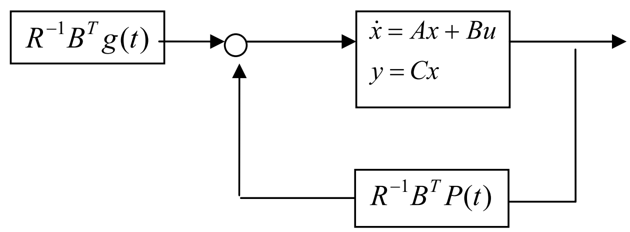


Figure 4. LQR control system

## IV. SIMULATION

Consider the following dynamic model which is given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -2 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \ C = [1 \quad 1 \quad 1]$$

The output to be tracked is

$$y_r(t) = [10 \quad 0 \quad 0]^T$$

Matrixes $F$, $Q$, $R$ are given by (4) (5) (6)

Usually it is hard to find the analytical solution of Riccati matrix differential equation in (5). However, we can get numerical solutions by applying Euler Scheme in an offline way.

$$\dot{P}(t) = \frac{dP(t)}{dt} \approx \frac{P(t+\Delta t) - P(t)}{\Delta t}$$

$$\dot{g}(t) = \frac{dg(t)}{dt} \approx \frac{g(t+\Delta t) - g(t)}{\Delta t}$$

Thus, we can rewrite (5) (6) as the following difference equations

$$\begin{aligned} P(t+\Delta t) \approx P(t) + \Delta t[-P(t)A - A^T P(t) \\ + P(t)BR^{-1}B^T P(t) - C^T QC] \end{aligned}$$

$$\begin{aligned} g(t+\Delta t) \approx g(t) + \Delta t[A - BR^{-1}B^T P(t)]^T g(t) \\ + \Delta t C^T Q y_r] \end{aligned}$$

So we can compute $P(t)$ from $P(t_f)$ with a small negative increment, because $P(t)$ is independent with $x(t)$. Let $\Delta t$=-0.05 here.

Figure 5 shows the trajectories of system output. We can see the trolley stops at the destination after an acceleration process and a deceleration process. The response has a good dynamic performance. Figure 6 shows the result of conventional speed planning method. The trolley stops at the destination in a very slow process.

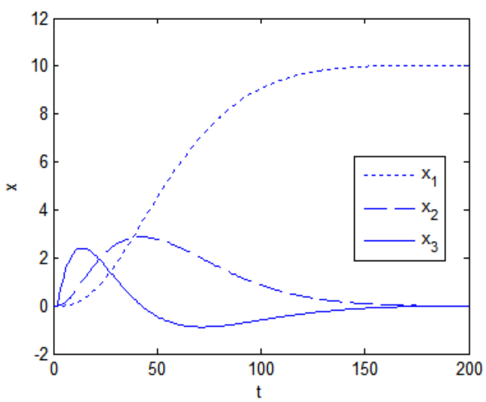


Figure 5. Optimal trajectory of states

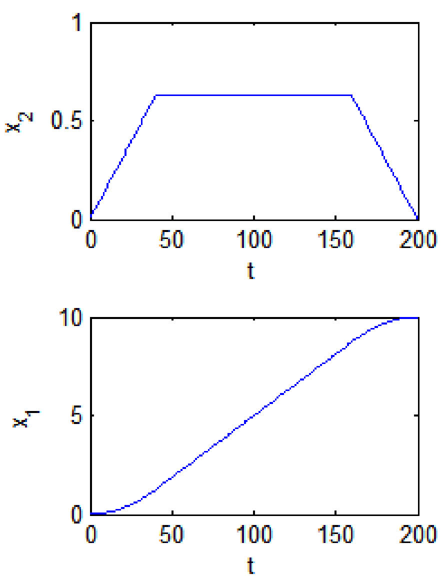


Figure 6. Optimal trajectory of states

The trolley in Fig.5 shows a good dynamic performance because it runs a variable accelerated motion, and the optimal controller with quadratic cost function makes full use of the accelerating ability. At the same time, the second order dynamic element used as the speed planning ensures a smooth accelerating and decelerating process. This also exemplifies that closed-loop control systems usually have a better performance than open-loop control systems.

## V. CONCLUSION

In this paper, we have developed a control approach for automated trolleys. An optimal controller is designed under quadratic cost function after linearization of the system by replacing the conventional acceleration and deceleration curve with the response of a second order dynamic element. The developed control system can be used to promote the dynamic performance of automated trolleys.

There are also some uncertainties not considered in this paper, such as lateral force, load varying, and so on. A robust controller against these uncertainties is the direction of the following research.

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