Proceedings of the 2009 IEEE International Conference on Systems, Man, and Cybernetics
San Antonio, TX, USA - October 2009

On the Equivalence of Single Input Type Fuzzy Inference Methods

Hirosato Seki*† and Masaharu Mizumoto‡
*Graduate School of Information Science and Technology,
Osaka University, Osaka, 565-0871 Japan.
Email: h.seki@ist.osaka-u.ac.jp
†Japan Society for the Promotion of Science (JSPS), Japan.
‡Department of Engineering Informatics,
Osaka Electro-Communication University, Osaka, 572-8530 Japan.
Email: mizumoto@mzlab.osakac.ac.jp

***Abstract*—This paper addresses equivalence of fuzzy inference methods. It first presents single input type fuzzy inference methods: the single input rule modules connected type fuzzy inference method (SIRMs method) and single input connected fuzzy inference method (SIC method). Secondly, the equivalence conditions of the SIRMs method and SIC method are shown. Finally, this paper also discusses the equivalence conditions between the single input type fuzzy inference methods and the conventional fuzzy inference methods like the simplified fuzzy inference method, product–sum–gravity method and fuzzy singleton-type inference method which are all widely used as fuzzy control methods.**

***Index Terms*—Fuzzy inference, single input type fuzzy inference method, simplified fuzzy inference method, product–sum–gravity method, fuzzy singleton-type inference method, equivalence.**

I. Introduction

Since this paper is a continuation of [1], we assume that the readers are already familiar with [1]. In this paper, we focus on equivalence of fuzzy inference methods [1]–[10].

This paper shows that the single input rule modules connected fuzzy inference method (SIRMs method) [11]–[16] can be reduced to the single input connected fuzzy inference method (SIC method) [17]–[19]. Moreover, it is shown that the SIC method is reduced to the simplified fuzzy inference method [20]–[22] is a special case of the T–S inference method [23], [24] in which the consequent part of the T–S inference method is replaced to constant. Similarly, it is shown that the SIC method is reduced to the product–sum–gravity method [25]–[29] whose consequent parts consist of fuzzy sets and fuzzy singleton-type inference method [30] by multiplying fuzzy sets in the antecedent part of the each inference method by the area and weight of the product–sum–gravity method and fuzzy singleton-type inference method, respectively.

Furthermore, from these results, this paper also shows the SIRMs method and SIC method are special case of the above conventional fuzzy inference methods.

II. Fuzzy Inference Methods

In this section we review the SIRMs method, SIC method and simplified fuzzy inference method.

A. SIRMs Connected Fuzzy Inference Method

We briefly review the single input rule modules connected fuzzy inference method (SIRMs method) [11]–[16]. The SIRMs method has n rule modules, and final inference result by the SIRMs method is obtained by the weighted sum of n inference results from rule modules and n weights. Rule modules of the SIRMs method are given as

$$\begin{aligned} &\text{Rules-1} : \{x_1 = A^1_j \longrightarrow y_1 = y^1_j\}^{m_1}_{j=1} \\ &\qquad\vdots \\ &\text{Rules-}i : \{x_i = A^i_j \longrightarrow y_i = y^i_j\}^{m_i}_{j=1} \\ &\qquad\vdots \\ &\text{Rules-}n : \{x_n = A^n_j \longrightarrow y_n = y^n_j\}^{m_n}_{j=1} \end{aligned} \tag{1}$$

where Rules-i stands for the ith single input rule module, the ith input item x_i is the sole variable of the antecedent part of the Rules-i, and y_i stands for the variable of its consequent part. A^i_j means the fuzzy set of the jth rule of the Rules-i, y^i_j is real value of consequent part, and $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m_i$. m_i is the number of rules in Rules-i.

The degree of the antecedent part in the jth rule of Rules-i is obtained by (2) for input x^0_i, and the inference result y^0_i from Rules-i is given as (3) (see Fig. 1).

$$h^i_j = A^i_j(x^0_i) \tag{2}$$

$$y^0_i = \frac{\sum_{j=1}^{m_i} h^i_j y^i_j}{\sum_{j=1}^{m_i} h^i_j} \tag{3}$$

The final inference result y_0 of the SIRMs method is given by (4), where importance degree of each input item x_i ($i = 1, 2, \ldots, n$) is set as v_i. That is,

$$y_0 = \sum_{i=1}^{n} v_i y^0_i \tag{4}$$

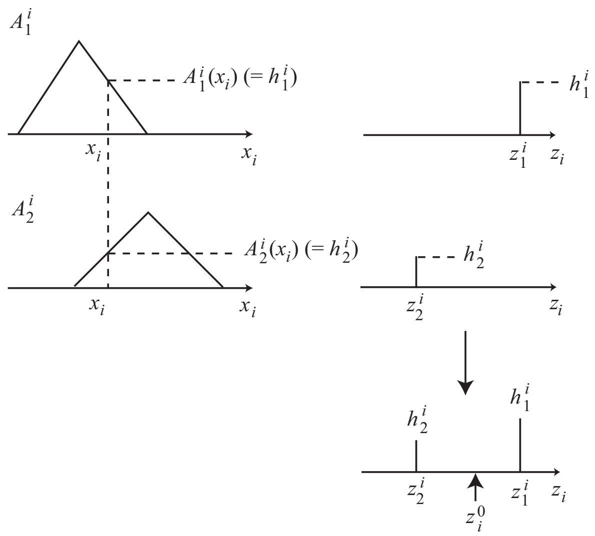

Fig. 1. Fuzzy inference method in Rules-i.

B. Single Input Connected (SIC) Fuzzy Inference Method

In this paper we tentatively call "Single Input Connected (SIC) fuzzy reasoning method" for single input type fuzzy reasoning method proposed by Hayashi et al. [17]–[19]. The SIC method sets up rule modules to each input item as well as SIRMs method. The final inference result of SIC method is obtained by the weighted average of the degrees of the antecedent part and consequent part of each rule module.

Namely, rule modules and degree h_j^i of the antecedent part of SIC method are given as those of SIRMs method in (1) and (2), respectively. The final inference result y^0 is given as follows by using degrees of antecedent part and consequent part from each rule module.

$$\begin{aligned} y^0 &= \frac{\sum_{j=1}^{m_1} h_j^1 y_j^1 + \cdots + \sum_{j=1}^{m_n} h_j^n y_j^n}{\sum_{j=1}^{m_1} h_j^1 + \cdots + \sum_{j=1}^{m_n} h_j^n} \\ &= \frac{\sum_{i=1}^{n}\sum_{j=1}^{m_i} h_j^i y_j^i}{\sum_{i=1}^{n}\sum_{j=1}^{m_i} h_j^i} \end{aligned} \tag{5}$$

III. Equivalence of SIRMs method and SIC method

Example 1: We consider the following rule modules as a simple example.

$$\text{Rules-1} = \begin{cases} x_1 = A_1^1 \longrightarrow y_1 = 1 \\ x_1 = A_2^1 \longrightarrow y_1 = 2 \\ x_1 = A_3^1 \longrightarrow y_1 = 1 \end{cases}$$

$$\text{Rules-2} = \begin{cases} x_2 = A_1^2 \longrightarrow y_2 = 1 \\ x_2 = A_2^2 \longrightarrow y_2 = 3 \\ x_2 = A_3^2 \longrightarrow y_2 = 5 \end{cases} \tag{6}$$

where fuzzy sets A_1^i, A_2^i, A_3^i $(i = 1, 2)$ in Fig. 2 are used. The inference results of (6) in the SIRMs method and SIC method is obtained as shown in Fig. 3 and Fig. 4, respectively, where $v_1 = w_2 = 0.5$. □

As mentioned above, generally, the inference results of the SIRMs method and SIC method do not obtain even if these methods use same rules. Therefore, in this section, in order to clarify the property between the SIRMs method and SIC method, we discuss the equivalent condition between SIRMs method and SIC method.

We have the following theorem on the SIRMs method and SIC method.

Theorem 1: Let each weight of SIRMs method be obtained from the following equation.

$$v_i = \frac{\sum_{k=1}^{m_i} h_k^i}{\sum_{i=1}^{n}\sum_{k=1}^{m_i} h_k^i} \tag{7}$$

Then, the inference results by SIRMs method and SIC method are equivalent. □

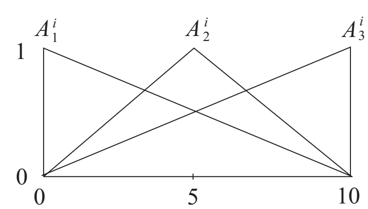

Fig. 2. Fuzzy sets (1)

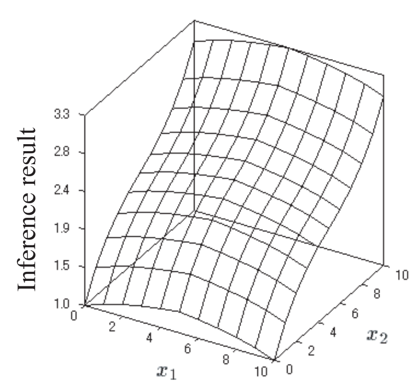

Fig. 3. Inference result by the SIRMs method

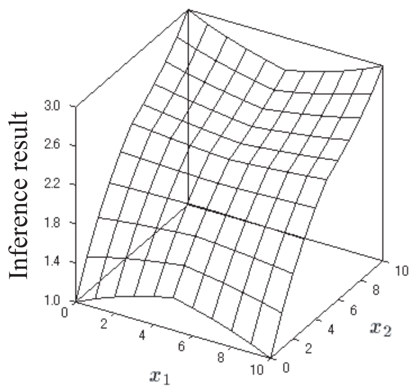

Fig. 4. Inference result by the SIC method

Proof: We show that the inference results by the SIRMs method and SIC method are equal if each weight of SIRMs method is assumed as in (7).

The inference result by the SIC method is obtained from (1) and (5) as follows.

$$y^0 = \frac{\sum_{i=1}^{n}\sum_{k=1}^{m_i} h_k^i y_k^i}{\sum_{i=1}^{n}\sum_{k=1}^{m_i} h_k^i} \tag{8}$$

On the other hand, the inference result by SIRMs method is obtained from (1), (3), (4) as follows.

$$y' = \sum_{i=1}^{n}\left(v_i \cdot \frac{\sum_{k=1}^{m_i} h_k^i y_k^i}{\sum_{k=1}^{m_i} h_k^i} \right) \tag{9}$$

(9) can be expanded to

$$y' = v_1 \cdot \frac{\sum_{k=1}^{m_1} h_k^1 y_k^1}{\sum_{k=1}^{m_1} h_k^1} + \cdots + v_n \cdot \frac{\sum_{k=1}^{m_n} h_k^n y_k^n}{\sum_{k=1}^{m_n} h_k^n} \tag{10}$$

Since each weight of SIRMs method is assumed in (7), (10) is transformed to

$$\begin{aligned} y' &= \frac{\sum_{k=1}^{m_i} h_k^1}{\sum_{i=1}^{n}\sum_{k=1}^{m_i} h_k^i} \cdot \frac{\sum_{k=1}^{m_1} h_k^1 y_k^1}{\sum_{k=1}^{m_1} h_k^1} + \cdots + \frac{\sum_{k=1}^{m_n} h_k^n}{\sum_{i=1}^{n}\sum_{k=1}^{m_i} h_k^i} \cdot \frac{\sum_{k=1}^{m_n} h_k^n y_k^n}{\sum_{k=1}^{m_n} h_k^n} \\ &= \frac{\sum_{k=1}^{m_1} h_k^1 y_k^1 + \cdots + \sum_{k=1}^{m_n} h_k^n y_k^n}{\sum_{i=1}^{n}\sum_{k=1}^{m_i} h_k^i} \\ &= \frac{\sum_{i=1}^{n}\sum_{k=1}^{m_i} h_k^i y_k^i}{\sum_{i=1}^{n}\sum_{k=1}^{m_i} h_k^i} \end{aligned} \tag{11}$$

Therefore, (9) is shown to be equal to (8), and thus Theorem 1 holds.

□

Example 2: We consider the inference result under rules of (6) and fuzzy sets in Fig. 5, where weights of SIRMs method are $w_1 = w_2 = 0.5$. These fuzzy sets satisfy the following condition:

$$A_1^i(x) + A_2^i(x) + A_3^i(x) = 1 \tag{12}$$

i.e. "fuzzy partition". Therefore, (12) satisfies condition (7).

From (6), the inference results by the SIRMs method and SIC method are obtained as shown in Fig. 6. Therefore, the

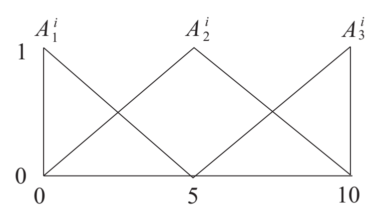

Fig. 5. Fuzzy sets (2)

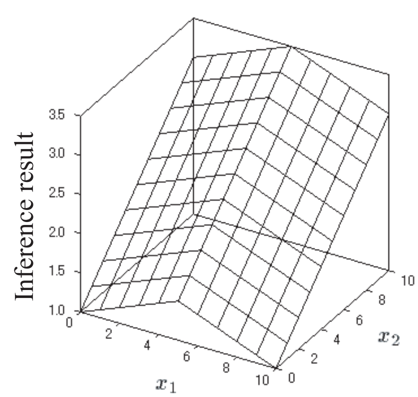

Fig. 6. Inference result (SIRMs, SIC)

inference results by two inference methods are same. □

As mentioned above, it turns out that the inference results by the SIRMs method and SIC method are equivalent if each weight of the SIRMs method satisfies the condition (7).

Remark 1: Generally, the SIRMs method and SIC method are not equivalent if each weight of the SIRMs method does not satisfy the condition (7).

IV. Equivalence of Single Input Type and Conventional Fuzzy Inference Method

In this section, we clarify the equivalence conditons of the SIC method and conventional fuzzy inference methods like the simplified fuzzy inference method, product–sumu–gravity method and fuzzy singleton-type inference method.

We have the following theorem regarding the simplified fuzzy inference method and SIC method.

Theorem 2: The inference results by simplified fuzzy inference method and SIC method are equivalent when the rules of simplified fuzzy inference method obtained by SIC method are used. □

Proof: In order to distinguish subscript j of Rules-i in (1), j_i is used, and thus (1) is rewritten as

$$\text{Rules-}i : \{x_i = A^i_{j_i} \longrightarrow y_i = y^i_{j_i}\}^{m_i}_{j_i=1}. \tag{13}$$

We can obtain the following rules of the simplified fuzzy inference method which corresponds to (13) of the rules of the SIC method by

$$\begin{gathered} x_1 = A^1_{j_1}, x_2 = A^2_{j_2}, \dots, x_n = A^n_{j_n} \\ \longrightarrow y = \frac{\sum_{j_i=1}^{m_1} h^1_{j_i}}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h^i_{j_i}} \cdot y^1_{j_1} + \cdots + \frac{\sum_{j_i=1}^{m_n} h^n_{j_i}}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h^i_{j_i}} \cdot y^n_{j_n} \end{gathered} \tag{14}$$

where $j_i = 1, 2, \dots, m_i$.

Inference result y' of the simplifief fuzzy inference method becomes as follows from (14):

$$y' = \frac{\sum_{j=1}^{M} \left\{ h_j \left(\frac{\sum_{j_i=1}^{m_1} h^1_{j_i}}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h^i_{j_i}} \cdot y^1_{j_1} + \cdots + \frac{\sum_{j_i=1}^{m_n} h^n_{j_i}}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h^i_{j_i}} \cdot y^n_{j_n} \right) \right\}}{\sum_{j=1}^{M} h_j} \tag{15}$$

where M is the total number of rules.

On the other hand, the conclusion y' by the SIC method becomes as follows from (5) and (13):

$$y' = \sum_{i=1}^{n} \left(\frac{\sum_{j_i=1}^{m_i} h^i_{j_i}}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h^i_{j_i}} \cdot \frac{\sum_{j_i=1}^{m_i} h^i_{j_i} y^i_{j_i}}{\sum_{j_i=1}^{m_i} h^i_{j_i}} \right) \tag{16}$$

(16) can be expanded to

$$\begin{aligned} y' = {} & \frac{\sum_{j_i=1}^{m_1} h^1_{j_i}}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h^i_{j_i}} \cdot \frac{\sum_{j_1=1}^{m_1} h^1_{j_1} y^1_{j_1}}{\sum_{j_1=1}^{m_1} h^1_{j_1}} + \cdots \\ & + \frac{\sum_{j_i=1}^{m_n} h^n_{j_i}}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h^i_{j_i}} \cdot \frac{\sum_{j_n=1}^{m_n} h^n_{j_n} y^n_{j_n}}{\sum_{j_n=1}^{m_n} h^n_{j_n}} \end{aligned} \tag{17}$$

When reducing (17) to a common denominator, the numerator of (17) is obtained as

$$\begin{aligned} & \frac{\sum_{j_i=1}^{m_1} h^1_{j_i}}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h^i_{j_i}} \cdot \sum_{j_1=1}^{m_1} h^1_{j_1} y^1_{j_1} \cdot \prod_{i=2}^{n}\sum_{j_i=1}^{m_i} h^i_{j_i} + \cdots \\ & + \frac{\sum_{j_i=1}^{m_n} h^n_{j_i}}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h^i_{j_i}} \cdot \sum_{j_n=1}^{m_n} h^n_{j_n} y^n_{j_n} \cdot \prod_{i=1}^{n-1}\sum_{j_i=1}^{m_i} h^i_{j_i} \end{aligned} \tag{18}$$

After (18) is expanded and rearranged, we find that it contains the following commmon item

$$(h^1_{j_1} \cdots h^n_{j_n}) \left(\frac{\sum_{j_i=1}^{m_1} h^1_{j_i}}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h^i_{j_i}} \cdot y^1_{j_1} + \cdots + \frac{\sum_{j_i=1}^{m_n} h^n_{j_i}}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h^i_{j_i}} \cdot y^n_{j_n} \right) \tag{19}$$

Since it has $M = \prod_{i=1}^{n} m_i$ kind of rules in total number, (19) becomes as follows :

$$\sum_{j=1}^{M} \left\{ h_j \left(\frac{\sum_{j_i=1}^{m_1} h^1_{j_i}}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h^i_{j_i}} \cdot y^1_{j_1} + \cdots + \frac{\sum_{j_i=1}^{m_n} h^n_{j_i}}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h^i_{j_i}} \cdot y^n_{j_n} \right) \right\} \tag{20}$$

Moreover, the denominator of (17) is transformed to (21)

$$\begin{aligned} & \prod_{i=1}^{n}\sum_{j_i=1}^{m_i} h^i_{j_i} \\ = {} & \sum_{j_1=1}^{m_1} \cdots \sum_{j_k=1}^{m_k} \cdots \sum_{j_n=1}^{m_n} h^1_{j_1} \cdots h^k_{j_k} \cdots h^n_{j_n} \\ = {} & \sum_{j=1}^{M} h_j. \end{aligned} \tag{21}$$

Therefore, (16) becomes as follows and it is exactly the same as (15)

$$y' = \frac{\sum_{j=1}^{M}\left\{h_j\left(\frac{\sum_{j_i=1}^{m_1} h_{j_i}^1}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h_{j_i}^i}\cdot y_{j_1}^1+\cdots+\frac{\sum_{j_i=1}^{m_n} h_{j_i}^n}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h_{j_i}^i}\cdot y_{j_n}^n\right)\right\}}{\sum_{j=1}^{M} h_j} \tag{22}$$

Therefore, Theorem 2 holds. □

Next, we show the equivalence conditions of the SIC method, product–sum–gravity method and fuzzy singleton-type inference method.

We have the following theorem regarding the product–sum–gravity method and SIC method.

Theorem 3: Let the area of the consequent part of the product–sum–gravity method be the weight for the antecedent part of the SIC method. Then, the SIC method is equivalent to the product–sum–gravity method. □

Proof: In order to prove Theorem 3, we use the equivalence in [1] and Theorem 2.

We can obtain the following rules of the product–sum–gravity method which corresponds to (13) of the rules of the SIC method:

$$x_1 = A_{j_1}^1, x_2 = A_{j_2}^2, \ldots, x_n = A_{j_n}^n \longrightarrow y = B_j \tag{23}$$

where B_j is fuzzy set. Moreover, let the center of gravity of fuzzy set B_j of consequent part in the product–sum–gravity method be

$$c_j = \frac{\sum_{j_i=1}^{m_1} h_{j_i}^1}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h_{j_i}^i}\cdot y_{j_1}^1+\cdots+\frac{\sum_{j_i=1}^{m_n} h_{j_i}^n}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h_{j_i}^i}\cdot y_{j_n}^n \tag{24}$$

[1] shows the inference result by the product–sum–gravity method can be obtained from the area as inference result of each rule and center. Therefore, the inference result y' of the product–sum–gravity method is obtained as follows:

$$y' = \frac{\sum_{j=1}^{M}\left\{h_j S_j\left(\frac{\sum_{j_i=1}^{m_1} h_{j_i}^1}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h_{j_i}^i}\cdot y_{j_1}^1+\cdots+\frac{\sum_{j_i=1}^{m_n} h_{j_i}^n}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h_{j_i}^i}\cdot y_{j_n}^n\right)\right\}}{\sum_{j=1}^{M} h_j S_j} \tag{25}$$

where S_j is the area of fuzzy sets in jth rule.

On the other hand, the inference result by the SIC method is (15). Here, let the area S_i of the product–sum–gravity method be the weight for the antecedent part of the SIC method. Then, the SIC method is equivalent to the product–sum–gravity method. Therefore, Theorem 3 holds. □

By the same token, we have the following theorem on the fuzzy singleton-type inference method and SIC method.

Theorem 4: Let the weight of the fuzzy singleton-type inference method be the weight for the antecedent part of the SIC method. Then, the SIC method is equivalent to the fuzzy singleton-type inference method. □

Proof: The consequent parts in rules of fuzzy singleton-type inference method assume as in (14). Then, the inference result by the fuzzy singleton-type inference method is obtained as follows:

$$y' = \frac{\sum_{j=1}^{M}\left\{h_j w_j\left(\frac{\sum_{j_i=1}^{m_1} h_{j_i}^1}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h_{j_i}^i}\cdot y_{j_1}^1+\cdots+\frac{\sum_{j_i=1}^{m_n} h_{j_i}^n}{\sum_{i=1}^{n}\sum_{j_i=1}^{m_i} h_{j_i}^i}\cdot y_{j_n}^n\right)\right\}}{\sum_{j=1}^{M} h_j w_j} \tag{26}$$

Let the weight w_j of the fuzzy singleton-type inference method be the weight for the antecedent part of the SIC method as same as Theorem 3. Then, the SIC method is equivalent to the fuzzy singleton-type inference method. Therefore, Theorem 4 holds.

□

Remark 2: As mentioned above, the SIC method can be transformed to the simplified fuzzy inference method, product–sum–gravity method and fuzzy singleton-type inference method, but not vice versa. □

Remark 3: The SIC method is not necessarily equivalent to the product–sum–gravity method and fuzzy singleton-type inference method if the fuzzy sets are limited as normal fuzzy sets. □

V. Conclusion

This paper has shown the equivalence of the fuzzy inference methods. At first, the equivalence of single input type fuzzy inference methods is shown. Next, the equivalence of the SIC method, simplified fuzzy inference method, product–sum–gravity method and fuzzy singleton-type inference method as conventional fuzzy inference methods.

From above results, the SIC method can be transformed to the simplified fuzzy inference method, product–sum–gravity method and fuzzy singleton-type inference method, but not vice versa. Moreover, the SIC method is not necessarily equivalent to the product–sum–gravity method and fuzzy singleton-type inference method if the fuzzy sets are limited as normal fuzzy sets. Therefore, it turns out that the SIRMs method and SIC method are special case of the conventional fuzzy inference methods.

Acknowledgment

This work was partially supported by a Grant-in-Aid for Scientific Research (Project No. 20·797) from the Japan Society for the Promotion Science (JSPS).

References

[1] M. Mizumoto, "Product–sum–gravity method = fuzzy singleton-type reasoning method = simplified fuzzy reasoning method," Proc. 1996 IEEE Int. Conf. Fuzzy Syst., vol. 3, pp. 2098–2102, New Orleans, USA, 1996.

[2] J. R. Jang and C. T. Sun, "Functional equivalence between radial basis function networks and fuzzy inference systems," *IEEE Trans. Neural Networks*, vol. 4, no. 1, pp. 156–159, 1993.

[3] K. J. Hunt, R. Haas, and M. Brown, "On the functional equivalence of fuzzy inference systems and spline-based networks," *Int. J. Neural Syst.*, vol. 6, no. 2, pp. 171–184, 1995.

[4] J. M. Benitez, J. L. Castro, and I. Requena, "Are artifical neural networks black boxes?," *IEEE Trans. Neural Networks*, vol. 8, no. 5, pp. 1156–1164, September 1997.

[5] J. L. Castro, C. J. Mantas, and J. M. Benitez, "Interpretation of artifical neural networks by means of fuzzy rules," *IEEE Trans. Neural Networks*, vol. 13, no. 1, pp. 101–116, January 2002.

[6] E. Kolman and M. Margaliot, "Are artificial neural networks white boxes?," *IEEE Trans. Neural Networks*, vol. 16, no. 4, pp. 844–852, July 2005.

[7] H. Seki, H. Ishii, and M. Mizumoto, "On the property of single input rule modules connected type fuzzy reasoning method," in *Proc. 2007 IEEE Int. Conf. Fuzzy Syst.*, London, UK, pp. 1080–1185, July 2007.

[8] C. J. Mantas and J. M. Puche, "Artificial neural networks are zero-order TSK fuzzy systems," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 3, pp. 630–643, June 2008.

[9] H. Seki, H. Ishii, and M. Mizumoto, "On the generalization of single input rule modules connected type fuzzy reasoning method," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 5, pp. 1180–1187, October 2008.

[10] H. Seki, "On the equivalence of SIRMs connected fuzzy inference method," in *Proc. 2009 IEEE Int. Conf. Fuzzy Syst.*, Jeju Island, Korea, August 2009. (to appear)

[11] N. Yubazaki, J. Yi, and K. Hirota, "SIRMs (Single Input Rule Modules) connected fuzzy inference model," *Journal of Advanced Computational Intelligence*, vol. 1, no. 1, pp. 23–30, 1997.

[12] J. Yi, N. Yubazaki, and K. Hirota, "Upswing and stabilization control of inverted pendulum and cart system by the SIRMs dynamically connected fuzzy inference model," in *Proc. 1999 IEEE Int. Conf. Fuzzy Syst.*, Korea, vol. 1, pp. 400–405.

[13] J. Yi, N. Yubazaki, and K. Hirota, "A proposal of SIRMs dynamically connected fuzzy inference model for plural input fuzzy control," *Fuzzy Sets Syst.*, vol. 125, no. 1, pp. 79–92, 2002.

[14] J. Yi, N. Yubazaki, and K. Hirota, "A new fuzzy controller for stabilization of parallel-type double inverted pendulum system," *Fuzzy Sets Syst.*, vol. 126, no. 1, pp. 105–119, 2002.

[15] J. Yi, N. Yubazaki, and K. Hirota, "Anti-swing fuzzy control of overhead traveling crane," in *Proc. 2002 IEEE Int. Conf. Fuzzy Syst.*, Hawaii, HI, vol. 2, pp. 1298–1303.

[16] S. Khwan-on, T. Kulworawanichpong, A. Srikaew, and S. Sujitjorn, "Neuro-tabu-fuzzy controller to stabilize an inverted pendulum system," in *Proc. 2004 IEEE Region 10 Conf. TENCON 2004*, Chiang Mai, Thailand, vol. D, pp. 562–565.

[17] K. Hayashi, A. Otsubo, S. Murakami, and M. Maeda, "Realization of nonlinear and linear PID controls using simplified indirect fuzzy inference method," *Fuzzy Sets Syst.*, vol. 105, pp.409–414, 1999.

[18] K. Hayashi, A. Otsubo, and K. Shiranita, "Improvement of conventional method of PI fuzzy control," *IEICE Trans. Fundamentals*, vol. E84-A, no. 6, pp. 1588–1592, June 2001.

[19] H. Seki, H. Ishii, and M. Mizumoto, "On the monotonicity of single input type fuzzy reasoning methods," *IEICE Trans. on Fundamentals*, vol. E90-A, no. 7, pp. 1462–1468, July 2007.

[20] M. Maeda and S. Murakami, "An automobile tracking control with a fuzzy logic," (in Japanese) in *Proc. 3rd Fuzzy System Symposium*, pp. 61-66, 1987.

[21] H. Ichihashi and T. Watanabe, "Learning control by fuzzy models using a simplified fuzzy reasoning," (in Japanese) *Journal of Japan Society for Fuzzy Theory and Systems*, vol. 2, no. 3, pp. 429–437, 1990.

[22] M. Sugeno, "On stability of fuzzy systems expressed by fuzzy rules with singleton consequents," *IEEE Trans. Fuzzy Syst.*, vol. 7, no. 2, pp. 201–224, 1999.

[23] M. Sugeno, "An introductory survey of fuzzy control," *Inf. Sci.*, vol. 36, pp. 59–83, 1985.

[24] T. Takagi and M. Sugeno, "Fuzzy identification of systems and its Applications to modeling and control," *IEEE Trans. Syst., Man, Cybern.*, vol. SMC-15, no. 1, pp. 116–132, 1985.

[25] M. Mizumoto, "Fuzzy controls under various fuzzy reasoning methods," *Inf. Sci.*, vol. 45, pp. 129–151, 1988.

[26] M. Mizumoto, *Fuzzy controls under product–sum–grvity method and new fuzzy control methods*, Fuzzy Control Systems (ed. A. Kandel and G. Langholz), CRC Press, pp. 275–294, 1993.

[27] B. Kosco, *Neural Networks and Fuzzy Systems*, Prentice Hall, 1992.

[28] B. -G. Hu, G. K. I. Mann, and R. G. Gosine, "A systematic study of fuzzy PID controllers—function-based evaluation approach," *IEEE Trans. Fuzzy Syst.*, vol. 9, no. 5, pp. 699–712, October 2001.

[29] M. Mizumoto, "Realization of PID controls by fuzzy control methods," *Fuzzy Sets Syst.* vol. 70, pp. 171–182, 1995.

[30] M. Mizumoto, "Fuzzy controls by fuzzy singleton-type reasoning method," in *Proc. 5th IFSA World Congress*, Seoul, Korea, pp. 945–948, 1993.