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# Modeling and evaluation of harbor crane work

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*Abstract*—**Currently there is a strong demand of increasing the efficiency of the work chain in container harbors. Several methods aiming at improving the logistics of the container yard have been proposed. Nevertheless, the efficiency of an individual container crane has not been investigated, even though it has a great impact on the efficiency of the whole work chain. This paper proposes a method for modeling and evaluation of the work of a single harbor crane. In the method, the work is first divided into comprehensible tasks. A sequence of tasks produces a work cycle. The work cycle is modeled by a Hidden Markov Model (HMM). Each task of the work cycle corresponds to a state of the HMM. The HMM recognizes the states of the work cycle by using measurements readily available in the database of the crane. The resulting task time distribution directly reveals the bottlenecks of the work.**

## I. Introduction

Human-machine systems have been investigated at least over 30 decades. Interactions between humans and computers have become natural parts of our everyday lives [1]. Understanding the basic interactions between human and machine facilitates the development of intelligent human-machine systems. In industry a human operator has a significant role, for example, in efficiency, fuel consumption and quality of the product.

Sea transportations have increased significantly over the past years and the trend seems similar in the future. For example in Finland, the container transportation is estimated to almost threefold over the next 22 years [2]. One essential reason is that the container transportation provides an inexpensive way to transfer goods. This increment has grown both container ships and container harbors. Due to the increased traffic, the work efficiency at the harbors has become crucial. As the cranes and the ships are already powerful enough for efficient work, one would not benefit much by building more powerful equipment. In the harbor, several machines work together in a chain. All of which usually have human operators in control. Each part of the logistic chain need to work seamlessly in order to obtain the optimal performance. Therefore, a method for evaluating the performance of a single harbor crane is needed. Once each machine is equipped with the performance evaluation method, the efficiency of the whole work chain can be monitored and optimized.

Currently the harbor operators have measurements about how many containers are transfered between ship and shore per hour. This is often the most important number, since it is usually used as a basis for billing. If the container transfer rate is not at a desirable level, there are no methods to analyze the work efficiency. Therefore, this paper aims at finding a method to provide more specific information about the work efficiency of the individual vehicles at the harbor. Well-defined work phases and time distribution of the phases would reveal additional information about the nature of the work of the cranes.

In this paper a structure of a work cycle of a Ship-to-Shore container crane (STS) is constructed. The work cycle consists of tasks which are comprehensible for a human operator. The work cycle is modeled with a Hidden Markov Model (HMM) so that each state of the HMM corresponds to a task. By using the available measurement information from the database of the crane, the tasks of the work cycle are recognized by the proposed HMM. The state sequence given by the HMM can be used to calculate a task time distribution of the crane. The task time distribution provides valuable information about the crucial phases of the work. In addition, an indicator of the goodness of the modeling is formulated. The indicator reveals if the model has succeeded to model the work cycle properly. The method can be adapted easily to model the work of other crane types as well.

This paper is organized as follows. A literature review of the related research is given in Section II. The structure of the work cycle of the STS, and the HMM-based work cycle recognition method are introduced in Section III. In addition, the section proposes a measure for recognition of anomalous work cycles based on state transition patterns. The experimental setup and the results are analyzed in Section IV. Section V concludes the paper.

## II. Related Work

Several methods for efficient container handling at container terminals have been proposed. Space allocation [3], remarshaling of the containers [4], [5], [6], best combination of space and the number of yard cranes [7], [8] and estimations of the required handling of the containers [9] have been studied. Efficient container yard enables the STS to work efficiently while loading and unloading ships.



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Anti-sway of the crane have also been widely investigated. At least a nonlinear controller [10], a simple proportional derivative (PD) controller [11], a fuzzy logic control system with Linear Quadratic Gaussian Control [12], a nonlinear tracking control [13], an output feedback PD controller [14], and nonlinear control schemes [15] have been applied to control the sway of the cranes. Efficient and fast working of the STS can be made easier by different control aids.

The methods developed earlier to increase the efficiency of the harbor work do not take into account the effect of the human operator. This can be overcome by using the work cycle recognition. The HMM-based work cycle recognition of forestry machines, harvesters and forwarders, has been studied with good results [16]. In addition, the HMM has been applied in several other areas, such as speech recognition [17] [18], fault detection [19] and skill evaluation [20]. Skill evaluation of human operators based on the task sequence recognition by the HMM has been studied [21]. Due to the success in analyzing the work cycles of the forestry machines and the skills of the machine operators, the HMM is a promising tool in analyzing the harbor crane work as well.

## III. Work cycle modeling method

### A. Description of the work cycle

The STS is designed to load and unload ships at the harbor. When a ship arrives to the harbor, the STS unloads and loads containers between ship and shore. Containers are moved between the STS and the container yard by straddle carriers (SC) or road trucks (RT). Containers wait for delivery at the container yard. Container yard is managed by Rubber Tired Gantry cranes (RTGs), Rail Mounted Gantry cranes (RMGs) or straddle carriers. From the container yard the containers are delivered to another ship or to the land transportation. Fig. 1 illustrates vehicles and cranes at the harbor. The containers are colored red and the container cranes and the vehicles black. In addition to all the vehicles, there are also staff on the land who assist the crane operators to carry out their jobs.

The work of the STS is periodic, which makes it possible to construct a work cycle for the STS. In short, the work cycle includes the following tasks: moving to a container, catching the container, moving with the container and placing the container. A more detailed partition, shown in Fig. 2, was made to reveice additional information about the work of the STS. There are some boundary conditions to the tasks, i.e. the states, of the work cycle. They need to be discrete and comprehensible for the crane operator. In addition, the states of the work cycle should occur in a logical and fixed order during normal work. To be able to separate adjacent states of the work cycle, the states need to have dissimilar characteristic set of events.

### B. HMM applied to the crane

The HMM is an extension of a Markov model. In the HMM the states of the Markov model cannot be observed, i.e. the states are hidden. Thus, the state process is observed only through a stochastic process which produces the observations.

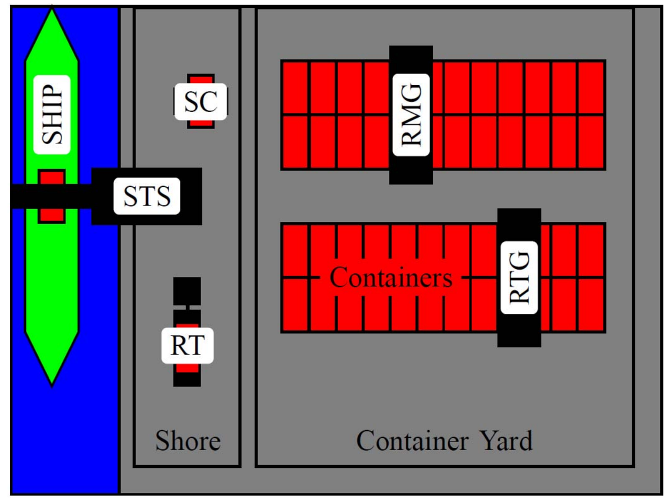


Figure 1. The vehicles and the cranes for container handling at the harbor.

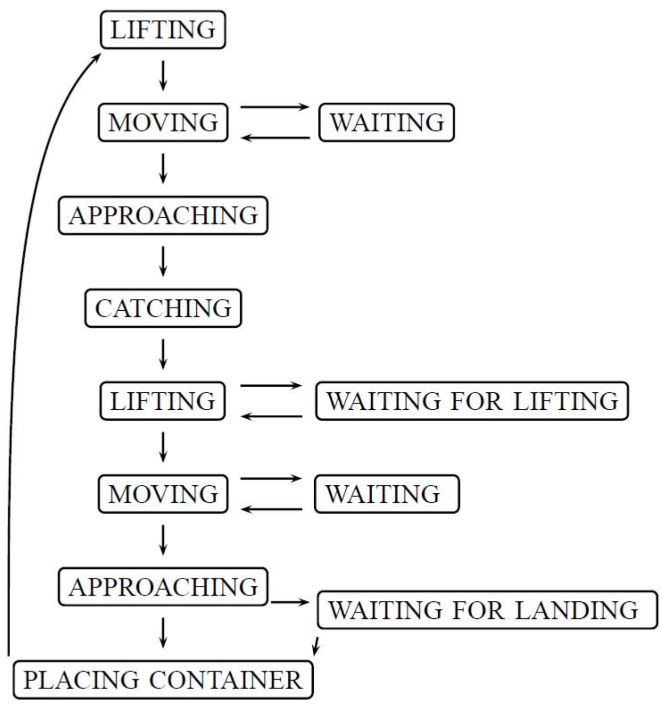


Figure 2. A flow chart of the model. Only the most probable state transitions are shown.

Each state has a probability distribution for the observations to appear. Hence, the most probable state sequence can be calculated based on the sequence of observations. [18]

A HMM can be defined as follows [18]:

1) $N$, the number of the states. Individual states are referred as $S = \{S_1, S_2, \ldots, S_N\}$ and a state at time $t$ is $q_t$.
2) $M$, the number of observation symbols per state. Individual symbols are marked as $V = \{v_1, v_2, \ldots, v_M\}$ and a sequence of observations as $O = O_1O_2 \ldots O_T$.
3) The state transition probability from state $S_i$ to state $S_j$ is $A = \{a_{ij}\}$, where
$a_{ij} = P[q_{t+1} = S_j | q_t = S_i], \quad 1 \leq i, j \leq N$

4) The observation symbol probability distribution in state $j$ is $B = \{b_j(k)\}$, where
$b_j(k) = P[v_k \text{ at } t|q_t = S_j], \quad 1 \leq 1 \leq N, \ 1 \leq k \leq M$
5) The initial state distribution is $\pi = \{\pi_i\}$, where
$\pi_i = P[q_1 = S_i], \quad 1 \leq N$

For convenience, the complete parameter set of the model is often notated as $\lambda = (A, B, \pi)$.

The tasks of the work cycle correspond to the states of the HMM. Observations for the HMM are generated from raw data of the crane. The parameters $\lambda = (A, B, \pi)$ are trained by the Viterbi re-estimation algorithm [22]. Once the parameters are trained, the observations and the HMM-model can be used to obtain the most probable sequence of states by the Viterbi algorithm [23].

The STS has two different basic task types; loading and unloading ships, and work cycles are divided into these task types. Both task types have same structure. Thus, only one parameter set $\lambda$ is required.

### C. Recognition of anomalous work cycles

The model does not always produce a decent state path according to the constructed flow chart shown in Fig. 2. Thus, a method is needed to separate properly and improperly modeled work cycles. The HMM produces a logarithmic probability (LogP) which is used to choose the most probable state path for the given observations. It is defined as follows

$$\text{LogP} = \sum_{i=1}^{n} \log(b_{q_i}(O_i)) + \sum_{i=1}^{n-1} \log(a_{q_i,q_{i+1}}) \qquad (1)$$

Thus, it reveals the mutual order of all possible state paths for the given observations $O$. Nevertheless, it does not provide information about the goodness of the best state path according to the flow chart in Fig. 2.

A state transition probability (StateP) was constructed as an indicator to recognize improperly modeled work cycles. It is defined as follows

$$\text{StateP} = \sum_{i=1}^{n-1} \log(a_{q_i,q_{i+1}} \mid q_i \neq q_{i+1}) \qquad (2)$$

Thus, StateP adds only logarithmic state transition probabilities where the following state $i+1$ differs from the preceding state $i$. It notices event probabilities $b_{q_t}(O_t)$ only indirectly, i.e. if observations induce a state transition.

A threshold can be defined for StateP, so that all properly modeled work cycles can be identified, since

$$MinProperPath - MaxImproperPath > \zeta \qquad (3)$$

where MinProperPath is the smallest probability to obtain a decent state path, MaxImproperPath is the greatest probability to obtain an improper state path and $\zeta > 0$. The threshold can be chosen from range $(MaxImproperPath, MinProperPath)$.

## IV. Experimental results

### A. Setting up the experiment

During normal operation, a large number of variables is measured and saved to the database of the STS. The raw data include control signals of the STS operator, and position information of the crane. Time resolution of the data is very high, which enables good temporal resolution for the model.

Raw data from two different cranes from two different harbors were available for both training and testing the model. In the following, the cranes are referred as Harbor 1 and Harbor 2. Containers are transported from the STS to the container yard by straddle carriers in the Harbor 1, and by road trucks in the Harbor 2. One month data from the Harbor 2 were selected as training data to obtain the parameter set $\lambda$. Thus, one month data from the Harbor 2 and two months data from the Harbor 1 were available for testing the model.

The prerequisite for the work cycle recognition by HMMs is to extract the observations for the model. The events need to describe the features of the tasks in the work cycle. Therefore, one needs to become familiar with the work of the crane as well as the available measurements in order to obtain decent events. Events and their time stamps for the HMM are constructed from raw data of the crane and they are combined into a one dimensional observation vector. The events are then fed to the HMM algorithm, which produces the state path, the task time distribution and StateP of the work cycle. Because the event definitions depend highly on the available measurements and the measurement resolution, this paper does not provide a detailed description of the events used in the experiments.

### B. Modeling the work cycle

In order to obtain a good description of the work cycle, two visits to the Harbor 1 were arranged to learn the harbor environment and the work of the STS. As the main duty of the STS is to load and unload ships, as mentioned in Section III, only these two task types were considered.

The work cycle of the STS can be described as follows. First, the STS moves to the container. When approaching the container, more skills are usually required from the operator. The spreader needs to hit the container close enough so that the container can be caught. After the container is attached, the STS starts to move to the landing place of the container. Most of the distance the operator drives with full speed, but when approaching the landing place, more careful steering is needed. Finally, the STS places the container to a desired location.

The work of the STS was divided into 12 states as shown in Fig. 2. In addition, the work cycle can be categorized in six basic tasks: lifting, moving, waiting, approaching, catching/placing the container and waiting for lifting/landing. A time distribution of the six basic tasks shows what are the most time consuming tasks. More detailed time distribution of the 12 states specifies, for example, the nature of the detected waiting times.

To achieve appropriate HMM parameters $\lambda$, the initial values $\lambda_0$ for the Viterbi re-estimation algorithm need to be chosen carefully. The reason is that the parameter estimation algorithm can converge to several local minima. Thus, the initial values define which parameter set is achieved. Appropriate state path in Fig. 2 provided a good starting point to the initial matrix $A$ in $\lambda$, since the allowed transitions are known. One state generates usually several events, so all $a_{ii}$ are close to one in matrix $A$, except the last diagonal element $a_{12,12}$, which is 1. Thus, probabilities in the diagonal of the matrix $A$ are set close to one and smaller probabilities for the other allowed transitions. Forbidden transitions have zero probabilities in the training phase. Matrix $B$ includes the event probabilities for every state. When combining the state descriptions and the event descriptions, the initial values for matrix $B$ can be formulated. The sum of an individual row in matrix $B$ is one. Probabilities for allowed events in a row are uniformly distributed:

$$\begin{aligned} &\beta_{ij} \sim Unif(0,1) \\ &\sum_{j}^{n} \beta_{ij} = 1 \end{aligned} \tag{4}$$

Matrices $A$ and $B$ obtained by the Viterbi re-estimation algorithm need to be modified before they can be used in real modeling. A small probability, $\epsilon$, is substituted in the illicit state transitions in the lower triangular matrix of $A$. In addition, all zero probabilities in matrix $B$ are substituted with the same small probability $\epsilon$. This construction prevents the HMM from stopping during the Viterbi algorithm, for example because of an undesired raw data or an unorthodox work cycle. In addition, small probabilities in matrices $A$ and $B$ ensure that the information, when the model successfully models the work cycle, is obtained.

### C. Recognition of anomalous work cycles

In order to separate anomalous work cycles from normal work, a threshold for StateP needs to be defined. The threshold must fulfill the criterion (3). StateP was calculated for every work cycle and Fig. 3 shows StateP values of the modeled work cycles in both harbors. There is a clear gap between about $-5.80$ and $-4.30$. This might be a good threshold for the properly/improperly modeled work cycles. Nevertheless, this threshold has to be confirmed reliably. An algorithm was generated to determine if a state path of an individual work cycle produced by the model corresponds to the flow chart in Fig. 2. As a result, work cycles can be categorized as properly or improperly modeled and compared to StateP values of the work cycles.

The results were that the smallest StateP for the properly modeled work cycles was $-4.29$ and the greatest StateP for the improperly modeled work cycles was $-5.89$. Thus, properly modeled work cycles always produce greater StateP values than $-4.30$ and improperly modeled work cycles produce always smaller StateP values than $-5.80$. Value $-4.50$ from the range $(-5.80, -4.30)$ was chosen as the threshold for StateP in the model to categorize properly and improperly modeled work cycles.

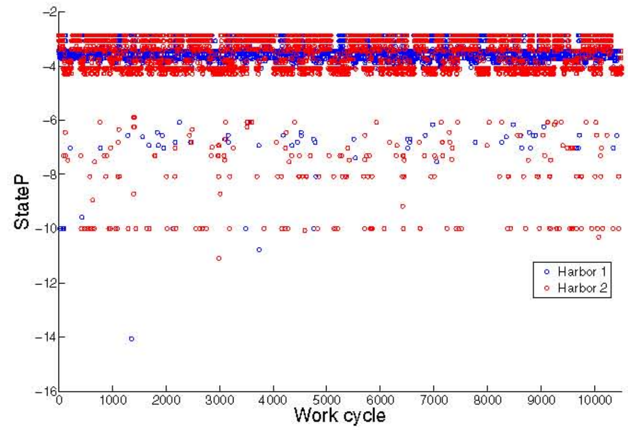


Figure 3. StateP values of the work cycles.

### D. Model validation

StateP informs if the model has succeeded to produce a decent state path based on the flow chart in Fig. 2. Nevertheless, the state path needs to be compared also to the actual work of the STS. This comparison confirms if the state path corresponds to the real work phases of the STS in a desired manner. Thus, the results of the modeling were verified visually.

Video material of the work of the STS was recorded from four different angles. A presentation from the four video signals and subtitles showing the current state of the work cycle produced by the model, was edited. A screen shot from the presentation is provided in Fig. 4. The plot in the upper left shows how the operator controls the crane, and the upper right plot shows the view to the container from the trolley. Lower plots provide more general views of the crane, the ship and the straddle carriers. The current state of the work cycle by the HMM is at the bottom. In this manner the state path produced by the model can be compared to the movements of the crane during the work cycle.

The presentation confirms that the model produces state paths which match nicely to the actual work of the STS.

### E. Analysis of task time distribution

Based on the classification given by StateP, the HMM managed to model from 97.51 to 99.7% of the detected work cycles properly. Coefficients of determinations in the Harbor 1 and 2 are presented in Table I. According to StateP only small percentage of the detected work cycles were improperly modeled.

The most interesting results enabled by the developed model are the task time distributions of the work cycles. Fig. 5 presents the average task time distributions for loading and unloading task types from the Harbor 1. Fig. 6 illustrates same distributions from the Harbor 2.

The task time distributions for both task types have approximately the same structure in both harbors. When considering the six basic tasks, moving, waiting and approaching are the

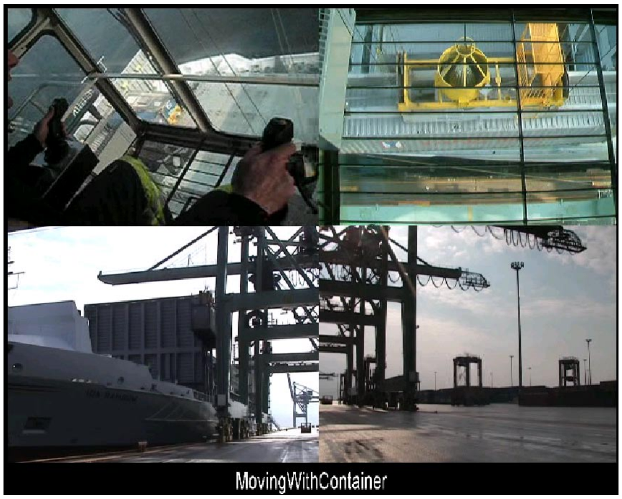


Figure 4. A screen shot from the video presentation.

Table I
COEFFICIENTS OF DETERMINATIONS IN THE HARBOR 1 AND 2.

| Harbor | Task type | Containers | Properly modeled | % |
|---|---|---|---|---|
| Harbor 1 | Unloading | 5222 | 5208 | 99.73 |
| | Loading | 5032 | 4989 | 99.15 |
| Harbor 2 | Unloading | 2849 | 2778 | 97.51 |
| | Loading | 3217 | 3147 | 97.82 |

three main time consumers taking from 79% to 83% of the total work cycle time.

Total task time durations of the task types in the Harbor 1 are 144 s for unloading and 162.5 s for loading. In the Harbor 2 total task time durations of the task types are 223.9 s for unloading and 192.3 s for loading. It is evident that work in the Harbor 1 is quicker than in the Harbor 2, since unloading consumes 79.9 s and loading 29.8 s more in the Harbor 2. Examination of the six basic tasks reveals, that during the unloading process waiting takes 39.3 s, approaching 20.6 s and catching/placing the container 17.8 s more in the Harbor 2, in total 77.7 s. This explains the total difference, 79.9 s almost completely. The same review for the loading task type shows that approaching takes 14.3 s and catching/placing the container 13.2 s more in the Harbor 2, in total 27.5 s. This number is close to the total difference, 29.8 s.

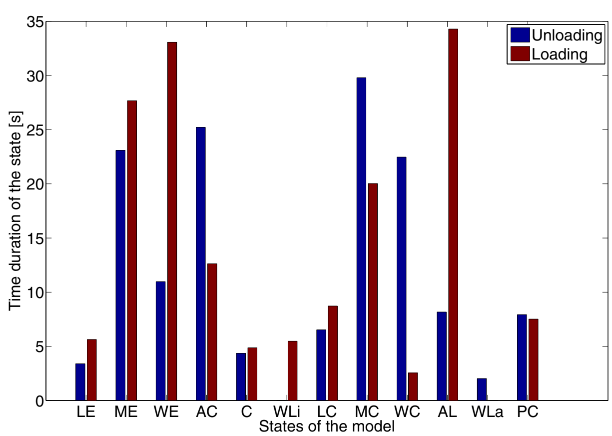


Figure 5. Task time distributions in the Harbor 1.

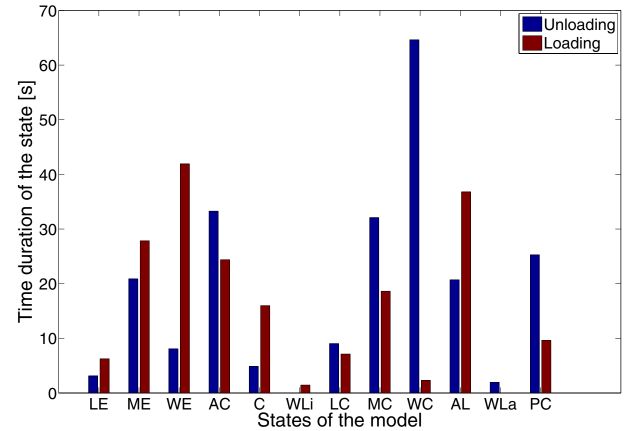


Figure 6. Task time distributions in the Harbor 2.

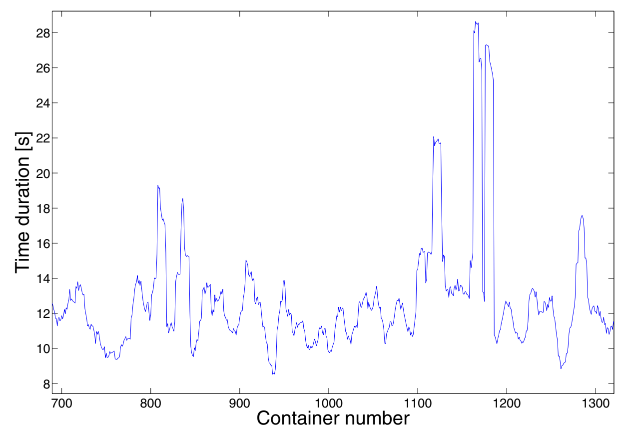


Figure 7. Moving average of the duration of the Approaching Container state in the Harbor 1.

In the Harbor 2 the crane waits 29.0 s longer per container than in the Harbor 1. Greater waiting times in the Harbor 2 suggest, that rationalization of the work chain might improve the performance, since slow container transportation between the STS and the container yard increases waiting times of the STS. Road trucks could explain the greater approaching and catching/placing the container times at the shore. Nevertheless, road trucks do not explain greater approaching time at the ship side.

Comparison between two harbors illustrates differences between working methods. The results reveal that straddle carriers and road trucks affect the working techniques of the STS. As a result, the task time distributions of the different working techniques are different. Unfortunately, comparison data of other possible environmental aspects, which could influence to the efficiency of the STS, were not available. For example, type of the ship and more extreme weather conditions could affect to the task time distribution and the total work cycle time.

In addition to the average task time distributions of the work cycles, also a time consumption of an individual state can be investigated over time. A crane operator and a task type of the work cycle affect to the time durations of the individual states. In Fig. 7 moving average of the time duration of the Approaching Container state in the Harbor 1 is presented. Moving average is calculated from 10 previous work cycles. In some periods the average is below 10 s, while there are periods where the average is around 14 s. Thus, for example, the impact of the operator and the task type can be examined based on the time durations of the individual states.

## V. Conclusion

The working phases of the harbor crane were studied and modeled using the HMM. Also an indicator for the separation of the proper and improper work cycles, StateP, was developed. StateP takes into account only state transitions where the following state differs from the preceding state. The HMM produces the task time distribution of the work cycle.

Raw data from two STS from different harbors were used to initialize the parameters of the model and to validate the model. The results obtained by using raw data from two different working environments show that the model is generalizable. The experimental results show that the model managed to model the work cycle of the STS remarkably well. Only from $0.25\%$ to $2.5\%$ of the detected work cycles were improperly modeled. The task time distributions reveal that from the six basic tasks waiting, moving and approaching correspond approximately $80\%$ of the total work cycle time. Also differences between crane operators and harbor environments were observed.

The time distributions of the work cycles provide valuable information about the character of the work of the STS. Also the efficiency of the STS can be measured. Different crane operators and harbors can be compared. Moreover, if the modeling is extended to other harbor cranes, also the efficiency of the different harbor cranes can be compared. Harbor operators can optimize the whole work chain in the harbor, since the bottlenecks of the work can be spotted. In addition, the machine operators' skills can be evaluated based on the proposed model. The machine operators can also compare their task time distribution to an ideal task time distibution and optimize their work.

Because the state transitions of the model were validated only qualitatively, a more detailed quantitative study is needed. State transitions . Also the detection algorithm of the work cycles needs to be validated in more detail. Tools for the harbor operators can be developed based on the task time distributions.

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