Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

Stability Criteria for A Class of MIMO Networked Control Systems with Network Constraints

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***Abstract*—This paper is concerned with the stability analysis for a class of MIMO networked control systems (NCSs) with network constraints. In view of MIMO NCSs where network is of limited access channels, a discrete-time switched delay model is formulated. By constructing a novel piecewise Lyapunov-Krasovskii functional, a new stability criterion is developed in terms of linear matrix inequalities. A numerical example is given to show the effectiveness of the proposed method.**

***Index Terms*—Networked control systems; Switched delay system; Piecewise Lyapunov functional**

I. Introduction

In recent years, networked control systems (NCSs) have received much attention due to many advantages in practice such as reduced wiring and power requirements, ease of system diagnosis and maintenance, and flexibility of operations [1]–[4]. However it is well known that network insertion also makes analysis and design of NCSs more complex.

Networked-induced delay and data dropout are two main issues in NCSs. Various kinds of control methodologies for dealing with these two issues have been proposed. For example, based on remote control and local control strategy, a class of hybrid multi-rate control models with uncertainties and multiple time-varying delays was formulated in [8], and their robust stability properties were also investigated. Closed-loop NCSs have been modeled as systems with time-varying delay [18]–[20] and the design problems of state feedback controller [20] were obtained by solving a set of linear matrix inequalities. The problem of stabilisation of NCSs with packet dropout was studied in [6], and time-varying optimal control with packet dropout was studied in [7]. Special cases where systems occasionally have extreme large delays or intermittent controller failures were discussed in [9], [10]. A networked predictive control method [13], [14] was proposed to compensate the network-induced delay. Other methodologies can also be found in [11], [12], [15], [17] and references therein.

Another fundamental issue in networked control systems is that of network constraints [21], because it can only provide at the same time limited number of simultaneous access channels for all applications. As a consequence, in an NCS, only limited number of sensors and actuators are allowed to communicate with the controller via the network at any one time.

In this paper, the problem of stabilization and controller design for a class of MIMO NCSs with both network constraints and network delays is investigated. In Section 2, the closed-loop NCS is modeled as a switched delay system. In Section 3, new delay dependent stability criteria using a novel piecewise Lyapunov functional and the average dwell time method are proposed to study the stabilization of NCS. In Section 4, an illustrative example is given to demonstrate the effectiveness of the proposed method. Finally the conclusions are given.

II. System formulation and preliminaries

Similar to the models considered in [5], [21], a class of MIMO NCSs with both network constraints and network delays are considered in this section. The closed-loop NCS is illustrated in Fig. 1, which is composed of a plant G_p, a controller G_c, and a network with constraints and delays. The plant G_p has r independent sensors and n independent

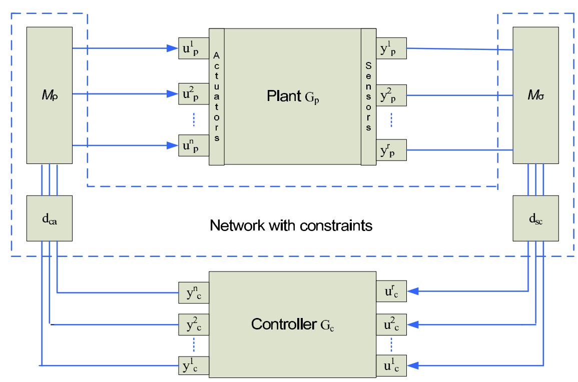

Fig. 1. MIMO networked control system with multiple time-delays

actuators, and the discrete time-invariant dynamics of plant G_p can be described by:

$$\begin{cases} x_p(k+1) = Ax_p(k) + Bu_p(k) \\ y_p(k) = Cx_p(k) \end{cases} \tag{1}$$

where $x_p(k) \in R^m, u_p(k) \in R^n, y_p(k) \in R^r$ are the plant state, input and output vector, respectively. m, n, r are their corresponding dimensions, and $A \in R^{m\times m}, B \in R^{m\times n}, C \in R^{r\times m}$ are known real constant matrices. The controller G_c is implemented by computer, and can be described by:

$$y_c(k) = Ku_c(k) \tag{2}$$

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where $u_c(k) \in R^r, y_c(k) \in R^n$ are the controller input and output vector, respectively, and K are known real constant matrices.

From Fig.1, due to the network constraints, only m_r sensors and m_n actuators can communicate with the controller at the same time, and d_{sc} represents the network delay from sensor to controller while d_{ca} is the delay from controller to actuator. The controller process delay can be considered constant and included either in d_{sc} or d_{sc}. The plant output $y_p^i, i = 1, 2, ..., r$ is transformed into $\hat{y}_p^i$ due to network constraints, and then acts as the controller input u_c^i with the sensor-to-controller delay d_{sc}, whereas the plant input $u_p^j, j = 1, 2, ..., n$ is obtained from $\hat{u}_p^j$, which comes form y_c^j with the controller-to-actuator delay d_{ca}. Define:

$$\sigma_i = \begin{cases} 1, & \text{sensor } i \text{ gains access} \\ 0, & \text{if not} \end{cases} \tag{3a}$$

$$\rho_j = \begin{cases} 1, & \text{actuator } j \text{ gains access} \\ 0, & \text{if not} \end{cases} \tag{3b}$$

then, the controller input u_c^i and the plant input u_p^j can be written as:

$$\begin{cases} u_c^i(k) = \hat{y}_p^i(k - d_{sc}) = \sigma_i(k) y_p^i(k - d_{sc}) & i = 1, 2, ..., r \\ u_p^j(k) = \rho_j(k)\hat{u}_p^j(k) = \rho_j(k) y_c^j(k - d_{ca}) & j = 1, 2, ..., n \end{cases} \tag{4}$$

We can rewrite $u_p(k), u_c(k)$ in vector form:

$$u_c(k) = \begin{bmatrix} \sigma_1(k) y_p^1(k - d_{sc}) \\ \sigma_2(k) y_p^2(k - d_{sc}) \\ \vdots \\ \sigma_r(k) y_p^r(k - d_{sc}) \end{bmatrix}, u_p(k) = \begin{bmatrix} \rho_1(k) y_c^1(k - d_{ca}) \\ \rho_2(k) y_c^2(k - d_{ca}) \\ \vdots \\ \rho_n(k) y_c^n(k - d_{ca}) \end{bmatrix} \tag{5}$$

Define

$$\begin{cases} M_\sigma = diag\{\sigma_1, \sigma_2, \cdots, \sigma_r\} \\ M_\rho = diag\{\rho_1, \rho_2, \cdots, \rho_n\} \end{cases} \tag{6}$$

where M_σ switches between $\binom{r}{m_r} = m_\sigma$ constant matrices; and M_ρ switches between $\binom{n}{m_n} = m_\rho$ constant matrices. Then we have:

$$\begin{cases} u_c(k) = M_{\sigma(k)} y_p(k - d_{sc}) \\ u_p(k) = M_{\rho(k)} y_c(k - d_{ca}) \end{cases} \tag{7}$$

From (1)-(7), we can easily obtain:

$$x_p(k+1) = A x_p(k) + B M_{\rho(k)} K M_{\sigma(k)} C x_p(k - d_{sc} - d_{ca}) \tag{8}$$

For simplicity, define: $K_{\delta(k)} = M_{\rho(k)} K M_{\sigma(k)}$, $E_{\delta(k)} = B K_{\delta(k)} C$, then $K_{\delta(k)}$ (also $E_{\delta(k)}$) switches between $\binom{n}{m_n} \cdot \binom{r}{m_r} = m_\rho \cdot m_\sigma$ matrices. Without loss of generality, define $d = d_{sc} + d_{ca}$, and assume d_1, d_2 are the lower and upper bound of d, respectively; then the closed-loop NCS can be rewritten as:

$$\Sigma : \begin{cases} x(k+1) = A x(k) + E_{\delta(k)} x(k - d(k)) & d_1 \leqslant d(k) \leqslant d_2 \\ x(k) = \Phi(k) & k \in [-d_2, \ 0] \end{cases} \tag{9}$$

III. Main Results

First let us consider the stability of the non-switched delay system:

$$\hat{\Sigma} : \begin{cases} x(k+1) = A x(k) + E x(k - d(k)) & d_1 \leqslant d(k) \leqslant d_2 \\ x(k) = \Phi(k) & k \in [-d_2, \ 0] \end{cases} \tag{10}$$

Lemma 3.1: Given scalars $d_2 \geqslant d_1 > 0$, $\alpha > 0$, system (10) is exponentially stable if there exist matrices $P = P^T > 0$, $Q_i = Q_i^T > 0$, $(i = 1, 2)$, $Z_i = Z_i^T > 0$, $(i = 1, 2)$, N, M, S, such that

$$\begin{bmatrix} \Phi_1 + \Phi_2 + \Phi_2^T & \Phi_3 \\ * & \Phi_4 \end{bmatrix} < 0 \tag{11}$$

where

$$\Phi_1 = \begin{bmatrix} -e^{-\alpha} P + Q_1 + Q_2 & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & -e^{-\alpha d_1} Q_1 & 0 \\ * & * & * & -e^{-\alpha d_2} Q_2 \end{bmatrix}$$

$$\Phi_2 = [N, \quad -N + M - S, \quad S, \quad -M]$$

$$\Phi_3 = \left[\Xi_1^T P, d_2 \Xi_2^T Z_1, (d_2 - d_1)\Xi_2^T Z_2, \ \sqrt{c_1} N, \ \sqrt{c_2} M, \ \sqrt{c_1} S\right]$$

$$\Phi_4 = diag\left[-P, -d_2 Z_1, -(d_2 - d_1) Z_2, -Z_1, -(Z_1 + Z_2), -Z_2\right]$$

$$\Xi_1 = [A, \quad E, \quad 0, \quad 0]$$

$$\Xi_2 = [A - I, \quad E, \quad 0, \quad 0]$$

$$c_1 = \frac{e^{\alpha(d_2+1)} - e^{\alpha}}{e^{\alpha} - 1}, \quad c_2 = \frac{e^{\alpha(d_2+1)} - e^{\alpha(d_1+1)}}{e^{\alpha} - 1}$$

Proof: Define $\eta(k) = x(k+1) - x(k)$, it is clear that

$$\eta(k) = x(k+1) - x(k) = (A - I)x(k) + E x(k - d(k)). \tag{12}$$

Construct a Lyapunov functional as follows:

$$\begin{aligned} V(k) &= V_1(k) + V_2(k) + V_3(k) \\ V_1(k) &= x^T(k) P x(k) \\ V_2(k) &= \sum_{\theta=-d_2+1}^{0} \sum_{l=k-1+\theta}^{k-1} \eta^T(l) e^{\alpha(l-k+1)} Z_1 \eta(l) \\ &\quad + \sum_{\theta=-d_2+1}^{-d_1} \sum_{l=k-1+\theta}^{k-1} \eta^T(l) e^{\alpha(l-k+1)} Z_2 \eta(l) \\ V_3(k) &= \sum_{l=k-d_1}^{k-1} x^T(l) e^{\alpha(l-k+1)} Q_1 x(l) \\ &\quad + \sum_{l=k-d_2}^{k-1} x^T(l) e^{\alpha(l-k+1)} Q_2 x(l) \end{aligned}$$

where $P = P^T > 0$, $Q_i^T = Q_i \geqslant 0$ $(i = 1, 2)$, and $Z_j^T = Z_j \geqslant 0$ $(j = 1, 2)$ are to be determined. Define $\Delta V(k) = V(k+1) - e^{-\alpha} V(k)$, $\zeta(k) = [x^T(k) \quad x^T(k - d(k)) \quad x^T(k - d_1) \quad x^T(k - d_2)]$, then along the solution of system (10), we have:

$$\Delta V_1(k) = x^T(k+1) P x(k+1) - x^T(k) e^{-\alpha} P x(k)$$

$$= \zeta^T(k)\Xi_1^T P\Xi_1\zeta^T(k) - x^T(k)e^{-\alpha}Px(k)$$

$$\begin{aligned}\Delta V_2(k) &= \sum_{\theta=-d_2+1}^{0}\sum_{l=k+\theta}^{k}\eta^T(l)e^{\alpha(l-k)}Z_1\eta(l) \\ &+ \sum_{\theta=-d_2+1}^{-d_1}\sum_{l=k+\theta}^{k}\eta^T(l)e^{\alpha(l-k)}Z_2\eta(l) \\ &- \sum_{\theta=-d_2+1}^{0}\sum_{l=k-1+\theta}^{k-1}\eta^T(l)e^{\alpha(l-k)}Z_1\eta(l) \\ &- \sum_{\theta=-d_2+1}^{-d_1}\sum_{l=k-1+\theta}^{k-1}\eta^T(l)e^{\alpha(l-k)}Z_2\eta(l) \\ &= d_2\eta^T(k)Z_1\eta(k) - \sum_{l=k-d_2}^{k-1}\eta^T(l)e^{\alpha(l-k)}Z_1\eta(l) \\ &+(d_2-d_1)\eta^T(k)Z_2\eta(k) - \sum_{l=k-d_2}^{k-d_1-1}\eta^T(l)Z_2\eta(l) \\ &= \zeta^T(k)\Xi_2^T\left(d_2Z_1+(d_2-d_1)Z_2\right)\Xi_2\zeta(k) \\ &- \sum_{l=k-d(k)}^{k-1}\eta^T(l)e^{\alpha(l-k)}Z_1\eta(l) \\ &- \sum_{l=k-d(k)}^{k-d_1-1}\eta^T(l)e^{\alpha(l-k)}Z_2\eta(l) \\ &- \sum_{l=k-d_2}^{k-d(k)-1}\eta^T(l)e^{\alpha(l-k)}(Z_1+Z_2)\eta(l) \\ \Delta V_3(k) &= \sum_{l=k-d_1+1}^{k}x^T(l)e^{\alpha(l-k)}Q_1x^T(l) \\ &+ \sum_{l=k-d_2+1}^{k}x^T(l)e^{\alpha(l-k)}Q_2x^T(l) \\ &- \sum_{l=k-d_1}^{k-1}x^T(l)e^{\alpha(l-k)}Q_1x^T(l) \\ &- \sum_{l=k-d_2}^{k-1}x^T(l)e^{\alpha(l-k)}Q_2x^T(l) \\ &= x^T(k)(Q_1+Q_2)x(k) - x^T(k-d_1)e^{-d_1}Q_1x(k-d_1) \\ &-x^T(k-d_2)e^{-d_2}Q_2x^T(k-d_2) \qquad (13)\end{aligned}$$

From (12), the following equations hold for any matrices N, M, and S with appropriate dimensions:

$$\begin{aligned} 0 &= 2\zeta^T(k)N\left[x(k)-x(k-d(k)) - \sum_{l=k-d(k)}^{k-1}\eta(l)\right] \\ 0 &= 2\zeta^T(k)M\left[x(k-d(k))-x(k-d_2) - \sum_{l=k-d_2}^{k-d(k)-1}\eta(l)\right] \\ 0 &= 2\zeta^T(k)S\left[x(k-d_1)-x(k-d(k)) - \sum_{l=k-d(k)}^{k-d_1-1}\eta(l)\right] \qquad (14)\end{aligned}$$

Using the inequality

$$2x^Ty \leqslant x^TPx + y^TP^{-1}y, \forall x,y\in R^n \ and \ P\in R^{n\times n}$$

in [16], the following inequalities can be obtained:

$$\begin{aligned} 2\zeta^T(k)N\sum_{l=k-d(k)}^{k-1}\eta(l) &\leqslant \sum_{l=k-d(k)}^{k-1}\eta^T(l)e^{\alpha(l-k)}Z_1\eta(l) \\ &+ \sum_{l=k-d(k)}^{k-1}\zeta^T(k)e^{-\alpha(l-k)}NZ_1^{-1}N^T\zeta(k) \\ &\leqslant \sum_{l=k-d(k)}^{k-1}\eta^T(l)e^{\alpha(l-k)}Z_1\eta(l) \\ &+c_1\zeta^T(k)NZ_1^{-1}N^T\zeta(k) \\ 2\zeta^T(k)M\sum_{l=k-d_2}^{k-d(k)-1}\eta(l) &\leqslant \sum_{l=k-d_2}^{k-d(k)-1}\eta^T(l)e^{\alpha(l-k)}(Z_1+Z_2)\eta(l) \\ &+ \sum_{l=k-d_2}^{k-d(k)-1}\zeta^T(k)e^{-\alpha(l-k)}M(Z_1+Z_2)^{-1} \\ &M^T\zeta(k) \\ &\leqslant \sum_{l=k-d_2}^{k-d(k)-1}\eta^T(l)e^{\alpha(l-k)}(Z_1+Z_2)\eta(l) \\ &+c_2\zeta^T(k)M(Z_1+Z_2)^{-1}M^T\zeta(k) \\ 2\zeta^T(k)S\sum_{l=k-d_2}^{k-1}\eta(l) &\leqslant \sum_{l=k-d_2}^{k-1}\eta^T(l)e^{\alpha(l-k)}Z_2\eta(l) \\ &+ \sum_{l=k-d_2}^{k-1}\zeta^T(k)e^{-\alpha(l-k)}SZ_2^{-1}S^T\zeta(k) \\ &\leqslant \sum_{l=k-d_2}^{k-1}\eta^T(l)e^{\alpha(l-k)}Z_2\eta(l) \\ &+c_1\zeta^T(k)SZ_2^{-1}S^T\zeta(k) \qquad (15)\end{aligned}$$

Using (13)-(15) allows us to write $\Delta V(k)$ as:

$$\begin{aligned}\Delta V(k) &\leqslant \zeta^T(k)[\Phi_1+\Phi_2+\Phi_2^T+c_1NZ_1^{-1}N^T \\ &+c_2M(Z_1+Z_2)^{-1}M^T + c_1SZ_2^{-1}S^T]\zeta(k) \qquad (16)\end{aligned}$$

Thus, if $\Phi_1+\Phi_2+\Phi_2^T+c_1NZ_1^{-1}N^T+c_2M(Z_1+Z_2)^{-1}M^T+c_1SZ_2^{-1}S^T<0$, which is equivalent to (11) by Schur complements, then $\Delta V(k)<0$. That is, system (10) is exponentially stable, and along the trajectory of system (10), the convergence rate can be estimated as:

$$V(k)\leqslant e^{-\alpha(k-k_0)}V(k_0) \qquad (17)$$

This completes the proof. ■

Based on the Lemma obtained above, a new delay-dependent stability criterion for the switched delay system (9) is developed using a piecewise Lyapunov-Krasovskii functional.

Theorem 3.1: Given scalars $d_2\geqslant d_1>0$, $\alpha>0$, system (9) is exponentially stable if there exist matrices $P_{ij}=P_{ij}^T>0$, $Q_{1_{(ij)}}=Q_{1_{(ij)}}^T>0$, $Q_{2_{(ij)}}=Q_{2_{(ij)}}^T>0$, $Z_{1_{(ij)}}=Z_{1_{(ij)}}^T>0$, $Z_{2_{(ij)}}=Z_{2_{(ij)}}^T>0$, N_{ij}, M_{ij}, S_{ij}, $(i=1,2,\cdots,m_\rho; j=1,2,\cdots,m_\sigma)$, such that

$$\begin{bmatrix}\Phi_{1_{(ij)}}+\Phi_{2_{(ij)}}+\Phi_{2_{(ij)}}^T & \Phi_{3_{(ij)}} \\ * & \Phi_{4_{(ij)}}\end{bmatrix}<0 \qquad (18)$$

where

$$\Phi_{1_{(ij)}} = \begin{bmatrix} -e^{-\alpha}P_{ij} & 0 & 0 & 0 \\ * & 0 & 0 & 0 \\ * & * & -e^{-\alpha d_1}Q_{1_{(ij)}} & 0 \\ * & * & * & -e^{-\alpha d_2}Q_{2_{(ij)}} \end{bmatrix}$$

$$\Phi_{2_{(ij)}} = \begin{bmatrix} N_{ij}, & -N_{ij}+M_{ij}-S_{ij}, & S_{ij}, & -M_{ij} \end{bmatrix}$$

$$\Phi_{3_{(ij)}} = [\Xi_{1_{(ij)}}^T P_{ij}, d_2\Xi_{2_{(ij)}}^T Z_{1_{(ij)}}, (d_2-d_1)\Xi_{2_{(ij)}}^T Z_{2_{(ij)}}, \sqrt{c_1}N_{ij}, \sqrt{c_2}M_{ij}, \sqrt{c_1}S_{ij}]$$

$$\Phi_{4_{(ij)}} = diag[-P_{ij}, -d_2Z_{1_{(ij)}}, -(d_2-d_1)Z_{2_{(ij)}}, -Z_{1_{(ij)}}, -(Z_{1_{(ij)}}+Z_{2_{(ij)}}), -Z_{2_{(ij)}}]$$

$$\Xi_{1_{(ij)}} = \begin{bmatrix} A, & E_{ij}, & 0, & 0 \end{bmatrix}$$

$$\Xi_{2_{(ij)}} = \begin{bmatrix} A-I, & E_{ij}, & 0, & 0 \end{bmatrix}$$

$$c_1 = \frac{e^{\alpha(d_2+1)}-e^{\alpha}}{e^{\alpha}-1}, \quad c_2 = \frac{e^{\alpha(d_2+1)}-e^{\alpha(d_1+1)}}{e^{\alpha}-1}$$

then, system (9) is exponentially stable for any switching signal with average dwell time satisfying

$$T_a > T_a^* = \frac{ln\mu}{\alpha} \tag{19}$$

and an estimate of state decay is given by

$$\| x(k) \| \leqslant \sqrt{\frac{b}{a}} e^{-\lambda(k-k_0)} \| x(k_0) \| \tag{20}$$

where $\mu \geqslant 1$ satisfies

$$\begin{aligned} & P_{ij} \leqslant \mu P_{st},\ Q_{1_{(ij)}} \leqslant \mu Q_{1_{(st)}},\ Q_{2_{(ij)}} \leqslant \mu Q_{2_{(st)}}, \\ & Z_{1_{(ij)}} \leqslant \mu Z_{1_{(st)}},\ Z_{2_{(ij)}} \leqslant \mu Z_{1_{(st)}}, \\ & \forall\ i,s = 1,2,\cdots,m_\rho, i \neq s; j,t = 1,2,\cdots,m_\sigma, j \neq t \end{aligned} \tag{21}$$

and

$$\begin{aligned} \lambda &= \frac{1}{2}\left(\alpha - \frac{ln\mu}{T_a}\right) \\ a &= \min_{\substack{1\leqslant i\leqslant m_\rho \\ 1\leqslant j\leqslant m_\sigma}} \lambda_{min}(P_{ij}) \\ b &= \max_{\substack{1\leqslant i\leqslant m_\rho \\ 1\leqslant j\leqslant m_\sigma}} \Bigg(\lambda_{max}(P_{ij}) + \frac{d_2(d_2+1)}{2}\lambda_{max}(Z_{1_{(ij)}}) \\ &\quad + \frac{d_2^2-d_1^2+d_2-d_1}{2}\lambda_{max}(Z_{2_{(ij)}}) \\ &\quad + d_1\lambda_{max}(Q_{1_{(ij)}}) + d_2\lambda_{max}(Q_{2_{(ij)}})\Bigg) \end{aligned} \tag{22}$$

Proof: Define the following piecewise Lyapunov functional

$$V(k) = V_{\delta(k)}(k) = V_{1_{\delta(k)}}(k) + V_{2_{\delta(k)}}(k) + V_{3_{\delta(k)}}(k) \tag{23}$$

where

$$\begin{aligned} V_{1_{\delta(k)}}(k) &= x^T(k)P_{\delta(k)}x(k) \\ V_{2_{\delta(k)}}(k) &= \sum_{\theta=-d_2+1}^{0}\sum_{l=k-1+\theta}^{k-1}\eta^T(l)e^{\alpha(l-k+1)}Z_{1_{\delta(k)}}\eta(l) \\ &\quad + \sum_{\theta=-d_2+1}^{-d_1}\sum_{l=k-1+\theta}^{k-1}\eta^T(l)e^{\alpha(l-k+1)}Z_{2_{\delta(k)}}\eta(l) \\ V_{3_{\delta(k)}}(k) &= \sum_{l=k-d_1}^{k-1}x^T(l)e^{\alpha(l-k+1)}Q_{1_{\delta(k)}}x(l) \\ &\quad + \sum_{l=k-d_2}^{k-1}x^T(l)e^{\alpha(l-k+1)}Q_{2_{\delta(k)}}x(l) \end{aligned}$$

At any switching interval $[k_{i-1}, k_i]$, using (18), (23) and Lemma 3.1, we have

$$V(k) = V_{\delta(k)}(k) \leqslant e^{-\alpha(k-k_i)}V_{\delta(k_i)}(k_i) \tag{24}$$

From (21), at switching instant k_i, it holds

$$V_{\delta(k_i)}(k_i) \leqslant \mu V_{\delta(k_i^-)}(k_i^-) \tag{25}$$

Therefore, it follows from (24) and (25) that

$$\begin{aligned} V(k) &= V_{\delta(k)}(k) \\ &\leqslant e^{-\alpha(k-k_i)}V_{\delta(k_i)}(k_i) \\ &\leqslant e^{-\alpha(k-k_i)}\mu V_{\delta(k_i^-)}(k_i^-) \\ &\leqslant \cdots \leqslant e^{-\alpha(k-k_0)}\mu^{N_\delta(k-k_0)}V_{\delta(k_0)}(k_0) \\ &\leqslant e^{-(\alpha-\frac{ln\mu}{T_a})(k-k_i)}V_{\delta(k_0)}(k_0) \end{aligned} \tag{26}$$

where $N_\delta(k-k_0)$ is the number of switching over $[k, k_0]$. According to (22) and (23), it can be obtained that

$$a \| x(k) \|^2 \leqslant V_{\delta(k)}(k),\ V_{\delta(k_0)}(k_0) \leqslant b \| x(k_0) \|^2 \tag{27}$$

Combining (26) and (27) leads to

$$\| x(k) \|^2 \leqslant \frac{1}{a}V(k) \leqslant \frac{b}{a}e^{-(\alpha-\frac{ln\mu}{T_a})(k-k_0)} \| x(k_0) \|^2$$

This completes the proof. ■

IV. NUMERICAL EXAMPLE

Example 4.1: Consider the following unstable system:

$$A = \begin{bmatrix} 0.7547 & 0.4717 \\ 0.0943 & 0.8491 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

stabilized by:

$$K = \begin{bmatrix} -0.1 & 0.01 \\ 0.1 & -0.1 \end{bmatrix},$$

with network delay $2 \leqslant d(k) \leqslant 10$. Given $\alpha = 0.005$, $\mu = 1.05$, we have feasible solutions of (18) and (21). According to Theorem 3.1, $T_a^* = \frac{ln\mu}{\alpha} = 9.76$. Taking $T_a = 10$, and for the initial condition of the state $x_0 = [1, -1]$, the response of the system, network delay and random switching sequence are shown in Fig. 2, Fig. 3 and Fig. 4, respectively. It can be seen that the system is exponentially stable which illustrates the effectiveness of the above mentioned method.

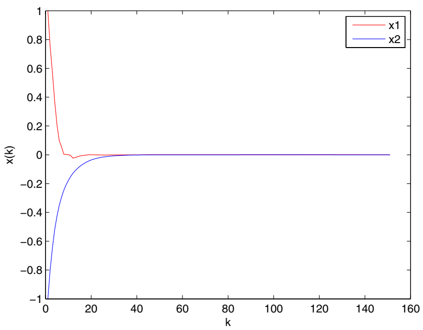

Fig. 2. System response with initial states

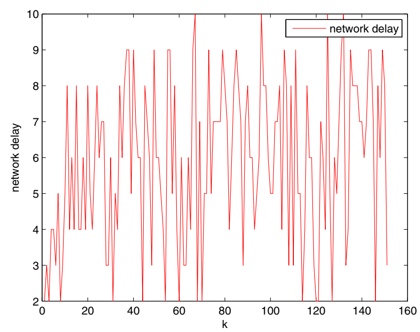

Fig. 3. Network delay and switching sequence

V. CONCLUSIONS

The stability analysis for a class of MIMO NCSs with network constraints has been investigated in this paper. In view of MIMO NCSs where network is of limited access channels, a discrete-time switched delay model is formulated. By constructing a new piecewise Lyapunov functional and using average-dwell time method, novel delay-dependent stability criteria have been derived. A numerical example has illustrated the merits of the proposed method.

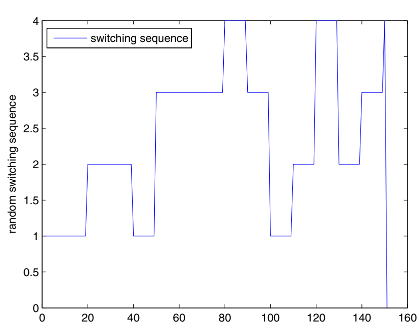

Fig. 4. Network delay and switching sequence

ACKNOWLEDGMENT

This work is partially supported by National Natural Science Foundation of China under Grant 60504020 and the Beijing Education Committee Cooperation Building Foundation Project.

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