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Game Theoretic Validation of Air Combat Simulation Models

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***Abstract*—The paper presents a new game theoretic approach towards the validation of discrete event air combat simulation models. In the approach, statistical techniques are applied for estimating game models based on simulation data. The estimation procedure is presented in cases involving games with both discrete and continuous decision variables. The validity of the simulation model is assessed by comparing the properties of the estimated games to actual practices in air combat. The approach enhances existing methods for the validation of discrete event simulation models by incorporating the inherent game setting of air combat into the analysis. The estimated games also provide a novel game theoretic perspective to simulation metamodeling which is used to facilitate simulation analysis. The utilization of the game theoretic approach is illustrated by analyzing simulation data produced with an existing air combat simulation model.**

***Index Terms*—Air combat, discrete event simulation, game theory, military decision making, validation.**

I. Introduction

The application of constructive simulation is often the most convenient as well as the least money and time consuming way to obtain information about the performance of systems used in air combat (AC) or the value of new ways for conducting air combat missions [1], [2]. A realistic AC simulation model requires components representing aircraft, weapons, radars, and other apparatus. The simulation model has to also adequately represent the decision making [3]–[5] and situational awareness [6] of pilots. Furthermore, uncertainties affecting AC must be taken into account. A suitable way for modeling the above features as well as the dynamic nature of AC is offered by the discrete event simulation methodology (e.g., [7]) and thus there exists several AC simulation models based on this methodology (e.g., [3], [8]–[10]).

A discrete event AC simulation model is controlled with input parameters and variables that affect the components describing the pilots' decision making as well as the properties of aircraft and other hardware. Uncertainty related to AC is represented by random factors in the simulation model whose effect on simulation output, e.g., the number of aircraft shot down, is analyzed using the Monte Carlo method (e.g., [7]). In this method, each AC scenario is replicated several times with different realizations of random factors determined by non-overlapping pseudorandom number streams to produce statistical estimates for the simulation output.

In practice, the nature of a large-scale discrete event air combat simulation model may be almost black box due to its high complexity. Therefore establishing that it performs as intended, i.e., the validating of the simulation model, is a challenging task (e.g., [11]). Once the simulation model has been validated, it can be utilized in simulation based optimization that offers a powerful tool for comparing available tactics or hardware configurations.

In the validation of the AC simulation model, the action of the opponent must be taken into account in a rational and realistic manner. In this paper, this issue is tackled by presenting a novel approach to the simulation analysis of AC that utilizes game theory (e.g., [12], [13]). In general, game theory gives a systematic way for analyzing decision problems with several players pursuing their own objectives. A game model consists of players, their decision variables and payoffs that depend on the decision variables and evaluate the attainment of players' objectives. Decision alternatives available to the players are presented by the ranges of the decision variables. The outcome of the game, i.e., the values of the players' payoffs, is determined based on the decision alternatives selected by the players. Using the game model, one can identify the players' best responses to the opponents' decisions, i.e., the game optimal value of the player's decision variable when the action of the opponent is fixed. Together, the players' best responses are used to find the equilibrium solution of the game.

The game theoretic approach introduced in this paper consists of four phases. First, the AC scenario is determined. In the scenario, the sides of AC are assumed to have a set of tactical alternatives related to available tactics or hardware configurations. The objectives that sides are trying to achieve are represented by measures of effectiveness (MOEs). Second, the scenario is simulated by inputting the combinations of tactical alternatives into the AC simulation model in order to calculate MOE estimates from the simulation output. Third, a suitable game model is estimated to capture the dependence between the tactical alternatives and MOE. In the game model, the tactical alternatives are represented by decision variables and the MOE estimate is used as the payoff. This paper presents the estimation procedures for games involving continuous or discrete decision variables. In the discrete case, one obtains a matrix game in which the MOE estimates are classified by applying analysis of variance (e.g., [14]). With continuous variables, a multivariate regression model (e.g., [15], [16]) is

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fitted to the simulation data.

In the simulation literature, there exists a versatile set of validation methods for discrete event simulation models [7], [17]. Commonly used methods include comparing the simulation results with actual data. Alternatively, a subject matter expert can assess the validity of the model output. To aid the assessment, several techniques can be used to describe the model output such as calculating descriptive statistics and presenting the results graphically. One can also perform a sensitivity analysis with respect to the model input to see how it affects the simulation output. While such methods are also suitable for the validation of an AC simulation model, they omit the game setting of AC which is taken as an integral part of the approach presented in this paper.

In the validation, the structures and solutions of the estimated game models are compared with the actual AC scenarios. This comparison focuses on the following properties of the estimated games. First, symmetric scenarios should result in symmetric game models. Second, the payoff of the game should depend on the decision variables in a manner that is consistent with the tactical alternatives and the MOE in actual AC. The analysis of these two properties is straightforward and can be carried out even without a subject matter expert. If an expert is available, also the best responses and equilibrium solutions of the games can be analyzed. They should be justifiable based on the corresponding MOE and tactical alternatives of the scenario. If these properties of the games are considered plausible, this is taken as positive evidence on the validity of the simulation model.

Game models estimated from simulation data can also be seen as a new type of simulation metamodel [18], [19]. In the simulation literature, metamodels refer to simpler, analytical models auxiliary to the simulation model [18]. There exists a number of metamodel types [20], [21] for various purposes that are, in general, understanding and validating of a simulation model as well as optimizing the simulation output [22]. Clearly, the estimated games are also applicable for all these purposes.

In this paper, the game theoretic approach is illustrated by representing the analysis of a discrete event air combat simulation model, X-Brawler [3], [8]. In X-Brawler, aircraft, weapons, and other systems as well as pilot's decision making are modeled at a high level of detail which should provide a good approximation of actual AC. Validation examples are presented in two scenarios that study the effects of pilot aggressiveness as well as the action taking place after the launch of a medium range air-to-air missile.

II. The Game Theoretic Approach

The game theoretic approach to AC simulation proceeds as follows. First, the AC scenario of interest is defined at a suitable precision so that the definition contains all the necessary information for performing the simulation. This includes the number and types of aircraft, weapons, sensory and other systems as well as the initial geometry of AC. The definition of the scenario also includes the objectives of the flights, the measure of the attainment of these objectives which is called a MOE, and the flights' means for achieving these objectives, i.e., their available tactics or hardware configurations. Here, these means are called the flights' tactical alternatives.

The MOE represents the outcome of the combat as perceived by the flights and it can be selected in many ways depending on the aim of the analysis and the objectives of the flights. In general, AC related decision problems contain multiple objectives but now it is assumed that the objectives can be presented with a scalar valued MOE. The MOE can be, e.g., mean of kills, mean of losses, or a linear combination of the former. One can also study probabilities of scenario specific AC events. In defensive scenarios, success can be measured by, e.g., the probability of taking down some pre-specified aircraft or destroying the entire attacking bomber fleet. On the other hand, in offensive scenarios, the MOE can be, e.g., the probability of reaching a given route point unharmed or destroying an important ground target.

To study how tactical alternatives affect a MOE, the AC scenario is simulated with suitable combinations of the tactical alternatives that are selected using design of experiments [14], [19]. The alternatives are entered to the simulation model using input variables and other scenario information is presented by input parameters. For each combination of input variable values, a MOE estimate is obtained from the simulation output. Once the simulation data are collected, a suitable game model is estimated using statistical techniques. In the estimated game model, the players' decision variables and payoffs are associated with the simulation input variables and the MOE estimates. The nature of the tactical alternatives determines whether the decision variables of the game model are discrete or continuous.

A. Estimation of Game Models

1) Simulation Data: To estimate game models one needs AC simulation data that consist of values of simulation input variables and resulting MOE estimates. If the tactical alternatives of the AC scenario are discrete, e.g., different maneuvers or missile types, they are presented by discrete valued input variables for blue and red, denoted by $x_1, \dots, x_n$ and $y_1, \dots, y_m$, respectively. Then, the scenario is simulated using the input variable values (x_i, y_j) to gather the needed data.

Continuous tactical alternatives are entered into a simulation model using input variables that have a continuous range of values and are denoted by x for blue and y for red. Examples of such alternatives include, e.g., missile launch distances or missile support times. For continuous input variables, it is not possible to simulate the AC scenario using all feasible values of input variables. Therefore, a discrete set of values (x_i, y_j) is chosen according to a suitable experimental design (e.g., [14]) to produce the necessary simulation data.

Because of random factors in a simulation model, a MOE can be regarded as a random variable with an unknown probability distribution. Similarly, a given AC event associated with the MOE takes place during simulation with an unknown probability. When the AC scenario is simulated with input variable values (x_i, y_j), simulation output gives a random sample from the MOE distribution or observations of the occurrence of the AC event. These are used to produce MOE estimates denoted by $\hat{U}(x_i, y_j)$. In practice, the MOE estimate is the mean of the sample estimating the expectation of the MOE distribution or the relative frequency of the occurrence of the AC event estimating the probability of the event. The values of the input variables and the MOE estimates form the simulation data $(x_i, y_j, \hat{U}(x_i, y_j))$.

2) Discrete Decision Variables: When tactical alternatives are discrete, the AC scenario is converted into a matrix game where the decision alternatives of the players coincide with the input variable values (x_i, y_j) and the payoffs are based on the MOE estimates $\hat{U}(x_i, y_j)$. Due to the random factors of the simulation model, the MOE estimates may not be entirely accurate and have to be classified statistically using variance analysis methods (e.g., [14], [15]). In these methods, the estimates are compared pairwise to find out which pairs are statistically significantly different. In this paper, the MOE estimates are classified using the Tukey-Kramer procedure [14] that carries out all the comparisons simultaneously to avoid the multiple comparisons problem. The Tukey-Kramer procedure also allows unequal sample sizes and variances for the MOE estimates. This is practical as some of the simulation replications may crash resulting in unequal sample sizes and there is no guarantee that different input variable values produce MOE estimates with equal variances.

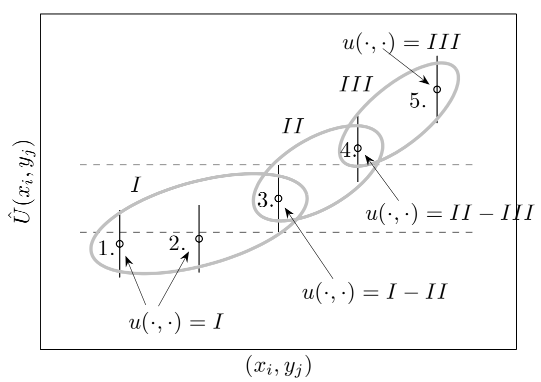


Fig. 1. Classification of MOE estimates for a matrix game. Simulation with input variable values (x_i, y_j) produces MOE estimates $\hat{U}(x_i, y_j)$ that are grouped into classes I, II, and III. The payoff has four values, i.e., $u(x_i, y_j) \in \{I, I-II, II-III, III\}$.

In the Tukey-Kramer procedure, the MOE estimates are grouped into classes. Two estimates belong to the same class if they do not differ from each other statistically significantly. Now, the classes are indexed in ascending order so that the lowest estimates belong to class I, the second lowest belong to class II, and so on. The classification gives the payoff, denoted by $u(x_i, y_j)$, that maps the combinations of decision alternatives (x_i, y_j) to the payoff values associated with classes I, II, III, etc. Then, the player maximizing the payoff prefers the outcome of the game with the higher payoff value. Note that a MOE estimate can simultaneously belong in two classes. Thus, the payoff value can be, e.g., $I - II$. When comparing two payoff values they are considered as equal if they share a common class, e.g., $I - II$ and $II - III$. It should also be noted that the classification gives only ordinal information about the ranking of the payoff values. That is, the payoff value I is considered smaller than II but the magnitude of the difference between the payoff values is unknown.

Fig. 1 shows an example of classification of five MOE estimates $\hat{U}(x_i, y_j)$ into three classes I, II, and III. The MOE estimates are marked with circles and each MOE estimate is associated with a vertical line that corresponds to a simultaneous 95% confidence interval that is used in the comparison of the estimates. If two vertical lines overlap, the corresponding estimates do not differ from each other statistically significantly. For example, in Fig. 1, the horizontal dashed lines represent the comparison of the third estimate with others. The first three estimates do not significantly differ from each other. Thus, they are regarded as equal and belonging to class I. The third and fourth estimates are also deemed equal and belonging to class II. Finally, the fourth and fifth estimates form class III.

In Fig. 1, the payoff $u(x_i, y_j)$ has four values I, $I - II$, $II - III$, and III. Again, two combinations of decision alternatives are considered to provide equal payoff values if the corresponding values overlap, i.e., they share a common class. For example, the third payoff value is $I - II$ and therefore it is considered equal to the first payoff value I and the fourth payoff value $II - III$. On the other hand, the fifth payoff value III is statistically significantly greater than the third $I - II$.

3) Continuous Decision Variables: In the case of continuous tactical alternatives, one needs a payoff function that approximates the dependence between the tactical alternatives and the MOE. There exists random variation in the MOE estimates that has to be accounted for in the approximation. Therefore, the payoff function $u(x, y; \boldsymbol{\beta})$ is constructed as a regression model [14], [15] based on the simulation data $(x_i, y_j, \hat{U}(x_i, y_j))$ as presented in Fig. 2. Now, the simulation input variables (x, y) are taken as the decision variables of blue and red. In order to achieve the best possible approximation, the payoff function is fitted to simulation data by selecting an appropriate type of regression model and estimating its parameter vector $\boldsymbol{\beta}$ with the method of least squares. The correctly constructed regression model then describes the dependence between the decision variables and the payoff of the game as well as approximates the payoff values for all values of the decision variables.

In practice, there are no apparent limitations for the functional form of the regression model, as it can be, e.g., a linear, logistic, or some non-linear regression model [14]. The model needs to be complex enough to accurately capture the dependence between tactical alternatives and a MOE. The goodness of fit of the regression model is studied using residuals, coefficient of determination, and/or deviance of

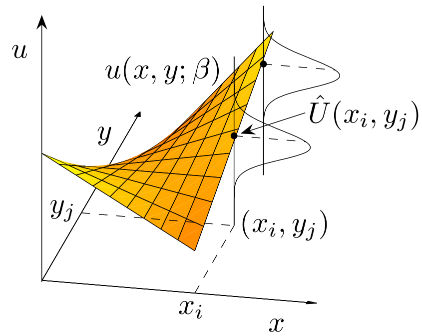


Fig. 2. Construction of the payoff function. Simulation with the input variable values (x_i, y_j) produces the MOE estimate $\hat{U}(x_i, y_j)$. The payoff function $u(x, y; \boldsymbol{\beta})$ is fitted to the data in order to approximate the MOE for all values of the variables x and y.

the model [14]–[16]. Simultaneously, the model needs to be as parsimonious as possible. Thus, statistically insignificant parameters are excluded from the model in order not to fit the model into random variation of the simulation data. The relevance of the parameters is analyzed with p-values [14]–[16]. The model selection is case dependent and the suitability of alternative models is also affected by the type of MOE. For example, logistic regression models [16] are ideal for modeling probabilities of AC events. Furthermore, several regression models can be combined to define more complex payoff functions, e.g., the difference between probabilities of two AC events.

B. Validation Analysis

The purpose of validation analysis is to ensure that the simulation model gives a satisfactory representation for AC. A widely-used validation approach is to study the simulation data with statistical methods [11] such as regression analysis and analysis of variance. The methods are used to analyze how different factors and variables affect the outcome of the simulated AC. Traditionally, this one-sided analysis is performed separately for each flight. If the simulation model is proper and functional, the statistical models should be consistent with the AC scenario under consideration.

Game models estimated from simulation data are utilized in validation in the same way as the one-sided statistical models. Now, validation is conducted in a two-sided setting from the view-point of both flights by studying whether the games are consistent with the actual AC scenario. If the game models are found unsatisfactory, this indicates a need for improvement in the simulation model or its settings.

In this paper, validation analysis concentrates on the following properties of games: MOE estimates, symmetry of game models, dependencies within models, best responses of players, and equilibrium solutions. In game models, the players' decision alternatives are represented by decision variables having a discrete or continuous range of values. Due to technical and practical reasons the players' decisions are in this paper limited into selecting exact values of the decision variables. In other words, the players are allowed to use only pure strategies and mixed strategies are excluded. That is, the players are not allowed to present decisions as probability profiles over the available decision alternatives. It should also be noted that the symmetry and, up to certain extent, the dependencies can be analyzed even without subject matter expertise whereas the analysis of best responses and equilibrium solutions requires a more profound familiarity with AC and its practices.

1) Symmetry: Symmetric AC scenarios are an integral part of the validation analysis. An AC scenario is said to be symmetric if the initial geometry is symmetric and the flights have similar aircraft and other hardware as well as similar tactical alternatives. In such a case, the estimated games are supposed to reflect this symmetry. Asymmetric game models, on the other hand, point towards problems in the simulation model or in the execution of simulation.

In a symmetric game, the payoff values are the same for each player under comparable circumstances. That is, the effect of a decision on the payoffs is independent of the player making the decision. The estimation of separate game models with the same payoff for blue and red should result in similar models that have same payoff values. For example, in a symmetric setting, the game models with payoffs equaling the number of blue and red kills should be alike. Furthermore, in the case of continuous decision variables, the symmetric payoff functions are expected to depend on the decision variables in a concurrent manner and the parameters of the payoff functions are supposed to mirror each other. In symmetric games, best responses and equilibrium solutions should also be symmetric. For example, if there is a Nash equilibrium where the players make certain decisions, also the decision combination where the decisions of the players are reversed should be a Nash equilibrium.

Validation analysis can also be extended to asymmetric scenarios to see how differences between the flights affect the simulation results. For example, one can start with a perfectly symmetrical AC scenario and gradually make it more uneven, e.g., by enhancing the characteristics of one flight's aircraft or by changing the initial geometry. By comparing corresponding games, it is possible to study the effect of increased asymmetry on the outcome of AC. Clearly, the outcome should favor the flight having superior aircraft or advantageous initial position.

2) Dependence between Decision Variables and Payoff: The game models show how the players' decisions affect the players' payoff values and the corresponding MOE estimates reflecting the outcome of the simulated AC scenario. Therefore, the payoffs and their dependence on the decision variables can be studied to see if they are reasonable, e.g., do more effective weapons systems result in a better outcome or do more defensive tactics reduce losses. Furthermore, the joint effects of players' decisions can be analyzed, e.g., to see what happens to the number of kills when both flights behave very aggressively.

3) Best Responses and Equilibrium Solutions: Best responses are a special case of the joint effects of the players' decisions which give the players' game optimal decisions when the opponent's decision is fixed. The best responses can be solved for all decision alternatives of the opponent and the logic behind these responses should concur with the respective AC scenario. If the estimated game models have dominating alternatives, this should also be justifiable on the basis of the scenario.

For example, if the AC scenario involves maximizing kills one may want to engage in a direct confrontation with the opponent. Therefore, best responses in this situation should represent aggressive behavior of the player, e.g., choosing a shorter missile launch range or maneuvering directly towards the opponent. In such a scenario, it would also be plausible to have a dominating alternative that performs best regardless of the opponent's decision.

Nash equilibria of the game models are also used in the validation analysis. If an estimated game model has one or more such equilibria, there should be an explanation based on the AC scenario to show why the players would behave in the given manner. This is compared to actual AC by considering whether similar decisions would be made by pilots.

III. Example of Validation Analysis

In the following, the use of the game theoretic approach in validation is demonstrated by analyzing scenarios that are based on simulations conducted with the AC simulation model X-Brawler [3], [8]. The aim of the analysis is to explore the decision making model of the simulated pilots and to ensure that settings and hardware models used in simulation are proper and functional. In the scenarios, flights engage in AC with identical aircraft and weapon systems. Furthermore, the initial geometry of the combat is symmetric. Such a situation should result in symmetric game models that can be analyzed without a subject matter expert.

A. Scenario 1: Level of Aggression

In this scenario, the effect of pilot aggressiveness is studied in 2 vs. 2 AC. The flights' tactical alternatives represent aggression levels for the simulated pilots. They are briefly summarized as follows:

- Low aggression level: The flights engage the opponent and launch a medium range air-to-air missile towards it. After the launch, the pilots support their missiles, i.e., relay state information about the opponent to the missile in order to increase the likelihood of a hit but not at the cost of being easy targets for the opponent. Finally, the flights retreat from the opponent. The distance to the opponent is maintained at beyond visual range during the entire engagement.
- Medium aggression level: Similar to the low aggression level, but now having retreated from the opponent the flights re-enter the combat. Should a pilot get caught within the visual range area, the engagement is continued as a dogfight.
- High aggression level: The flights engage the opponent, launch and support their missiles. Then, the aircraft are flown towards the opponent and a dogfight is engaged within visual range. During the engagement the pilots do not perform any defensive maneuvers, such as retreating.

MOEs analyzed in the scenario are the number of blue and red kills as well as the difference between kills. Here, the number of kills means the number of opposing aircraft shot down. To produce necessary MOE estimates, the scenario is simulated 2400 times for each of the nine combinations of input variable values representing the levels of pilot aggressiveness. The MOE estimates are classified using the procedure presented in Section II-A2 which results in matrix games.

		RED (*min*)		
		low	*medium*	*high*
BLUE (*max*)	*low*	*I* (0.179)	*III* (1.200)	*III* (1.205)
	medium	[*II*] (0.344)	*IV* (1.501)	*IV* (1.498)
	high	[*II*] (0.330)	*IV* (1.504)	*IV* (1.489)

Fig. 3. Matrix game with the payoff representing the number of blue kills. The Nash equilibria are circled.

		RED (*max*)		
		low	*medium*	*high*
BLUE (*min*)	*low*	*I* (0.257)	[*II*] (0.346)	[*II*] (0.320)
	medium	*III* (1.156)	*IV* (1.488)	*IV* (1.475)
	high	*III* (1.162)	*IV* (1.468)	*IV* (1.485)

Fig. 4. Matrix game with the payoff representing the number of red kills. The Nash equilibria are circled.

		RED (*min*)		
		low	*medium*	*high*
BLUE (*max*)	*low*	[*II*] (−0.077)	*IV* (0.855)	*IV* (0.885)
	medium	*I* (−0.811)	*III* (0.013)	*III* (0.023)
	high	*I* (−0.833)	*III* (0.036)	*III* (0.004)

Fig. 5. Matrix game with the payoff representing the difference of blue and red kills. The Nash equilibrium is circled.

The estimated games with the MOE estimates are presented in Figs. 3, 4, and 5. Blue maximizes its kills in Fig. 3 and the difference of kills in Fig. 5. In Fig. 4, blue minimizes red kills. All the games are zero sum and thus the objective of red is always opposite to blue. It should also be noted that *I* corresponds to the payoff value with the smallest MOE estimates and the estimates ascend according to the indexing.

To validate the simulation model, the symmetry of the games is considered first. The games for blue and red kills in Figs. 3 and 4 reflect the symmetry of the scenario as the game for blue kills is essentially the same as the game for red kills because the former game matrix can be obtained by transposing the latter. The game for the difference of kills in

Fig. 5 is also perfectly symmetric. Note that, payoff value I is the most advantageous for red while IV is preferred by blue. Additionally, when the players' decisions coincide the MOE estimates do not differ statistically significantly from zero. All the games discussed above represent the symmetric AC scenario and there is no reason to challenge the validity of the simulation model.

Next, the best responses of the players as well as the dependence between the decision variables and the payoff are analyzed. In Figs. 3 and 4, increase in the level of aggression leads to higher casualty rates for both flights. The best response for the player minimizing losses, i.e., blue in Fig. 4 and red in Fig. 3, is always the low aggression level whereas for the player maximizing kills, i.e., blue in Fig. 3 and red in Fig. 4, the best response is the medium or high aggression level. The game for the difference of kills (Fig. 5) implies that the low aggression level is the dominating alternative for both players as it gives always the most desirable payoff value. Thus, according to this game it is most effective to launch the missile and disengage. It should also be noted that the medium and high aggression levels produce identical payoff values. This could be explained, e.g., by ineffective implementation of evasive maneuvers or overtly effective missile models that render the maneuvering after the missile launch irrelevant.

To summarize, the symmetric AC scenario under consideration produces symmetric game models. In addition, the models are reasonable as increase of aggression increases the casualty rates for both flights which is compatible with the actual AC. These observations point towards the validity of the simulation model. However, the analysis also implies some shortcomings in the simulation results. The dominance of the low aggression level in Fig. 5 and the identical payoff values for the alternatives *medium* and *high* may not be entirely realistic. Therefore, further analysis of the decision making model of the simulated pilots as well as of the aircraft and missile models is recommended.

B. Scenario 2: Support Time of a Missile

The second scenario deals with a support time game [23] where the pilots face each other at the limit of the missile launch range. The pilots launch their missiles and support them, i.e., relay radar information about the opponent to their missiles to increase the likelihood of a hit. Tactical alternatives of the scenario are the support times of blue and red, denoted by $x, y \in [0, 15]$ (seconds). The main idea of the scenario is that supporting one's missile increases the hitting probability. On the other hand, longer support times take the pilot to a more disadvantageous position with regard to evading the opponent's missile and increase the probability of being hit.

Unlike the previous scenarios, now MOEs are the probability of blue kill, the probability of red kill, and the weighted sums of the probabilities. The tactical alternatives are presented by continuous input variables of the simulation model. The scenario is simulated for a set of input variable values that are selected according to a central composite design [14]. The used experimental design includes 12 combinations of the input variable values that are simulated 3000 times and a central combination that is simulated 12000 times.

Because the MOEs are probabilities, a logistic regression model [16] is fitted to the simulation data which gives regression models of the form

$$p(x, y; \boldsymbol{\beta}) = \frac{\exp(q(x, y; \boldsymbol{\beta}))}{1 + \exp(q(x, y; \boldsymbol{\beta}))}, \qquad (1)$$

where $q(x, y; \boldsymbol{\beta})$ is a quadratic function of the decision variables x and y, i.e.,

$$q(x, y; \boldsymbol{\beta}) = \beta_0 + \beta_1 x + \beta_2 y + \beta_3 x^2 + \beta_4 y^2 + \beta_5 xy. \qquad (2)$$

The functional form of $q(x, y; \boldsymbol{\beta})$ is selected to allow for the curvature of the regression model. Most importantly, the term xy links the players' decision variables together and turns the model into a game instead of two independent optimization problems.

Models for the probabilities of blue and red kill estimated from simulation data and the resulting parameter vectors $\boldsymbol{\beta}_B$ for blue and $\boldsymbol{\beta}_R$ for red are presented in Table I. The 95%-confidence intervals of the parameter estimates are given in parentheses. All the parameters are found to be statistically significant as their p-values are of magnitude 10^{-4} or smaller. The values of the model deviances are small compared to the respective degrees of freedom implying that they have no statistical significance and are probably result of random variation (see, Table I). Thus, the models fit the data well and there is no sign of lack-of-fit or need for the addition of higher order variables. The MOE estimates and the regression models are illustrated in Fig. 6.

TABLE I

PARAMETER VECTORS $\boldsymbol{\beta}_B$ AND $\boldsymbol{\beta}_R$ OF THE LOGISTIC REGRESSION MODELS REPRESENTING THE PROBABILITIES OF BLUE AND RED KILLS.

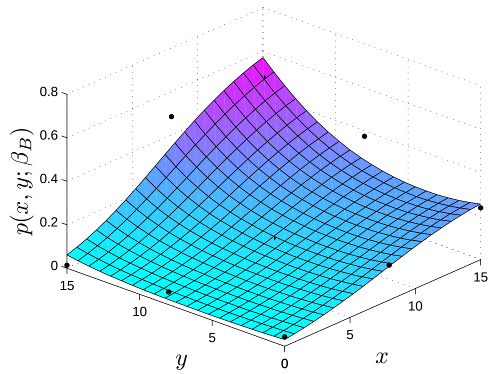
		Prob. of blue kill	Prob. of red kill
variable	parameter	$\boldsymbol{\beta}_B$	$\boldsymbol{\beta}_R$
constant	β_0	-3.444 (± 0.165)	-3.529 (± 0.169)
x	β_1	0.289 (± 0.027)	-0.126 (± 0.022)
y	β_2	-0.131 (± 0.021)	0.300 (± 0.028)
x^2	β_3	-0.009 (± 0.001)	0.011 (± 0.001)
y^2	β_4	0.012 (± 0.001)	-0.009 (± 0.001)
xy	β_5	0.003 (± 0.001)	0.003 (± 0.001)
model deviance		1331	1310
degrees of freedom		44991	44991
statistical significance		1.000	1.000

The payoff functions of the game are formulated by combining the probabilities of blue and red kills as weighted sums

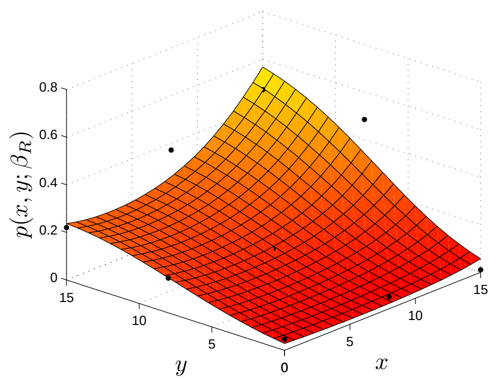
$$u_B(x, y) = w_B p(x, y; \boldsymbol{\beta}_B) + (1 - w_B)(1 - p(x, y; \boldsymbol{\beta}_R)) \qquad (3)$$

$$u_R(x, y) = w_R p(x, y; \boldsymbol{\beta}_R) + (1 - w_R)(1 - p(x, y; \boldsymbol{\beta}_B)), \qquad (4)$$

where the weights $0 \le w_B, w_R \le 1$. The payoff of blue $u_B(x, y)$ consists of the probability of blue kill $p(x, y; \boldsymbol{\beta}_B)$



(a) Probability of blue kill $p(x, y; \boldsymbol{\beta}_B)$ as the function of the blue support time x and the red support time y.



(b) Probability of red kill $p(x, y; \boldsymbol{\beta}_R)$ as the function of the blue support time x and the red support time y.

Fig. 6. Logistic regression models used in defining the payoffs of the support time game.

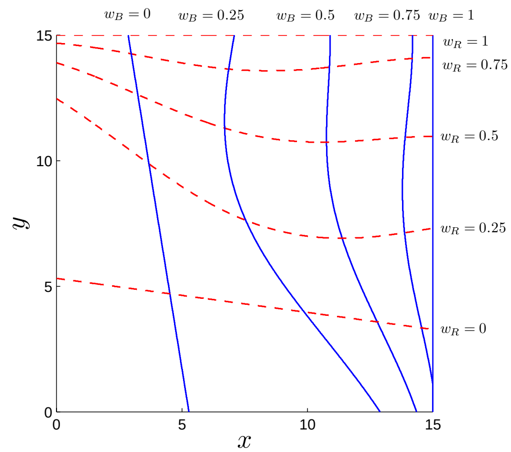


Fig. 7. Players' best response curves with different weights w_B and w_R. Blue's best response curves are marked with solid lines and red's with dashed lines.

and the probability of avoiding blue loss $(1 - p(x, y; \boldsymbol{\beta}_R))$. The larger the weight w_B, the more willing blue is to sustain losses, e.g., by setting $w_B = 1$, the payoff reduces to the probability of blue kill. Similarly, when $w_B = 0$, the payoff equals the probability of avoiding blue loss. Thus, the weight w_B can be interpreted as a measure of aggressiveness for blue. The payoff of red $u_R(x, y)$ is constructed in a similar manner.

Players' best response curves, i.e., the optimal support times against a given support time of the opponent, are solved by maximizing the payoffs (3) and (4) while holding the opponent's support time constant. The responses are presented in Fig. 7 for a set of weights w_B and w_R. For instance, if blue is only interested in the number of blue kills, i.e., $w_B = 1$, the best response of blue to all red's support times is to support for as long as possible, i.e., $x = 15$. As mentioned in the definition of the scenario, there is no reason for supporting the missile any longer due to its limitations. If blue wants to minimize its losses, i.e., $w_B = 0$, it is optimal for blue to support its missile for approximately 5 seconds and the optimal support time decreases linearly as a function of the red support time.

The payoff functions and the best response curves presented above are used in validation analysis by considering their symmetry and dependencies implied by them. The scenario is symmetric and therefore also the game model should be symmetric which holds for the presented game model. The parameters in $\boldsymbol{\beta}_B$ and $\boldsymbol{\beta}_R$ are symmetric (Table I). For example, the parameter of x in the model for the probability of blue kill is 0.289 ± 0.027 and the parameter of y in the model for the probability of red kill is 0.300 ± 0.028. These estimates do not differ statistically significantly. Hence, the decision variables affect the probabilities of kills in similar manner. This is also pointed out by the regression models in Fig. 6 which are perfect mirror images. Therefore, the payoff functions derived from the probabilities are also symmetric. Furthermore, the response curves in Fig. 7 are symmetric with respect to the line $x = y$ as expected.

The prolonging of the support times x and y generally results in higher kill probabilities (Fig. 6). Therefore, the estimated game model seems to be mostly reasonable as it shows that supporting one's missile increases both probabilities of kill. However, if the launcher of the missile does not support the missile at all, the probability of kill is very small regardless of the support time chosen by the opponent. One could argue that the zero probability of hit for an unsupported missile is not entirely realistic. The missile has also its own radar and the performance of the missile should not be entirely dependent upon the launching aircraft's radar. Therefore, the performance and behavior of the missiles in the simulation model should be further studied and confirmed.

The best response curves are realistic as increase in the weight assigned to the probability of kill leads to longer support times (Fig. 7). However, the best responses corresponding to the minimization of losses, i.e., the weights $w_B = 0$ or $w_R = 0$, deviate from the expected. Based on the actual scenario, the pilots should not support their missiles at all in order to minimize the probability of being hit. In the game model, the players' best response is to support their missiles for approximately 5 seconds before heading away from the opponent. This is an inconsistency compared to the actual scenario that warrants further analysis of the implementation of the evasive maneuvers as well as of the range of the missiles and the detection range of their radars. Nevertheless, in general, the estimated game model and the best response curves are in concordance with the actual AC scenario and the analysis supports the validity of the simulation model.

IV. Conclusions

This paper presented a new approach to the analysis of AC combining game theory and discrete event simulation. In the approach, data obtained by simulating AC scenarios are converted into game models using statistical techniques. The payoffs of the games containing discrete or continuous decision variables are estimated using analysis of variance and multivariate regression analysis, respectively. The estimated games are applied for validating simulation models. The game theoretic approach extends one-sided statistical validation methods by taking into account the game setting that is a critical part of AC. On the other hand, the estimated games can be considered as a new type of simulation metamodel.

The application of games in validation is based on comparison of their properties with actual AC practices. Such properties include symmetry of games, dependence between decision variables and payoffs, best responses, and equilibrium solutions. The utilization of these properties in validation is illustrated with the example analysis of an existing discrete event simulation model. The analysis gave both positive feedback on the accuracy of the simulation model and revealed some inconsistencies in the simulation data. These observations based on, e.g., best responses and equilibrium solutions, could not have been made with traditional one-sided validation methods. Thus, the use of the game theoretic approach gave additional insight into the validity of the simulation model. Overall, estimated games offer a way to present simulation data in an informative and easily interpretable form. Therefore, the game theoretic approach is capable of answering validation needs in a transparent manner.

In addition to validation, the estimated games could be used in simulation based optimization where the application of game theory would extend the existing simulation based optimization techniques into a two-sided setting by taking into account the joint effects of the both sides' decisions – instead of unilateral optimization. The game theoretic approach can also be applied to simulation analyses in other fields than AC. In addition to military problems, the approach lends itself naturally to all studies involving multiple actors with conflicting objectives and having a compelling need for simulation. Examples of such fields include problems in economics, computer science, and biology.

References

[1] C. A. Feuchter, *Air Force Analyst's Handbook: On Understanding the Nature of Analysis*, Office of Aerospace Studies, Air Force Material Command, Kirtland, NM, 2000.

[2] B. L. Stevens and F. L. Lewis, *Aircraft Control and Simulation*, 2nd ed. New York, NY: Wiley, 2003.

[3] E. Lazarus, "The application of value-driven decision-making in air combat simulation," in *Proc. IEEE Int. Conf. Systems, Man and Cybernetics*, vol. 3, Orlando, FL, 1997, pp. 2302–2307.

[4] K. Virtanen, T. Raivio, and R. P. Hämäläinen, "Decision theoretical approach to pilot simulation," *Journal of Aircraft*, vol. 36, no. 4, pp. 632–641, Jul.-Aug. 1999.

[5] ——, "Modeling pilot's sequential maneuvering decisions by a multistage influence diagram," *Journal of Guidance, Control, and Dynamics*, vol. 27, no. 4, pp. 665–677, Jul.-Aug. 2004.

[6] K. Virtanen, R. P. Hämäläinen, and V. A. Mattila, "Team optimal signaling strategies in air combat," *IEEE Transaction of Systems, Man, and Cybernetics - Part A: Systems and Humans*, vol. 36, no. 4, pp. 643–660, Jul. 2006.

[7] A. Law, *Simulation Modeling and Analysis*, 4th ed. New York, NY: McGraw-Hill Science/Engineering/Math, 2006.

[8] *The X-Brawler Air Combat Simulator Management Summary*, L-3 Communications Analytics Corporation, Vienna, VA, 2002.

[9] S. Glærum, "TALUS - an object oriented air combat simulation." in *Proceedings of the 1999 Winter Simulation Conference*, Phoenix, AZ, 1999, pp. 1160–1167.

[10] R. R. Hill, J. O. Miller, and G. A. McIntyre, "Simulation analysis: applications of discrete event simulation modeling to military problems," in *Proceedings of the 2001 Winter Simulation Conference*, Arlington, VA, 2001, pp. 780–788.

[11] R. G. Sargent, "Verification and validation of simulation models," in *Proceedings of the 2003 Winter Simulation Conference*, New Orleans, LA, 2003, pp. 37–48.

[12] T. Başar and G. J. Olsder, *Dynamic Noncooperative Game Theory*, 2nd ed. London, England: Academic Press, 1995.

[13] R. Gibbons, *A Primer in Game Theory*. Harlow, England: Pearson, 1992.

[14] D. C. Montgomery, *Design and Analysis of Experiments, 5th Edition*. New York, NY: Wiley, 2001.

[15] J. S. Milton and J. C. Arnold, *Probability and Statistics in the Engineering and Computing Sciences*. New York, NY: McGraw-Hill, 1986.

[16] S. Sharma, *Applied multivariate techniques*. New York, NY: Wiley, 1996.

[17] J. P. C. Kleijnen, "Validation of models: Statistical techniques and data availability," in *Proceedings of the 1999 Winter Simulation Conference*, Phoenix, AZ, 1999, pp. 647–654.

[18] L. W. Friedman, *The simulation metamodel*. Norwell, MA, USA: Kluwer Academic Publishers, 1996.

[19] J. P. C. Kleijnen, *Design and Analysis of Simulation Experiments*, 1st ed. New York, NY: Springer Science+Business Media, 2008.

[20] R. R. Barton, "Simulation metamodels," in *Proceedings of the 1998 Winter Simulation Conference*, Washington, DC, 1998, pp. 167–174.

[21] M. I. R. dos Santos and A. M. Porta Nova, "Statistical fitting and validation of non-linear simulation metamodels: A case study," *European Journal of Operational Research*, vol. 171, no. 1, pp. 53–63, May 2006.

[22] J. P. C. Kleijnen and R. G. Sargent, "A methodology for fitting and validating metamodels in simulation," *European Journal of Operational Research*, vol. 120, no. 1, pp. 14–29, Jan. 2000.

[23] J. Karelahti, K. Virtanen, and T. Raivio, "Game optimal support time of a medium range air-to-air missile," *Journal of Guidance, Control, and Dynamics*, vol. 29, no. 5, pp. 1061–1069, Sep.-Oct. 2006.