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Optimal Fuzzy Control of Piezoelectric Systems Based on Hybird Taguchi Method and Particle Swarm Optimization

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***Abstract*—This paper presents a novel evolutionary algorithm based on a hybrid of Taguchi method and particle swarm optimization (PSO), and thus is called HTPSO. First, the non-linear nano-positioning system is approximated by the Takagi-Sugeno (T-S) fuzzy model. Second, the parallel distributed compensation (PDC) is designed to control the piezoelectric system. Last, the parameters of the fuzzy controller are determined by the HTPSO approach. The HTPSO could search the optimal control force to improve the performance of the piezoelectric system. Computer simulation demonstrates the HTPSO possesses the best robustness against external disturbance.**

***Keywords*—T-S fuzzy model, piezoelectric system, Taguchi method, particle swarm optimization.**

I. INTRODUCTION

The nano-positioning control is the kernel technology among the nanoscience. The piezoelectric ceramic materials are often used as actuators in the nano-positioning control system [1]. The advantages of piezoelectric actuator include fast frequency response, small volume, large force, no friction, no backlash, high stiffness, and unlimited resolution, etc [2]. Thus, it is very important to study the control technology of piezoelectric systems.

Inverse piezoelectric effect is a process in electromechanical energy conversion that relates electrical energy to mechanical displacement [3]. The piezoelectricity relates electric polarization to mechanical strain such that the friction between molecules occurs and the response of piezoelectric systems exhibits hysteresis [4]. The hysteresis is a nonlinear phenomenon of passively dissipated energy. The nonlinear relationship between input voltage and output displacement limits the accuracy in nano-positioning control.

To analyze the hysteresis behavior of piezoelectric systems, there have been many researches in founding a mathematical model. The asymmetrical types of hysteretic model include polynomial model, Preisach's model and neural network model. The polynomial model is developed by using piecewise polynomial functions to approximate hysteretic curves [5]. The Preisach's model is built by hysteretic operating factors [6]. The neural network model is established according to input and out data [7]. The symmetrical types of hysteretic model include Maxwell model, Duhem model and Bouc-Wen model. The Maxwell model is developed according to energy parameter law [8]. The Duhem model is built by one order of differential equation [9]. The Bouc-Wen model is established by piezoelectric coefficients and dynamic equation of piezoelectric actuators [10]. In this paper, the Bouc-Wen model is analyzed and studied.

To make sure the positioning error is in the bound of the nanometer, it is necessary to compensate the hysteresis response of piezoceramic systems for the characteristic of nonlinear memory and non-minimum phase. In literature [11], a repetitive control algorithm is developed to design a PID controller for tracking periodic inputs. In literature [12], a self-tuning fuzzy controller is designed for piezoelectric systems. This paper uses the T-S fuzzy model to represent the nonlinear Boun-Wen equation via fuzzy if-then rules. In the following, the parallel distributed compensation (PDC) strategy is applied to design the nonlinear fuzzy controller. To obtain the optimal feedback gains, the hybrid of Taguchi method and particle swarm optimization are employed to search the best parameters in the T-S fuzzy control system.

The particle swarm optimization (PSO) algorithm is a population-based stochastic optimization technique developed by Dr. Eberhart and Dr. Kennedy [13] in 1995. It is inspired by the social behavior of bird flocks and fish schools. The PSO shares many similarities with evolutionary computation techniques such as Genetic Algorithms (GAs). The PSO algorithm is initialized with population of random particles. However, it is unlike GAs. The PSO has no evolution operators such as crossover and mutation. Compared with the GAs, the excellences of PSO are that it is easy to carry out and there are few parameters to adjust. The PSO has been widely used in many applications such as function optimization, fuzzy control system, and neural network training, etc.

However, the performance of PSO is depends on the setting of parameters. It is not easy to determine which the optimal parameters in the PSO are. In this paper, the Taguchi method is introduced into the traditional PSO such that the best parameters could be sought out. The Taguchi method has been widely used in industry such as manufacturing process, power systems, and automatic control engineering [14]. The Taguchi method involves the orthogonal array and the signal to noise ratio analysis that reveals which of the parameters are most effective in reaching the optimal fitness and the directions in

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which these parameters should be adapted to meliorate the performance. The most advantage of the Taguchi method is that it can use only few experiments to achieve the optimal solution. The Taguchi method effectively decreases the number of experiments. Thus, we use the HTPSO, which is a hybrid optimization approach that combines the concept of PSO with the Taguchi method, to design the optimal T-S fuzzy controller.

II. PIEZOELECTRIC SYSTEMS

A. *The Bouc-Wen Model*

This paper uses the Bouc-Wen model to describe the non-linear hysteretic curve of piezoelectric systems. The mathematical equations are described as follows:

$$\begin{cases} m\ddot{x} + b\dot{x} + kx = k(du - h) + \rho \\ \dot{h} = \alpha d\dot{u} - \beta|\dot{u}|h - \gamma\dot{u}|h| \\ \rho = kx_0 \end{cases} \quad (1)$$

where $\rho = kx_0$; m, b, k, d, and u are the tangent mass, damping, stiffness, effective piezoelectric coefficients, and input voltage of piezoelectric actuator. The α, β and γ are constants that control the shapes of the hysteretic curve. The ρ is a pre-compression strength. The displacement of piezoelectric actuator is x, and the variable h is from the nonlinear hysteresis equation.

To find these parameters, a voltage signal is inputted to the piezoelectric systems. The voltage signal is a triangular wave with amplitude 100 voltages and frequency 10Hz. After several experiments and computer simulations, the parameters of piezoelectric systems are identified as: m=0.148 Kg, b=129.5 N-s/m, $k = 3\times10^6$ N/m, $d = 3.5\times10^{-7}$, α=0.5, β=0.023, and γ=0.01.

B. *The T-S Fuzzy Model*

Then, the non-linear hysteretic mathematical equation is linearized by T-S fuzzy model. In order to approximate the Bouc-Wen equation, the state-space equation (2) is applied to represent the hysteretic dynamics. This paper uses three fuzzy rules to approach the non-linear mathematical model.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & -\frac{k}{m} \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{kd}{m} & 0 \\ 0 & \alpha d \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$+|u_2|\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\beta \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + u_2\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\gamma \end{bmatrix}\left|\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right| \quad (2)$$

$$y = [1 \ 0 \ 0]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

where $x_1 = x$, $x_2 = \dot{x}_1 = \dot{x}$, $x_3 = h$, input $u_1 = u$, and $u_2 = \dot{u}$.

Fig. 1 shows the triangular membership functions. The rules of the fuzzy model are shown as follows:

Model Rule 1 (M1):

IF $x_2(t)$ is about 0

THEN

$$\begin{aligned} \dot{x}(t) &= A_1 x(t) + Bu(t) \\ y(t) &= [1 \quad 0 \quad 0]x(t) \end{aligned} \quad (3)$$

Model Rule 2, 3 (M2, M3)

IF $x_2(t)$ is about $\pm 0.7e-3$

THEN

$$\begin{aligned} \dot{x}(t) &= A_2 x(t) + Bu(t) \\ y(t) &= [1 \quad 0 \quad 0]x(t) \end{aligned} \quad (4)$$

where $x(t) \in R^3$ is the state vector; $x_1 = x$, $x_2 = \dot{x}_1 = \dot{x}$, $x_3 = h$; $u(t) \in R^1$ is the input vector; $u_1 = u$, $y(t) \in R^1$ is the output vector. The subsystems matrices are defined as follows

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & -\frac{k}{m} \\ 0 & \frac{\alpha d u_2}{x_2(t)} & -(\beta|u_2| + \gamma u_2) \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & -\frac{k}{m} \\ 0 & 2\frac{\alpha d u_2}{x_2(t)} & -2(\beta|u_2| + \gamma u_2) \end{bmatrix} \quad (5)$$

$$B = \begin{bmatrix} 0 \\ \frac{kd}{m} \\ 0 \end{bmatrix}$$

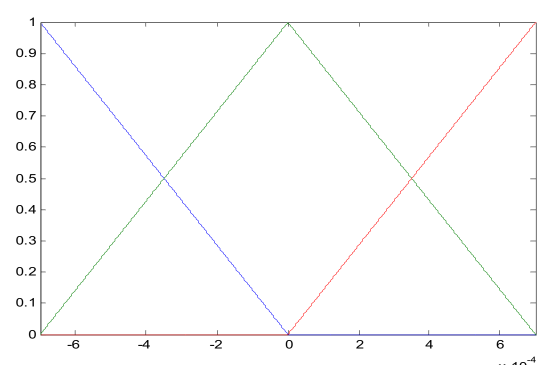

Figure 1. The membership functions.

III. THE HTPSO ALGORITHMS

A. The Parallel Distributed Compensation

Using the concept of parallel distributed compensation, the PDC controller is designed as

IF $z_1(t)$ is M_i,

THEN

$$u(t) = -F_i x(t), \quad i = 1, \ldots, 3. \tag{6}$$

where F_i is the state feedback gain.

The T-S fuzzy controller is expressed as follows:

$$u(t) = -\sum_{i=1}^{3} h_i(z(t)) F_i x(t) \tag{7}$$

To guarantee the nonlinear system is global stable, the lemma is proposed as follows [15]

Lemma: The closed-loop T-S fuzzy system is asymptotically stable if there exist positive-definite matrices P such that

$$\begin{aligned} & G_{ii}^T P + P G_{ii} < 0, \\ & \left(\frac{G_{ij} + G_{ji}}{2}\right)^T P + P\left(\frac{G_{ij} + G_{ji}}{2}\right) \le 0, \\ & \qquad i < j \text{ s.t. } h_i \cap h_j \ne \phi \end{aligned} \tag{8}$$

where $G_{ii} = A_i - B_i F_i$, $G_{ij} = A_i - B_i F_j$

B. The Particle Swarm Optimization

In recent years, the PSO algorithms have been widely used in many applications and research. It is an effective optimization method that belongs to the category of swarm intelligence. The PSO implements the simulation of social behavior. It has been demonstrated that the PSO algorithms get better performances as compared with other evolutionary computation techniques.

The PSO is inspired by particles moving in the searching space. Each particle is a candidate solution to the optimization problem. These particles, which are metaphor of bird flocks, are randomly initialized and fly through multi-dimensional space. During the movement, these particles update its velocity and position based on the experience of its own and the whole population. The updating procedure will drive the particle swarm to move toward region with better fitness value, and finally every particle is gathered around the point with the best fitness value. The operations of PSO are detailed as follows:

Step 1: Initialization

The initial velocity and initial position of every particle is randomly set in a pre-specified range.

Step 2: Updating Velocity

In each iterative procedure, the velocity of every particle is updated according to the following rule:

$$\begin{aligned} v_i^{new} = k * [v_i^{old} + c_1 * rand_1 * (p_i - x_i) \\ + c_2 * rand_2 * (p_g - x_i)] \end{aligned} \tag{9}$$

where the v_i and x_i are velocity and position of particle *i*, respectively; the p_i and p_g are the position with best fitness value so far by particle *i* and the whole population, respectively; the k is the constriction factor; the $rand_1$ and $rand_2$ are random variables from the range [0,1]; the c_1 and c_2 are the cognition parameter and the social parameter, respectively. After the updating, the v_i^{new} should be checked and clamped to pre-specified range.

Step 3: Updating Position

The position of every particle is updated according to the following rule:

$$x_i^{new} = x_i^{old} + v_i^{new} \tag{10}$$

After the updating, the x_i^{new} should be checked and clamped to pre-specified range to ensure legal solutions.

Step 4: Updating Memory

Update p_i and p_g when the following condition is met:

$$p_{i,best} \leftarrow p_i \quad if \quad f(p_i) > f(p_{i,best}) \tag{11}$$

$$p_{g,best} \leftarrow p_g \quad if \quad f(p_g) > f(p_{g,best}) \tag{12}$$

Step 5: Checking Termination

The step 2 to step 4 are repeated until the termination condition is matched. Once the stopping criterions are satisfied, the PSO will report the $p_{g,best}$ and $f_{g,best}$ as its solution.

C. The HTPSO Algorithms

To obtain the best parameters in PSO, such as cognition parameter, social parameter, iteration number, and population size, the Taguchi method is applied. Fig. 2 shows the flowchart of the HTPSO algorithm. The HTPSO is a hybrid optimization approach that combines the concept of PSO with the Taguchi selection method.

The orthogonal array (OA) and the signal to noise ratio (SNR) are two main tools in the Taguchi method that can decrease the number of experiments during optimization process. The OA has a fractional factorial characteristic. Each row describes the number of experiments and each column denotes the level of factors in the OA. The corresponding SNR is calculated by the fitness value

$$\eta = -10 \log(Fitness) \tag{13}$$

In this HTPSO design for the piezoelectric systems, Table I lists the corresponding parameters and their levels. Table II lists the $L_9(3^4)$ OA which represents 9 experiments with 4 three-level factors. Table III lists the response table. The fitness function is defined as

$$fitness = \min\left\{\sum_{l=1}^{n}[(p_i - x_{id}) + (p_g - x_{id})]\right\} \quad (14)$$

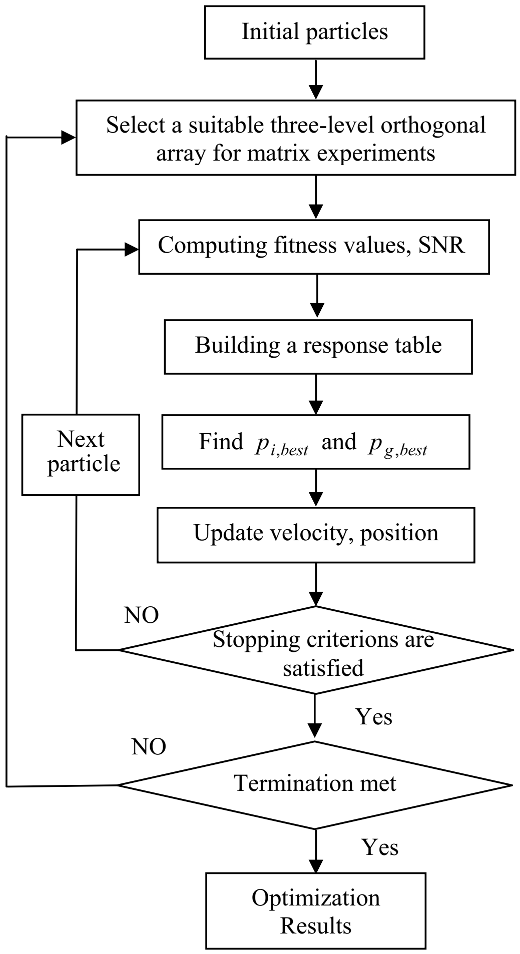

Figure 2. The flowchart of the HTPSO algorithm.

TABLE I. FACTORS AND LEVELS

	Level 1	Level 2	Level 3
A Cognition Parameter	2	2.1	2.2
B Social Parameter	2.2	2.1	2
C Population Size	60	80	100
A Generation Number	100	150	200

TABLE II. ORTHOGONAL ARRAY

Experiment Number	A	B	C	D	SNR
1	1	1	1	1	η_1
2	1	2	2	2	η_2
3	1	3	3	3	η_3
4	2	1	2	3	η_4
5	2	2	3	1	η_5
6	2	3	1	2	η_6
7	3	1	3	2	η_7
8	3	2	1	3	η_8
9	3	3	2	1	η_9

TABLE III. RESPONSE TABLE

Response Value				
R1	A1	B1	C1	D1
R2	A2	B2	C2	D2
R3	A3	B3	C3	D3

The response values are defined as follows:

A1=($\eta_1+\eta_2+\eta_3$)/3, A2=($\eta_4+\eta_5+\eta_6$)/3, A3=($\eta_7+\eta_8+\eta_9$)/3

B1=($\eta_1+\eta_4+\eta_7$)/3, B2=($\eta_2+\eta_5+\eta_8$)/3, B3=($\eta_3+\eta_6+\eta_9$)/3

C1=($\eta_1+\eta_6+\eta_8$)/3, C2=($\eta_2+\eta_4+\eta_9$)/3, C3=($\eta_3+\eta_5+\eta_7$)/3

D1=($\eta_1+\eta_5+\eta_9$)/3, D2=($\eta_2+\eta_6+\eta_7$)/3, D3=($\eta_3+\eta_4+\eta_8$)/3

IV. COMPUTER SIMULATION

After the HTPSO algorithms are implemented, the best parameters of PSO are searched as the cognition parameter: 2.1, the social parameter: 2.2, the population size: 60, and the generation number: 200. The optimal control gains are obtained as follows:

$$F1=10^9\times[1.638959 \quad 0.000015 \quad -0.001015]$$

$$F2=10^9\times[1.998194 \quad 0.000053 \quad -0.003125]$$

$$F3=10^9\times[1.133887 \quad 0.000149 \quad -0.002356]$$

A PSO algorithm is applied for comparison, the corresponding control gains are obtained as follows:

$$F1=10^9\times[2.000000 \quad 0.000100 \quad -0.000819]$$

$$F2=10^9\times[2.000000 \quad 0.000037 \quad 0]$$

$$F3=10^9\times[2.000000 \quad 0.000080 \quad 0]$$

Let the reference displacement command be a unit step signal with size of 10 μm . Fig. 3 and Fig. 4 show the simulation results of the PSO controller and the HTPSO controller, respectively. Fig. 5 and Fig. 6 show the control voltage of the PSO controller and the HTPSO controller, respectively. Table IV lists the performances of different controllers. The IAE means integral absolute error and the ITAE means integral of time multiplied absolute error. Moreover, the ISE represents integral square error and the ISTE represents integral of time multiplied square error, respectively. The equations of IAE, ITAE, ISE and ITSE are defined as:

$$\text{IAE} \equiv \lim_{t\to\infty} \int_0^t |e| dt$$

$$\text{ITAE} \equiv \lim_{t\to\infty} \int_0^t |e| \cdot t dt$$

$$\text{ISE} \equiv \lim_{t\to\infty} \int_0^t e^2 dt$$

$$\text{ITSE} \equiv \lim_{t\to\infty} \int e^2 \cdot t dt$$

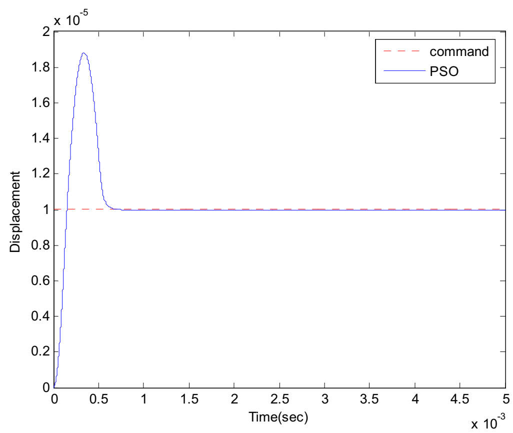

Figure 3. Step response of the PSO controller.

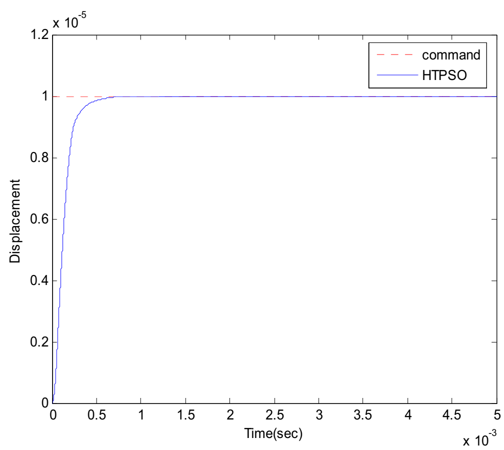

Figure 4. Step response of the HTPSO controller.

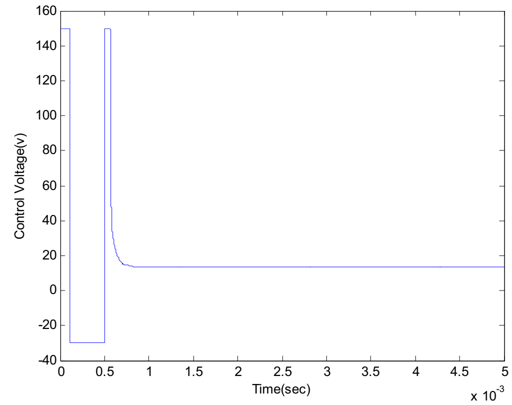

Figure 5. Control voltage of the PSO controller.

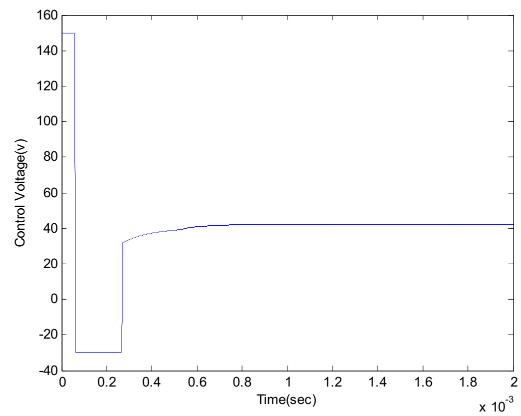

Figure 6. Control voltage of the HTPSO controller.

TABLE IV. COMPARSION OF PERFORMANCE

	PSO controller	TPSO controller
Rise time(ms)	958	0.199
Maximal overshoot (%)	88.0132	0
Settling time(ms)	0.61	0.4564
Steady-State Error(nm)	200.29	14.9
IAE(nm)	3.3104	1.445
ITAE(nm)	0.00095	0.00031
ISE(nm)	0.00002	0.000008
ITSE(nm)	0	0

In order to analysis the robustness of the PSO controller and TPSO controller, the disturbance of pulse wave is added to the piezoelectric systems during 0.001 sec to 0.0015 sec. The amplitude of the pulse wave is 300(V). Fig. 7 shows the step response of the PSO controller under external disturbance. Fig. 8 shows the step response of the TPSO controller under the same disturbance. Table V lists the robust performances of different controllers.

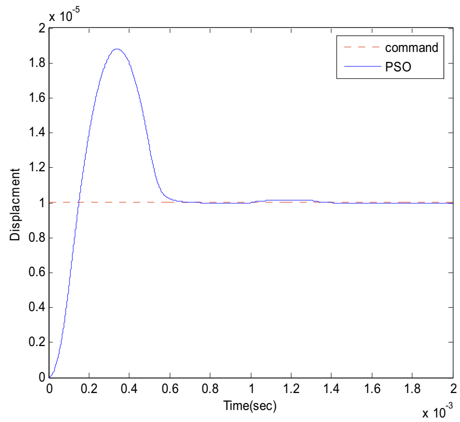

Figure 7. Step response of the PSO controller under disturbance.

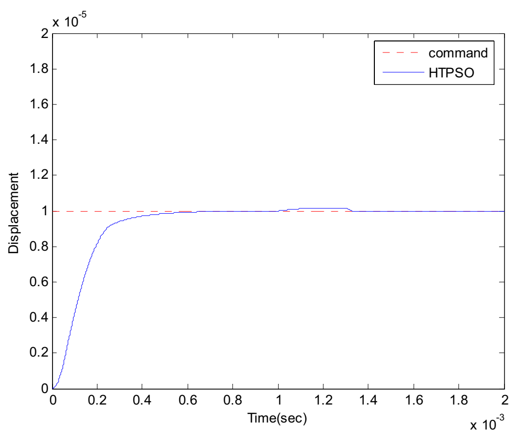

Figure 8. Step response of the HTPSO controller under disturbance.

TABLE V. COMPARSION OF ROBUST PERFORMANCE

	PSO	TPSO
Steady-State Error(nm)	200.29	19.3
IAE(nm)	3.3302	1.4395
ITAE(nm)	0.0009	0.00021
ISE(m)	0.00002	0.000008
ITSE(m)	0	0

V. CONCLUSION

The optimal T-S fuzzy controller has been designed for the piezoelectric systems. The best feedback gains are searched by the HTPSO algorithms that combine the concept of PSO with the Taguchi method. Computer simulations demonstrate that the HTPSO controller owns the robustness against the external disturbance.

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