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Generalized Fibonacci Numbers and Applications

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***Abstract*— The present paper relates to the methods for data encoding and the reading of coded information represented by colored (including monochrome/black, gray) symbols (bars, triangles, circles, or other symbols). It also introduces new algorithms for generating secure, reliable, and high capacity color barcodes by using so called weighted n-dimensional random Fibonacci number based representations of data. The representation, symbols, and colors can be used as encryption keys that can be encoded into barcodes, thus eliminating the direct dependence on cryptographic techniques. To supply an extra layer of security, one may encrypt given data using different types of encryption methods.**

***Keywords*—Color barcode, Weighted n-dimensional random Fibonacci sequences, Fibonacci numbers, Fibonacci p-code, Cellphones**

I. INTRODUCTION

Since the 20th century, barcodes have provided optical machine-readable representations of data. Initially used to label railroad cars, barcodes are now laid on almost every medium, from paper and cardboard to skin and DNA to plastics and metals [1]. Their widespread use can be attributed to their low cost, portability, and high information capacity [2-5]. In spite of a range of competing technologies, such as RFID, barcodes continue to offer the most cost effective means of tracking information: it costs about $0.005 to implement a barcode, while a passive RFID, for instance, may run from $0.07 to $0.30 per tag. Moreover, with constantly improving technology, the potential and demand for barcodes continues to increase.

The rise of inexpensive cameras in technologies such as cellphones or webcams has increased consumer use of barcodes. These technologies can incorporate built-in scanning software that allows the consumer to receive pertinent product information from a captured barcode image. The software includes a computer vision algorithm that localizes and segments the image, extracts the bits of information, and passes them on the appropriate decoder. Once the product is identified, software program retrieves its relevant information to the consumer [6].

Many companies that offer such technology have developed their own unique, proprietary barcode formats that are better geared toward mobile apps [7]. Such efforts are part of a larger aim to seek new ideas in making barcode systems more efficient, accessible, and secure. Traditionally, barcodes have represented data in the widths and spacings of black parallel lines. They are now referred to as linear or 1D (1 dimensional) barcodes or symbologies. Since such barcodes use two types of symbol elements, however, they carry a limited amount of information with few security features. As a result, other types of symbologies, most notably 2-D and color barcodes, have been developed to offer more capabilities than their linear counterpart [7].

2D barcodes can be broadly classified as either stacked symbology or matrix code. Stacked symbology, also called multi-row code, is created by “stacking” a series of linear barcode on top of each other. Matrix code is made up of black and white "cells.” These “cells” represent bits and are arranged in either a square or rectangular shape. Both types of 2D codes differ from their 1D counterpart in that they are not “vertically redundant.” In other words, while linear code can be truncated in height with any lost of information, 2D code contains essential data in both its length and height. Using both dimensions allows 2D code to store more information. However, vertical redundancy increases the probability that damaged barcodes will be read. The higher the height of the barcode, the more likely the scanner will be able to read one path along the lines [10]. As a result, 2D code uses some of its additional data capacity to prevent misreads and provide a satisfactory read rate, often by encoding extra data for error correction [8].

Variations on such symbologies have been examined and proposed in various studies [2]. One such variation is the color barcode, which was invented in [18-20]. Examples of color symbologies include the High Capacity Color Barcode (HCCB) of Microsoft Inc [11] and the Color Bar Code System of Imageid Ltd.[12]. HCCB, for instance, uses colored triangles instead of black and white lines or squares, each of which can be one of four colors: black, red, green and yellow. The color barcodes not only hold aesthetic value, but also store more information in the same physical size of the code [10]. However, the most immediate applications for such symbologies is limited to identifying commercial audiovisual works such as motion pictures, video games, broadcasts, digital video recordings and other media [10]. Figure 1 (a)-(d) provides examples of typical 2-D barcodes and their color variations. The primary criterion for judging the aforementioned barcodes is their performance. That is, given its size, how much information can the symbology store

Figure 1. Examples of Barcode: (a) PDF417 code 128; (b) QR; (c) Data Matrix; (d) HCCB [11]

and how efficiently and inexpensively can this information be machine read? In an increasingly complex society, however, many of our present technologies require information dense symbologies. In order to accommodate applications such as the labeling of semiconductor packages or credit card labeling, as well as labeling sight, sound, or software media, there exists a need for encoding several hundred to several thousand alphanumeric characters [21]. Present symbologies, in other words, do not have the information capacity required in today's Information Age.

Present symbologies also lack the security features to ensure the legitimacy of products. Counterfeit products, in the form of lottery tickets, casino chips, tokens, currency, coupons, etc., now divert billions of dollars annually from legitimate businesses and governments. Current attempted solutions to resolve the counterfeit problem, such as the use of holograms for credit cards and special paper and inks for currency, are based on the difficulty of counterfeiters to make exact copies. However, given the steady advance of technology, what can be made with authorization can be copied, for example by reverse engineering, or can simply be made with unauthorized assistance or unauthorized access to the pertinent technology. Recently, electronic devices have been incorporated into products and/or their packaging. Even so, electronic devices can be copied, and further, tend to add expense to a product [22]. Symbologies are thus needed to easily identify counterfeit products without incurring significant costs.

In this paper, we introduce a new concept of generating secure, reliable, and high capacity barcode symbols by using parametric representations of data. The rest of this paper is organized as followed. Section II introduces some necessary background. A concept of generating generation weighted n-dimensional random Fibonacci sequences are provided in Section III. A concept of generating color barcodes with security features is provided in Section IV. Section IV also presents an illustrative algorithm of generating weighted n-dimensional random Fibonacci sequences representation based barcodes. A conclusion is reached in Section V.

II. Number Representations and Bar Codes Background

In this section, a background of barcodes and number representation is introduced. Some background descriptions of barcode can be found in the following patents [4, 6, 8, 9, 23 -34, 36, 37].

Barcode performance is usually evaluated by three basic criteria: **reliability, density**, **first time read rate.** We will add here two more criteria **security and cost**.

First time read rate: Present barcodes and barcode reading devices/scanners suffer from poor first time read rates, i.e., the frequency with which a particular barcode using given apparatus may be required to pass a product over the scanner several times before the equipment accepts the barcoded information in order to input its information, often measured in percentage. An illustrative example of this problem is UPC code and supermarket scanners. ***A barcode's reliability***, or how well it avoids misreads, is a function of error exposure. Two distinct forms of error detecting have been used: individual character error detecting and overall symbol error detecting. ***Density:*** The number of characters per unit length represented by the barcode symbol is referred to as the density of the symbol. ***The density (high capacity) of a barcode*** is measured in characters per inch. It can be shown that density depends of encoding techniques and the printing quality of a barcode. ***Barcode security*** is defined as either a protection of barcodes against unauthorized access or a technique for ensuring that barcoded information cannot be read or compromised by any individuals without authorization.

Fibonacci and Lucas Numbers: Fibonacci and Lucas (Francois-Edouard-Anatole Lucas) numbers can be presented recursively as:

$$f_k = f_{k-1} + f_{k-2}, \quad l_k = l_{k-1} + l_{k-2}, \quad (1)$$

where

$f_0 = 0,\ f_1 = 1,\ l_0 = 2,\ l_1 = 1, \quad k > 1.$

Fibonacci P-codes: They are defined by the following recurrence:

$$F_n^{(p)} = \begin{cases} 0 \quad if\ n \le 0 \quad , \\ 1 \quad if \quad 0 < n \le p+1 \quad , \\ F_{n-1}^{(p)} + F_{n-p-1}^{(p)} \quad \text{if} \quad n>p+1 \end{cases} \qquad (2)$$

'p' is a non-negative integer such as 0,1,2,3,4….;

Lucas P-codes: They are defined by the following recurrence:

$$L_n^{(p)} = \begin{cases} p+1 \quad if\ n = 0 \quad , \\ 1 \quad if \quad 0 < n \le p \quad , \\ L_{n-1}^{(p)} + L_{n-p-1}^{(p)} \quad \text{if} \quad n>p \end{cases} \qquad (3)$$

'p' is a non-negative integer such as 0,1,2,3,4….;

Example: The initial sequences for the first five p-numbers are as follows:

P	**Fibonacci p-numbers**	**Lucas p-numbers**
0	0,1,2,4,8,16,32,64,128,512,1024…	1,1,2,4,8,16,32, 64,128,256,512..
1	0,1,1,2,3,5,8,13,21,34,55,89,143…	2,1,3,4,7,11,18,29,47,76…
2	0,1,1,1,2,3,4,6,9,13,19,28,41,60…	3,1,1,4,5,6,10,15,21,31,46…
3	0,1,1,1,1,2,3,4,5,7,10,14,19,26,36..	4,1,1,1,5,6,7,8,13,19,26…
4	0,1,1,1,1,1,2,3,4,5,6,8,11,15,20…	5,1,1,1,1,6,7,8,9,10,16,23…

According to the table, the result under Fibonacci p-numbers when p=0 gives the traditional powers of two sequence. Furthermore, given a value of p=1 results in the Fibonacci sequence. In summation, 'p' provides a large number of sequences and p-codes that all relate to the Fibonacci sequence. This recursive concept can also be used to generate Lucas p-numbers and p-codes [15].

Definition: Negofibonacci/Negolucas numbers can be defined as

$$f_{-n} = (-1)^n f_n, (or \quad f_{n-2} = f_n - f_{n-1}) \quad l_{-n} = (-1)^n l_n$$

Properties:

1. $f_{n+m} = f_n f_{m+1} + f_{n-1} f_m, \quad (-1)^{n+m} f_{n-m} = f_{n+1} f_m - f_n f_{m-1},$
$f_{n+m+k} = f_{n+1} f_{m+1} f_{k+1} + f_n f_m f_k - f_{n-1} f_{m-1} f_{k-1}$

2. ***Zeckendorf's Theorem:*** Any integer number 'X' can be uniquely represented as a sum of a finite canonical Fibonacci p-code. The canonical representation of a Fibonacci p-code is determined by using the following: X = $F^{(p)}_{n-1} + m$; Where $0 \le m \le F^{(p)}_{n-p}$; for an arbitrary given integer $p \ge 0$ and $n \ge p$, $F^{(p)}_{n-1}$ is the greatest Fibonacci p-number that is less than or equal to X.

3. The normal Fibonacci p-code is a redundant code in comparison with the usual binary code (case p=0): the number of bits for representation of numbers 0 through some N is O(1ogN). The redundancy of code leads to its noise stability [15]

III. N-DIMENSIONAL RANDOM FIBONACCI SEQUENCES BASED REPRESENTATIONS OF DATA.

Definition: We define weighted random n-dimensional (generalized) Fibonacci sequences (WFS) as:

$$F^{(p)}_n = \begin{cases} a \quad if\ n = 0 \quad , \\ b_n \quad if \quad 0 < n \le m, \quad b_n \quad are \quad constant \quad , \text{m= p or p+1} \\ c_1 F^{(p)}_{n-1} + c_2 F^{(p)}_{n-2} + \ldots + c_{p+1} F^{(p)}_{n-p-1} \qquad n > m \\ where \quad c_i\ , \quad i = 1,\ldots,m \quad are \quad constant \end{cases}$$

The coefficients c_i can be chosen randomly. For instance, by choosing c_i from the following classes: {0,1}, {-1,0,1}, {0,1,2}, {j,1,where j^2=-1}, we can generate many well known and newly developed number sequences. Some of these sequences are generated below.

1. $c_1 = c_{p-1} = b_i = 1, \quad c_2 = \ldots = c_{p-2} = 0$
$a = 0, \quad m = p+1, \quad n > p+1 \quad , i = 1\ldots m$

then the WFS are the Fibonacci p-numbers (2).

2. $c_1 = c_{p-1} = b_i = 1, \quad c_2 = \ldots = c_{p-2} = 0$
$a = p+1, \quad m = p, \quad n > p \quad , i = 1\ldots m$

then the WFS are the Lucas p-numbers (3).

3. $c_1 = c_2 = c_3 = 1, \qquad a = 0,$
$m = p = 2, \quad n > p, \quad b_1 = 0, \quad b_2 = 1$

then the WFS are the tribonacci numbers (The tribonacci numbers starts with three predetermined terms and each term afterwards is the sum of the preceding three terms):

$$t_n = t_{n-1} + t_{n-2} + t_{n-3}, \quad t_0 = t_1 = 0, t_2 = 1,$$

The first few tribonacci numbers are:

0,0, 1, 1, 2, 4, 7, 13, 24, 44, 81, ...

4. $c_q = q, where \quad q = 1,2,3, \quad a = 0$
$m = p = 2, \quad n > p, \quad b_1 = 0, \quad b_2 = 1$

then the WFS are the Trucas numbers, which satisfy the following recursion:

$$t_n = t_{n-1} + 2t_{n-2} + 3t_{n-3}, \quad t_0 = t_1 = 0, t_2 = 1,$$

5. $c_1 = c_2 = c_3 = c_4 = 1, \quad a = 0$
$m = p = 3, \quad n > p, \quad b_1 = b_2 = 0, \quad b_3 = 1$

then the WFS are the tetranacci numbers (the tetranacci numbers start with four predetermined terms, each term afterwards being the sum of the preceding four terms):

$$t_n = t_{n-1} + t_{n-2} + t_{n-3} + t_{n-4}, \quad t_0 = \ldots = t_2 = 0, t_4 = 1,$$

The first few tetranacci numbers are:

0,0,0, 1, 1, 2, 4, 8, 15, 29, 56, 108, ...

6. $c_1 = c_2 = \ldots = c_r = 1, \quad a = 0, \quad m = p = r,$
$n > p \quad b_r = 1, \quad b_i = 0, i = 1,2,\ldots,r-1$

then the WFS are the r-bonacci numbers, which satisfy the following recursion:

$$t_n = t_{n-1} + t_{n-2} + \ldots + t_{n-r}, \quad t_0 = \ldots = t_{r-1} = 0, t_r = 1,$$

7. $c_1 = \alpha_1, c_2 = \alpha_2, a = 0, \quad b_1 = 1$
$m = p = 1 \qquad n > p$

then the WFS are the weighted Fibonacci numbers, which satisfy the following recursion:

$$G^{(p)}_n = \alpha_1 G^{(p)}_{n-1} + \alpha_2 G^{(p)}_{n-2}$$

8. $$c_1 = c_{q-1} = c_{p-1} = b_i = 1, \quad c_2 = \ldots = c_{p-2} = 0 \quad q < p$$
$$a = p+1, \quad m = p, \quad n > p \quad , i = 1 \ldots m$$

then the WFS generate a new number sequence, which we call Fibonacci (q,p)-numbers, and satisfy the following recursion:

$$F_n^{(p)} = \begin{cases} 0 & if\ n \le 0 \quad , \\ 1 & if \quad 0 < n \le p+1 \quad , \\ F_{n-1}^{(p)} + F_{n-q-1}^{(p)} + F_{n-p-1}^{(p)} & \text{if} \quad n>p+1 \quad q < p \end{cases}$$

9. $$c_1 = c_{q-1} = c_{p-1} = b_i = 1, \quad c_2 = \ldots = c_{p-2} = 0 \quad q < p$$
$$a = p+1, \quad m = p, \quad n > p \quad , i = 1 \ldots m \quad b_q = -1$$

then the WFS generate a new number sequence, which we call signed Fibonacci (q,p)-numbers, and satisfy the following recursion:

$$F_n^{(p)} = \begin{cases} 0 & if\ n \le 0 \quad , \\ 1 & if \quad 0 < n \le p+1 \quad , \\ F_{n-1}^{(p)} - F_{n-q-1}^{(p)} + F_{n-p-1}^{(p)} & \text{if} \quad n>p+1 \quad q < p \end{cases}$$

10. $$c_1 = 1, c_2 = j = \sqrt{-1}, \qquad a = 0,$$
$$m = p = 1, \quad n > p, \quad b_1 = 1,$$

then the WFS are the Gaussian Fibonacci numbers, which satisfy the following recursion: $c_n = t_{n-1} + j t_{n-2}$

The first few Gaussian Fibonacci numbers are:

1,1+j,2+j,3+2j, 5+3j,…..,..

Definition: weighted random n-dimensional Negofibonacci/Negolucas numbers can be defined as:

$$F_{-n}^{(p)} = (-1)^n F_n^{(p)}, or, \quad F_{n\ \ n-2}^{(p)} = F_n^{(p)} - F_n^{(p)}$$

Secure barcodes can be developed by randomly choosing these coefficients. These randomly generated coefficients offer many more possible of uniquely number representation systems. For instance, any integer number 'X' can be uniquely represented as a sum of a finite weighted n-dimensional Fibonacci numbers and Fibonacci p-numbers (see Zeckendorf's and related theorems):

$$X = d_n F^{(p)}_n + d_{n-1} F^{(p)}_{n-1} + d_{n-2} F^{(p)}_{n-2} + \ldots + d_{p+1} F^{(p)}_{p+1} + d_p F^{(p)}_p$$

Where the coefficients d_n can belong to one of the following classes: {0,1}, {-1,0,1}, {0,1,2}, {j,1,where $j^2=-1$}. Such classes help generate well known as well as newly developed number representation systems. Note, increasing the number of elements in a given class provides greater possible coding mechanisms, which can be used in designing high-density, secure barcodes.

IV. N-DIMENSIONAL FIBONACCI SEQUENCES BASED REPRESENTATIONS OF DATA.

In this section we show that weighted n-dimensional Fibonacci numbers allow for a universal generation of bar code systems as they not only contain Fibonacci representation based barcodes but also a multitude of other classes of barcodes, including the commonly used binary barcodes. This universal representation system can be made more robust against errors using various error correction methods. Figure (2) presents a block diagram of generating and decoding of a color barcode with security features:

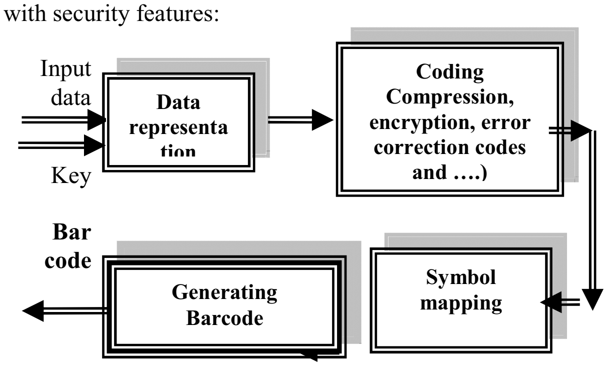


Figure (2): Block diagram of generating color barcode with security features.

One exemplary symbol mapping (in the RGB color space) might be the following:

Black	0	0	0	0
Blue	0	0	0	1
Green	0	1	0	0
Red	1	0	0	0
Yellow	0	0	1	0

Table 1. The five colors used in the base barcode

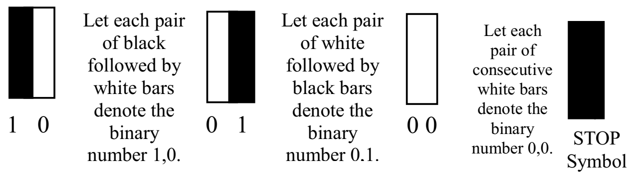
To provide an extra layer of security, one may encrypt the terms of a given system using different types of encryption methods.

Illustrative Examples: Let's generate the zip code, 78249 by using black-white and color approaches in both ternary (base 3) and Fibonacci representations.

Let each individual red bar denote the binary number '1'. Let each individual white bar denote the binary number '0'. Let each individual blue bar denote the binary number '2'.

The zip code, 78249, expressed in base-3, is represented by the following color UPC barcode:

Figure (3): The generated color UPC barcode.



The zip code, expressed in Fibonacci, can also be represented by the following black-white barcode. Note the comparison between the commonly used (binary) and presented methods of representation (an illustrative example).

The presented method stores 8 digits using 36 bits

The commonly used method stores 8 Digits using 64 bits

Figure (4): a) Presented method, b) Current method

Thus, the presented method generates a (secure) higher density barcode. Note that though we use rectangular symbols, other symbols such as circles, rectangular, ellipse, and squares, may be used in generating these barcodes.

V. CONCLUSION

In this paper, we have introduced a new concept for color barcodes. We have also introduced new algorithms for generating secure, reliable, and high capacity color barcodes by using so called weighted n-dimensional Fibonacci sequences based representations of data. The representation, symbols, and colors can be used as encryption keys that can be encoded into barcodes, thus eliminating the direct dependence on cryptographic techniques. To supply an extra layer of security, one may encrypt given data using different types of encryption methods. The advantage of using n-dimensional Fibonacci sequences in place of commonly used Fibonacci and binary codes lies in amount of possible sequences. Consequently, codes become more secure as there is no feasible way to extract the information within the barcode without knowing the exact set of parameters used. The addition n-dimensional Fibonacci sequences provide for a more universal generation of a barcode system as they not only contain Fibonacci representation barcodes but also a multitude of other classes of barcodes, including the commonly used binary barcodes.

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