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Stability and Stabilization for Discrete Systems with Time-varying Delays Based on the Average Dwell-time Method

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***Abstract*—In this paper, the problems of the exponential stability and stabilization for a class of discrete systems with time-varying delays are considered. By converting discrete systems with time-varying delays into switched systems and using the average dwell-time method, a new stability criterion is obtained and presented in terms of linear matrix inequality. Based on the obtained stability condition, a design method for the feedback controller to stabilize the system is also proposed. Finally, some numerical examples are given to show the effectiveness of the proposed method.**

***Index Terms*—Exponential stability, time-varying delay, feedback control, switched systems, linear matrix inequality, average dwell-time method.**

I. INTRODUCTION

Time-delay is often encountered in many practical systems. It is well known that time-delay is often one of the sources of poor control performance and instability. Therefore, the subject of stability analysis and stabilization of systems with delays has attracted considerable attention during the past few years [1], [2].

A typical example of a discrete system with time-varying delay is the networked control system where the delay from the sensor to the controller and the delay from the controller to the actuator are both time-varying [3], [4], [5]. So the study of stability and stabilization of discrete-time-delay systems is important both in theory and in practical applications. However, less attention has been given to discrete-time-delay systems [6], [7], [8], [9], [10] compared with continuous-time-delay systems [11], [12], [13], [14], [15], [16], [17], [18], [19]. Recently, a descriptor approach has been applied to the stability analysis of discrete-time-delay systems with norm-bounded or polytopic uncertainties [20], [21]. By considering some useful terms which were ignored in previous results, new less conservative stability criteria were obtained in [22]. However, in [22] the bounds on some terms were enlarged, which may lead to some conservatism. Using a new method to estimate the upper bound on the derivative of the Lyapunov functional without ignoring any terms, delay-dependent stability conditions less conservative than those in [22] were obtained in [23]. It should be noted that the above results for stability of discrete-time-delay systems are all based on the Lyapunov functional approach. In these results, a basic assumption is that absolute values of the eigenvalues of $A + A_1$ are all less than 1, that is, the system is stable without delay. In practice, there are some systems which are stable with nonzero delay but unstable without delay [24]. For such systems, all the above results will fail to obtain a feasible solution. To the best of the authors' knowledge, few results are available for such systems especially in discrete-time domain, which motivates this study.

A special feature of discrete-time-delay systems is that they can be transformed into systems without delay by the augmentation method. A drawback of this method is that it may result in a large dimension of the augmented system for large time-delays which makes the obtained stability conditions difficult to check. However, this drawback has been overcome to some extent due to the fast development of the high speed processor. Recently, this augmentation technique has been extended to discrete-time systems with time-varying delay in [25] where systems with time-varying delays are converted to switched systems and the common quadratic Lyapunov function approach is used to solve the problems of stability and stabilization for such systems. It is well known the common quadratic Lyapunov function method is conservative. Therefore, the result in [25] should be further improved.

In existing results, the asymptotic stability for discrete-time systems with time-varying delays has been investigated and little attention has been paid to the exponential stability for such systems. In existing results, the delay is often assumed to belong to a given interval $0 \le d_1 \le d(k) \le d_2$. If there are some delays larger than d_2 occurring in the delay sequence, existing results will fail to determine whether the system is stable or not. Basically, if the occurrence rate for the large delay is not high, the system will be still stable. To the best of the authors' knowledge, such a problem has not been well considered and only [26] has considered a similar problem. However, results in [26] are based on the Lyapunov-Krasovskii functional approach, so they cannot be applied to systems which are unstable without delay.

In this paper, discrete systems with time-varying delays

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are converted into switched systems using the augmentation method. Unlike existing results, the assumption that absolute values of the eigenvalues of $A + A_1$ are all less than 1 is removed. Using the average dwell-time method [27], we obtain a new exponential stability criterion. A method of designing a feedback controller to stabilize the system is also proposed. The controller gain can be obtained by solving a set of linear matrix inequalities. Finally, some numerical examples are given to illustrate the effectiveness of the proposed method.

II. Problem formulation and main results

Consider the following discrete-time systems with a time-varying delay

$$\begin{aligned} x(k+1) &= Ax(k) + A_1x(k-d(k)) + Bu(k) \\ x(k) &= \varphi(k), \quad k = -d_2, -d_2+1, \cdots, 0 \end{aligned} \tag{1}$$

where $x(k) \in \mathcal{R}^n$ is the state vector, $u(k) \in \mathcal{R}^m$ is the control input; A, A_1 and B are constant matrices with appropriate dimensions; $\varphi(k)$ is the initial condition sequence; $d(k)$ is an integer denoting the time-varying delay and satisfies

$$d_1 \le d(k) \le d_2 \tag{2}$$

where d_1 and d_2 are known positive integers.

The object of this paper is to derive a new stability criterion and design a state feedback controller $u(k) = Kx(k)$ to stabilize system (1).

A. Stability analysis

Firstly, consider the nominal system of (1)

$$\begin{aligned} x(k+1) &= Ax(k) + A_1x(k-d(k)) \\ x(k) &= \varphi(k), \quad k = -d_2, -d_2+1, \cdots, 0 \end{aligned} \tag{3}$$

Define $z(k) = \begin{bmatrix} x(k) \\ x(k-1) \\ \vdots \\ x(k-d_2+1) \\ x(k-d_2) \end{bmatrix}$ and system (3) can be rewritten as the following switched system

$$z(k+1) = \Xi_{\sigma(k)}z(k) \tag{4}$$

where $\sigma(k) \in \mathcal{I}$ is a piecewise constant function and denotes the active mode. $\mathcal{I} = \{d_1, d_1+1, \cdots, d_2\}$ and

$$\Xi_j = \begin{bmatrix} A & \overbrace{0\cdots0}^{j-1} & A_1 & 0 & \cdots & 0 \\ & & & & & 0 \\ & & \boldsymbol{I}_{\mathrm{d_2 n \times d_2 n}} & & & \vdots \\ & & & & & 0 \end{bmatrix}, \quad j \in \mathcal{I}$$

Remark 1: Switched systems have received much attention in the past several years. There are plentiful results for switched systems available in the literature [28], [29], [30]. By converting systems with time-varying delays into a switched system, we can use the results for switched systems to analyze and synthesize time-delay systems.

Let r_i, $i \in \mathcal{I}$, denote the occurrence rate of the ith subsystem of (4) during the interval $(0, k)$. r_i can be obtained by $r_i = n_i/k$ where n_i denotes the times that the ith subsystem is activated. Before moving on, the following definitions are introduced.

Definition 1: [31] [32] System (4) is said to be globally exponentially stable with a decay rate $\lambda > 1$, if the solution $x(k)$ of system (4) satisfies

$$\|x(k)\| \le c\lambda^{-k}\|x(0)\|, \quad \forall k \ge 0$$

where $c > 0$ is a constant.

Definition 2: [28], [31] For any $k \ge 1$, let $N_\sigma[0, k)$ denote the number of switchings of $\sigma(k)$ over the interval $[0,\ k)$. If $N_\sigma[0, k) \le N_0 + k/T_a$ holds for $N_0 \ge 0$ and $T_a > 0$, then T_a is called the average dwell time and N_0 is called the chatter bound and is often chosen as $N_0 = 0$.

For the subsystem of (4) choose the following Lyapunov function

$$V_i(k) = z^{\mathrm{T}}(k)P_iz(k) \tag{5}$$

The following lemma gives an estimation of the decay or increase rate of the above Lyapunov function.

Lemma 1: For given scalar $\lambda_i > 0$, if there exist matrices $P_i > 0$, and any matrices F_i, G_i, $i \in \mathcal{I}$, such that

$$\Sigma_i = \begin{bmatrix} \Sigma_i^{11} & \lambda_i\Xi_i^{\mathrm{T}}G_i^{\mathrm{T}} - F_i \\ * & P_i - G_i - G_i^{\mathrm{T}} \end{bmatrix} < 0, \quad i \in \mathcal{I} \tag{6}$$

where $\Sigma_i^{11} = -P_i + \lambda_iF_i\Xi_i + \lambda_i\Xi_i^{\mathrm{T}}F_i^{\mathrm{T}}$, then the Lyapunov function (5) has the following decay or increase property

$$V_i(k) \le \lambda_i^{-2(k-k_0)}V_i(k_0) \tag{7}$$

Proof: Following a similar line as in [32], define $\xi(k) = \lambda_i^{k-k_0}z(k)$, and one can obtain

$$\xi(k+1) = \lambda_i\Xi_i\xi(k) \tag{8}$$

For subsystems of (8), choose the Lyapunov function $\mathcal{V}_i(k) = \xi^{\mathrm{T}}(k)P_i\xi(k)$. It is easy to obtain

$$\begin{aligned} \triangle\mathcal{V}_i(k) &= \mathcal{V}_i(k+1) - \mathcal{V}_i(k) \\ &= \xi^{\mathrm{T}}(k+1)P_i\xi(k+1) - \xi^{\mathrm{T}}(k)P_i\xi(k) \end{aligned} \tag{9}$$

For any matrices F_i and G_i, the following equation holds

$$2\left[\xi^{\mathrm{T}}(k+1)G_i + \xi^{\mathrm{T}}(k)F_i\right]\left[-\xi(k+1) + \lambda_i\Xi_i\xi(k)\right] = 0 \tag{10}$$

From (9) and (10), one can obtain

$$\begin{aligned} \triangle\mathcal{V}_i(k) &= \xi^{\mathrm{T}}(k+1)P_i\xi(k+1) - \xi^{\mathrm{T}}(k)P_i\xi(k) \\ &\quad +2\left[\xi^{\mathrm{T}}(k+1)G_i + \xi^{\mathrm{T}}(k)F_i\right] \times \\ &\qquad \left[-\xi(k+1) + \lambda_i\Xi_i\xi(k)\right] \\ &= \begin{bmatrix} \xi(k) \\ \xi(k+1) \end{bmatrix}^{\mathrm{T}} \Sigma_i \begin{bmatrix} \xi(k) \\ \xi(k+1) \end{bmatrix} \end{aligned} \tag{11}$$

So, if $\Sigma_i < 0$ then $\triangle\mathcal{V}_i(k) < 0$ which implies $\mathcal{V}_i(k) < \mathcal{V}_i(k_0)$. It is clear that

$$\begin{aligned} V_i(k) &= \lambda_i^{-2(k-k_0)}\mathcal{V}_i(k) < \lambda_i^{-2(k-k_0)}\mathcal{V}_i(k_0) \\ &= \lambda_i^{-2(k-k_0)}V_i(k_0) \end{aligned}$$

The proof is completed. ■

On the basis of Lemma 1, the following theorem gives a sufficient exponential stability condition for system (3).

Theorem 1: For given positive scalars $\lambda_i > 0$, $r_i > 0$ and $\mu > 1$, if there exist matrices $P_i > 0$, and any matrices F_i and G_i, $i \in \mathcal{I}$ such that the following inequalities hold

$$\prod_{i=d_1}^{d_2} \lambda_i^{r_i} > \lambda > 1 \tag{12}$$

$$T_a > T_a^* = \frac{\ln \mu}{2 \ln \lambda} \tag{13}$$

$$\Sigma_i < 0, \quad i \in \mathcal{I} \tag{14}$$

$$P_i \le \mu P_j, \quad \forall\, i, j \in \mathcal{I} \tag{15}$$

then system (3) with a time-varying delay satisfying (2) is exponentially stable and the decay rate is estimated as

$$\|x(k)\| \le \sqrt{\frac{b}{a}} \lambda^{-k + \frac{\ln \mu}{2T_a \ln \lambda} k} \|x(0)\|$$

where $b = \max_{i \in \mathcal{I}} \lambda_{\max}(P_i)$, $a = \min_{i \in \mathcal{I}} \lambda_{\min}(P_i)$.

Proof: Choose the following piecewise Lyapunov function

$$V_{\sigma(k)}(k) = z^{\mathrm{T}}(k) P_{\sigma(k)} z(k) \tag{16}$$

Let $0 < k_1 < k_2 < \cdots < k_i$, $i \ge 1$ denote the switching instants over the interval $[0, k)$. Based on (15)-(16) and Lemma 1, the proof can be completed following a similar line to the proof of Theorem 1 in [32], which is omitted here. ■

Remark 2: Clearly in Theorem 1, it is not needed that every $\lambda_i > 1$, which means that some subsystems of (4) may be unstable. The relationship between the occurrence rate of all the possible delay, r_i, and the stability is also established.

Remark 3: By letting $\mu = 1$, we have $T_a^* = 0$, which means that the switching signals can be arbitrary, that is, the time-delay can be arbitrary varying within the given interval. Furthermore, if setting $\lambda_i = 1$, the results obtained by common quadratic Lyapunov function in [25] will be covered . So, results in [25] can be seen as a special case of Theorem 1. Moreover, only asymptotical stability problem was considered in [25]. In this paper, we give an estimation of the exponential decay rate in Theorem 1.

B. Controller design

Based on Theorem 1, we will give a design method of the state feedback controller of system (1).

Substitute $u(k) = Kx(k)$ into (1) and the closed-loop system can be obtained as

$$x(k+1) = (A + BK)x(k) + A_1 x(k - d(k)) \tag{17}$$

Similarly, we can rewrite system (17) as a switched system

$$z(k+1) = \hat{\Xi}_{\sigma(k)} z(k) \tag{18}$$

where $\sigma(k) \in \mathcal{I}$ is a piecewise constant function and denotes the active mode, and

$$\begin{aligned} \hat{\Xi}_j &= \begin{bmatrix} A + BK & \overbrace{0 \cdots 0}^{j-1} & A_1 & 0 & \cdots & 0 \\ & & & & & 0 \\ & & \boldsymbol{I}_{\mathrm{d_2 n \times d_2 n}} & & & \vdots \\ & & & & & 0 \end{bmatrix} \\ &= \Xi_j + \hat{B} K E_1, \quad j \in \mathcal{I} \end{aligned}$$

where $\hat{B}^{\mathrm{T}} = \begin{bmatrix} B^{\mathrm{T}} & 0 & \cdots & 0 & 0 \end{bmatrix}$, $E_1 = \begin{bmatrix} I & 0 & \cdots & 0 & 0 \end{bmatrix}$.

The following theorem presents a method to design the state feedback controller.

Theorem 2: For given positive scalars $\lambda_i > 0$, $r_i > 0$, $\mu > 1$ and δ_i, if there exist matrices $R_i > 0$ and any matrices M_i being of the form

$$M_i = \begin{bmatrix} M^{11} & M_i^{12} \\ 0 & M_i^{22} \end{bmatrix}, \quad i \in \mathcal{I}$$

and Y, such that the following inequalities hold

$$\prod_{i=d_1}^{d_2} \lambda_i^{r_i} > \lambda > 1 \tag{19}$$

$$T_a > T_a^* = \frac{\ln \mu}{2 \ln \lambda} \tag{20}$$

$$\begin{bmatrix} \Omega_i^{11} & \lambda_i M_i \Xi_i^{\mathrm{T}} + \lambda_i E_1^{\mathrm{T}} Y^{\mathrm{T}} B^{\mathrm{T}} - \delta_i M_i^{\mathrm{T}} \\ * & R_i - M_i - M_i^{\mathrm{T}} \end{bmatrix} < 0, \quad i \in \mathcal{I} \tag{21}$$

$$R_i \le \mu R_j, \quad \forall\, i, j \in \mathcal{I} \tag{22}$$

where $\Omega_i^{11} = -R_i + \delta_i \lambda_i M_i \Xi_i^{\mathrm{T}} + \delta_i \lambda_i E_1^{\mathrm{T}} Y^{\mathrm{T}} \hat{B}^{\mathrm{T}} + \delta_i \lambda_i \Xi_i M_i^{\mathrm{T}} + \delta_i \lambda_i \hat{B} Y E_1$, then system (1) with a time-varying delay satisfying (2) is exponentially stable, the controller gain is given by $K^{\mathrm{T}} = (M^{11})^{-1} Y^{\mathrm{T}}$ and the decay rate is estimated as

$$\|x(k)\| \le \sqrt{\frac{b}{a}} \lambda^{-k + \frac{\ln \mu}{2T_a \ln \lambda} k} \|x(0)\|$$

where
$b = \max_{i \in \mathcal{I}} \lambda_{\max}(M_i^{-1} R_i M_i^{-\mathrm{T}})$, $a = \min_{i \in \mathcal{I}} \lambda_{\min}(M_i^{-1} R_i M_i^{-\mathrm{T}})$.

Proof: Substituting $\hat{\Xi} = \Xi + \hat{B} K E_1$ into (14) yields

$$\begin{bmatrix} \hat{\Sigma}_i^{11} & \lambda_i \hat{\Xi}_i^{\mathrm{T}} G_i^{\mathrm{T}} - F_i \\ * & P_i - G_i - G_i^{\mathrm{T}} \end{bmatrix} < 0 \tag{23}$$

where $\hat{\Sigma}_i^{11} = -P_i + \lambda_i F_i \hat{\Xi}_i + \lambda_i \hat{\Xi}_i^{\mathrm{T}} F_i^{\mathrm{T}}$. Letting $F_i = \delta_i G_i$ yields

$$\begin{bmatrix} \bar{\Sigma}_i^{11} & \lambda_i \hat{\Xi}_i^{\mathrm{T}} G_i^{\mathrm{T}} - \delta_i G_i \\ * & P_i - G_i - G_i^{\mathrm{T}} \end{bmatrix} < 0 \tag{24}$$

where $\bar{\Sigma}_i^{11} = -P_i + \delta_i \lambda_i G_i \hat{\Xi}_i + \delta_i \lambda_i \hat{\Xi}_i^{\mathrm{T}} G_i^{\mathrm{T}}$. The feasibility of (24) guarantees that G_i is nonsingular. Denote $M_i = G_i^{-1}$. In order to obtain some LMI conditions, we restrict that M_i is of a special form, that is, $M_i = \begin{bmatrix} M^{11} & M_i^{12} \\ 0 & M_i^{22} \end{bmatrix}$. Pre- and post-multiply both sides of (24) with $\mathrm{diag}\{M_i,\ M_i\}$ and its transpose and introduce new matrices $R_i = M_i P_i M_i^{\mathrm{T}}$, $Y^{\mathrm{T}} = M^{11} K^{\mathrm{T}}$, and (21) will be obtained. This completes the proof. ■

Remark 4: Condition (21)-(22) in Theorem 2 are all LMI, so it is possible to obtain feasible solutions using some existing convex optimization toolboxes.

Remark 5: It can be seen that there are some tuning parameters δ_i, $i \in \mathcal{I}$, in the stability condition. Apply a numerical optimization algorithm, such as **fminsearch** in the optimization toolbox of Matlab, and a locally convergent feasible solution will be obtained.

III. Numerical Examples

Example 1: Consider the following discrete-time system with a time-varying delay:

$$A = \begin{bmatrix} 0.8 & 0.2 \\ 0 & 0.9 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix}$$

Assume the lower bound on the delay is $d_1 = 1$, and our objective is to determine the upper bound on the delay, d_2, which guarantees the above system is stable for all $d_1 \leq d(k) \leq d_2$. By applying Theorem 1 with $\mu = 1$, we can obtain that $d_2 = 5$. It means that the above system is stable for any time-varying delay $1 \leq d(k) \leq 5$. Moreover, we can see that Ξ_6 is unstable since its maximum eigenvalue is outside the unit circle. Therefore, if $d(k) = 6$ occurs in the system's delay sequence, the above system will be thought to be unstable according to the results in [22], [23]. However, it is assumed that the occurrence rates of the delays are $r_1 = 30\%$, $r_2 = 20\%$, $r_3 = 20\%$, $r_4 = 10\%$, $r_5 = 10\%$, $r_6 = 10\%$, respectively. Choose $\lambda_1 = 1.1$, $\lambda_2 = 1.05$, $\lambda_3 = 1.01$, $\lambda_4 = 0.95$, $\lambda_5 = 0.92$, $\lambda_6 = 0.87$, and $\prod_{i=1}^{6} \lambda_i^{r_i} = 1.013$. By Theorem 1, we obtain $T_a^* = 0.4$. It is easy to see that the maximum switching number of $\sigma(k)$ over the interval $[0, k)$ is $k-1$, so the minimum average dwell time $T_a = \frac{k}{k-1} > 1$. Therefore, the above system is exponentially stable under the above occurrence rates of the delays.

Example 2: Consider the following discrete-time system with a time-varying delay:

$$A = \begin{bmatrix} 0.3 & -0.9 \\ 0.5 & 0.9 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.4 \end{bmatrix}$$

Clearly, the maximum absolute value of the eigenvalue of $A+A_1$ is 1.005, so results in [6], [22], [23] are all unapplicable. By applying Theorem 1 with $\mu = 1$, we can see that the above system is stable for any time-varying delay satisfying $1 \leq d(k) \leq 3$.

Assume $d_1 = 1$ and $T_a = 1.04$, the results obtained by Theorem 1 are listed in Table I for different occurrence rates of the delay. Comparing the result for $r_4 = 10\%$, $r_5 = 0\%$ with the result for $r_4 = 0\%$, $r_5 = 10\%$, it is not difficult to see that the decay rate for $r_4 = 10\%$, $r_5 = 0\%$ is bigger than the decay rate for $r_4 = 0\%$, $r_5 = 10\%$. This result is reasonable since $d(k) = 5$ makes the system more unstable than $d(k) = 4$.

Example 3: Consider system (1) with:

$$A = \begin{bmatrix} 0.5 & 0.3 \\ 0.2 & 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0.1 & 0.4 \\ 0.2 & 0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

It is assumed that $1 \leq d(k) \leq 5$. Choose $\lambda_1 = 1.3$, $\lambda_2 = 1.3$, $\lambda_3 = 1.2$, $\lambda_4 = 1.2$, $\lambda_5 = 1$, $\delta_1 = 0.2$, $\delta_2 = 0.3$, $\delta_3 = 0.1$, $\delta_4 = 0$, $\delta_5 = 0$ and $\mu = 1.2$, the controller gain can be obtained as $K = \begin{bmatrix} -0.3117 & -0.7499 \end{bmatrix}$.

TABLE I
Results for different r_i

r_1	r_2	r_3	r_4	r_5	λ	μ	T_a^*
30%	30%	30%	10%	0%	1.0665	1.14	1.0180
30%	30%	30%	5%	5%	1.0377	1.079	1.0280
30%	30%	30%	0%	10 %	1.0096	1.02	1.0347

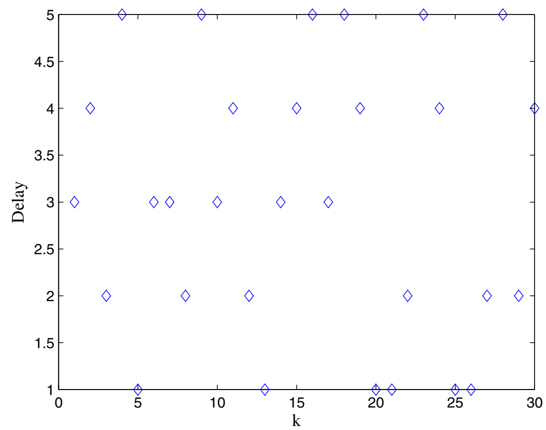

Fig. 1. Time-varying delay

For the purpose of simulation, the delay during the interval $[0, 30)$ is shown in Fig. 1. From Fig. 1, we can obtain $r_1 = 0.2$, $r_2 = 0.2$, $r_3 = 0.2$, $r_4 = 0.2$, $r_5 = 0.2$. In this case, $\prod_{i=1}^{6} \lambda_i^{r_i} = 1.1947$. Choose $\lambda = 1.194$ and we obtain $T_a^* = 0.514$. On the other hand, the switching number of $\sigma(k)$ during the interval $[0, 30)$ is 26 and thus the average dwell time $T_a = 1.1538$. Therefore, all the conditions in Theorem 1 are all satisfied. The closed-loop system is exponentially stable with a decay rate $\lambda^* = \lambda^{1-\frac{\ln \mu}{2T_a \ln \lambda}} = 1.103$. The response of the closed-loop system with the initial condition $x(0) = [1; 0]$ is shown in Fig. 2. Simulation results have demonstrated that the above system can be stabilized by the obtained feedback controller.

IV. Conclusions

In this paper, the problem of the exponential stability and stabilization for discrete systems with time-varying delays has been investigated. Using the average dwell-time method, a new stability criterion has been obtained. Based on the obtained stability condition, a design method of the feedback controller to stabilize the system has also been proposed. Through some numerical examples, the method proposed in this paper has been illustrated to be effective.

By converting a time-delay system into a switched system, plentiful results for switched systems can be used to analyze the time-delay systems. Especially, there are necessary and sufficient stability conditions available for switched system with arbitrary switching [30]. Using these stability conditions,

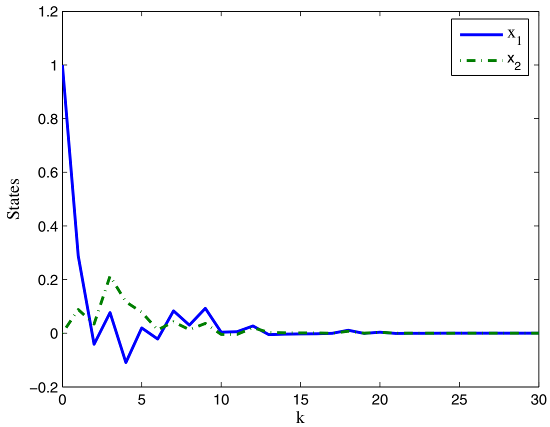

Fig. 2. State response of the closed-loop system

we can obtain some necessary and sufficient stability conditions for systems with time-varying delays. However, these conditions are only theoretically necessary and sufficient since the test is often inconclusive. How to make these necessary and sufficient conditions easy to check may be an important topic for future study.

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