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# Analysis of human's stabilization controller in a unicycle operation using Inverse Regulator Problem of Optimal Control

Takashi Shimamura,Hiroshi Ohsaki
Teruyoshi Sadahiro,Masami Iwase, Shoshiro Hatakeyama
Department of Robotics and Mechatornics
Tokyo Denki University
Ogawa, Kanda, Chiyoda,Tokyo 101-0052, Japan
shimamura@hatalab.k.dendai.ac.jp

***Abstract*—A purpose of this study is analyzing a human's controller to understand a mechanism of skill acquation processes for riding a unicycle. In order to analyze the controller, inputs which an operator gives to the unicycle and states of the unicycle are measured and a human-unicycle model as same as the experiment is derived. A weighting matrix to the state is obtained from the states, the inputs and the model solving Inverse Regulator Problem. The human's controller for riding the unicycle is discussed by the obtained weighting matrix.**

***Index Terms*—unicycle, analyze, Inverse Regulator Problem**

## I. Introduction

A unicycle is unstable, because it touches ground by one point. A human who can't ride on the unicycle must practice for the human learns to ride on the it. And the spent times are different for each human. However, a human have to redress the balance using whole body when the human rides on the unicycle. Consequently, the unicycle is excellent exercise machine.

In this research, a data of a human who can ride on the unicycle is analyzed from viewpoints of control engineering. A research for a unicycle is done widely. But the research is made mostly of paying attention only the unicycle or a unicycle robot[1][2][3][4]. A research of control of the unicycle with the human or skill upgrading of the human riding the unicycle is slight[5][6]. We research paying attention to a locomotion of a operator who can ride on the unicycle and the unicycle. The data is analyzed to figure out a reason for the operator can ride on the unicycle keeping stabilization of body. That is, the human's stabilization controller to ride the unicycle is analyzed.

In this paper, the data is measured by experiment. The human's controller is assumed oprimal regulator. And inverse regulator problem of linear optimal control[7][8](The following,inverse problem) is solved using the data. Accordingly, a weighting matrix of the quadratic performance index is obtained. Moreover, since the weighting matrix to the state for every time is obtained by solving an inverse problem for every data acquisition time, the dynamic change of a controller to change of the quantity of state of a unicycle is analyzed.

## II. Inverse Regulator Problem

### A. Problem of Linear Optimal Control

As opposed to Linear system

$$\dot{x} = Ax + Bu, x(0) = x_0; A \in \mathbf{R}^{n\times n}, B \in \mathbf{R}^{n\times m} \tag{1}$$

Solve for control input $u$ which makes linear quadratic performance index the minimum.

$$J = \int_0^\infty \left(x^T Q x + u^T R u\right) \mathrm{d}t; Q = Q^T \geq 0, R = R^T > 0 \tag{2}$$

The solution of this problem is obtained from solution $P$ of Riccati equation.

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{3}$$

$$F = R^{-1}B^T P \tag{4}$$

$$u = -Fx \tag{5}$$

However, generally the optimal control does not necessarily fulfill asymptotically stable conditions, unless the suitable restriction for weighting matrix $Q$ is imposed.

$$\lim_{t\to\infty} x(t) = 0 \tag{6}$$

That is, the optimal control is not necessarily stabilization control. Then, the stable regulator problem which restricted the control input so that it might become stabilization control is also considered.

If the difference between an optimal regulator and a stable regulator is shown directly, the former will minimize a performance index, but a closed loop system is not necessarily stabilized. Although the latter does not necessarily minimize a performance index instead, a closed loop system is always stabilized.

### B. Inverse Problem of Linear Optimal Control

Suppose that the state feedback gain (following gain $F$) is given for the system(1).

1) Solve a necessary and sufficient condition for gain $F$ to minimize a performance index (2) about suitable $Q \leq 0$ and suitable $R > 0$.
2) When gain $F$ minimizes a performance index(2), Solve all of weighting matrix $QandR$.

This problem will turn into an inverse problem of an optimal regulator problem, if the optimality of gain $F$ is considered in the control input defined previously, and if it thinks in stabilization control, it will turn into an inverse problem of a stable regulator problem.

In the case of the inverse problem of a stable regulator problem, the following three assumption is placed.

1) The system (1) is controllable.
2) The closed loop system (1),(5) are stable.
3) The weighting matrix $R$ of a performance index (2) is an identity matrix.

If this assumption is placed, a solution will be obtained by the following procedures.

1) It solves all $P$ that fills the following linear matrix inequality(LMI).

$$\Gamma(P) = \begin{bmatrix} PA + A^TP - F^TF & PB - F^T \\ B^TP - F & 0 \end{bmatrix} \quad (7)$$
$$\Gamma(P) \leq 0, \left(P = P^T\right)$$

2) It solves for $Q$ from $Q = F^TF - PA - A^TP$.

In the inverse problem of a stable regulator, it is $F$ the optimal necessary and sufficient condition that gain $F$ is stable. Therefore, the solution (7) to should just be obtained on this problem. In this paper, the Inverse problem is solved assuming man's stabilization controller to be a stable regulator, and weighting matrix $Q$ is obtained. Since $Q$ is weighting matrix which determines the performance of a regulator, a controller shall be analyzed in solving an inverse problem for every time of data acquisition, and analyzing obtained weighting matrix $Q$.

## III. UNICYCLE ENTRAINMENT EXPERIMENT

In order to conduct analysis using an inverse problem, the data at the time of unicycle entrainment is needed. Then, the unicycle entrainment experiment by a unicycle operation skillful person is conducted, and the data at the time of the unicycle operation of the input torque concerning the state where it was called the angle and angular velocity of a wheel and a saddle, and a wheel and a saddle is acquired.

### A. An experimental device and apparatus

In order to acquire the data at the time of unicycle operation, various sensors were installed in the unicycle. The installed sensor is shown below.

- The gyroscope sensor which measures the angular velocity of a saddle
- The acceleration sensor which measures the acceleration of advancing side by side
- The treading strength sensor for the treading strength measurement added to a pedal
- It is an encoder because of rotation angle acquisition of a pedal.
- 6DOF force sensor

The unicycle which actually installed the above-mentioned sensor is shown in Fig.1. Moreover, the coordinate system at the time of data acquisition is shown in Fig.2.

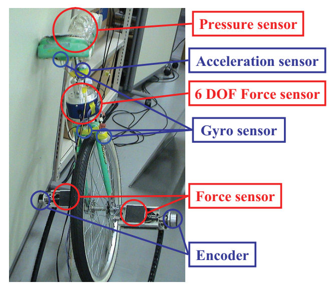



Fig. 1. The unicycle for an experiment

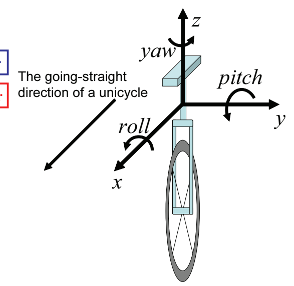



Fig. 2. The coordinate system at the time of data acquisition

Since various sensors are cables, when they conduct a run experiment of a unicycle, they will be restricted to the length of a line. For this reason, data is satisfactorily unacquirable. Then, experiment environment like Fig.3 is built.

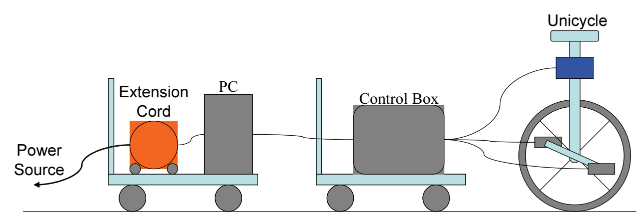



Fig. 3. Experiment environment

At Fig.3, they are moved to a cart by loading the control box for sensors, and the computer for data acquisition to compensate for a run of a unicycle. It is possible to acquire data by this, as long as the length of an extension cable allows.

### B. Experiment procedure

A procedure is shown below.

1) Ride and run to a unicycle (Fig. 1) by volunteer.
2) The course is a straight course of about 20 [m].
3) The data at the time of a run is acquired.
4) It is checked after a run whether data is acquirable.

Two volunteers are assembled and it is considered as volunteers A and B this time, respectively. Volunteers A and B are skillful persons who can ride on a unicycle more than 20 [m].

## C. *Experimental result*

Volunteers' A and B measurement result is shown in Fig.4 ∼Fig.13. In addition, the graph of blue, an orange, and red corresponds to a $x$ [ of figure 2 ], $y$, and $z$ axis about power, respectively. The graph of blue, an orange, and red corresponds to roll of its Fig.2, picth, and yaw about 3 axis torque. The graph of blue, an orange, and red corresponds to roll of Fig.2, picth, and yaw about the angle of three axes, and angular velocity, respectively.

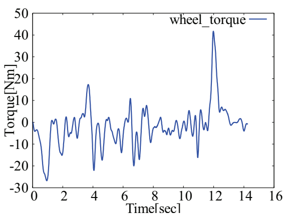



Fig. 4. Torque of a wheel (volunteer A)

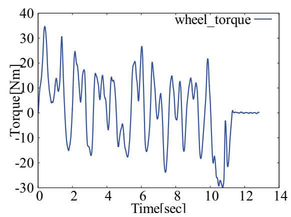



Fig. 5. Torque of a wheel (volunteer B)

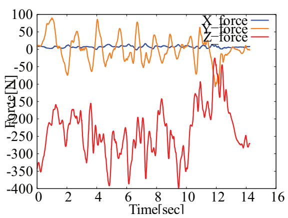



Fig. 6. Power in which it is added to a saddle (volunteer A)

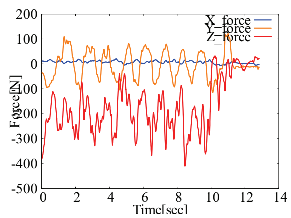



Fig. 7. Power in which it is added to a saddle (volunteer B)

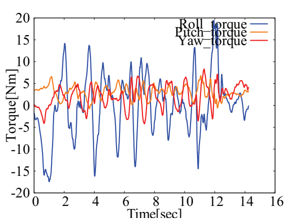



Fig. 8. Torque added to a saddle (volunteer A)

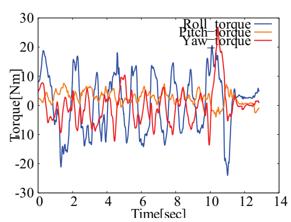



Fig. 9. Torque added to a saddle (volunteer B)

If Fig.8 about the torque of Volunteer A saddle is seen, the torque of pitch will vibrate near 3.0 [Nm]. The torque of roll and yaw is vibrating considering 0 [Nm] as a center. Moreover, the amplitude of the torque of roll is the largest. This tendency is looked at by Fig.9 about the torque of Volunteer B saddle.

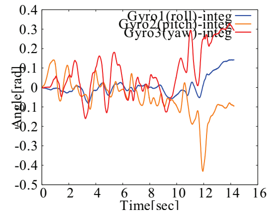



Fig. 10. The angle of a saddle (volunteer A)

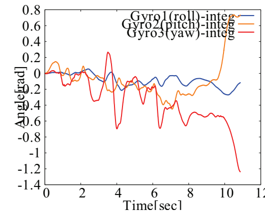



Fig. 11. The angle of a saddle (volunteer B)

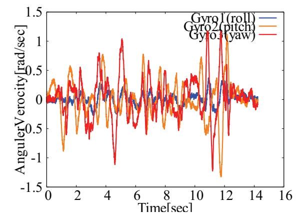



Fig. 12. Angular velocity of a saddle (volunteer A)

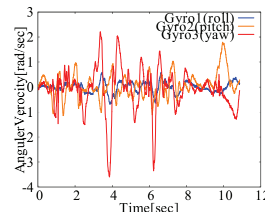



Fig. 13. Angular velocity of a saddle (volunteer B)

In Fig.10 Fig.11 about the angle of Volunteers A and B saddle, it is vibrating focusing on 0 [rad], and Volunteer A is running, without falling to near 10 [sec]. Similarly, Volunteer B is running, without falling to near 8 [sec]. The angular velocity at this time (Fig.12, 13) is vibrating focusing on 0 [rad/sec]. Moreover, Fig.12 and 13 tend [ common ] to say that the amplitude of the angular velocity of yaw is the largest.

For this reason, about torque, an angle, and angular velocity, Volunteers A and B tended to have been alike.

# IV. HUMAN-UNICYCLE MODEL

In order to analyze man's stabilization controller, in this paper, the thing of Fig.14 is considered as a model of the unicycle with which people rode. A parameter is taken as TableI. In addition, the volunteer's body mass was drawn using the estimate equation of the human body coefficient of Chandler.[10]

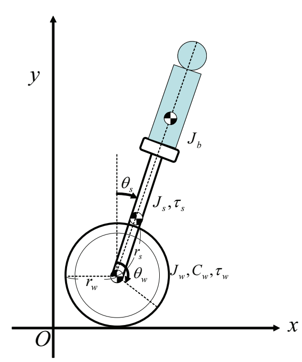



Fig. 14. Human-unicycle model

TABLE I

A PARAMETER OF THE MODEL

| | |
|---|---|
| acceleration of gravity[m/s$^2$] | $g = 9.81$ |
| moment of inertia of the wheel[kgm$^2$] | $J_w = 0.181558$ |
| moment of inertia of the saddle[kgm$^2$] | $J_s = 0.147715$ |
| mass of the wheel[kg] | $m_w = 3.641$ |
| mass of the saddle[kg] | $m_s = 2.159$ |
| volunteer A's body mass[kg] | $m_b = 35.0578$ |
| volunteer B's body mass[kg] | $m_b = 36.186$ |
| length from a saddle to volunteer A center of gravity[m] | $r_b = 0.251$ |
| length from a saddle to volunteer B center of gravity[m] | $r_b = 0.262$ |
| volunteer A's body moment of inertia[kgm$^2$] | $J_b = 28.356$ |
| volunteer B's body moment of inertia[kgm$^2$] | $J_b = 29.268$ |
| viscous coefficient of friction of an axle[Ns/rad] | $C_w = 0.0677$ |
| radius of the wheel[m] | $r_w = 0.281229$ |
| length from an axle to the center of gravity of the saddle[m] | $r_s = 0.453051$ |
| length of the saddle[m] | $\ell_s = 0.6325$ |
| angle of the wheel[rad] | $\theta_w$ |
| angle of the wheel[rad] | $\theta_s$ |
| angle of the volunteer[rad] | $\theta_b$ |

### A. Equation of motion

From Fig.14, an equation of motion is as follows.

$$M\ddot{q} + N\dot{q} + Gq = \tau \tag{8}$$

It is considered as $q = [\theta_w \quad \theta_s]^T$.

$$M = \begin{bmatrix} \alpha_1 & \beta_1 r_w \cos\theta_s \\ \beta_1 r_w \cos\theta_s & \gamma_1 \end{bmatrix}$$
$$N = \begin{bmatrix} C_w & -C_w \\ -C_w & C_w \end{bmatrix}$$
$$G = \begin{bmatrix} 0 & -\beta_1 r_w \dot{\theta}_s^2 \frac{\sin\theta_s}{\theta_s} \\ 0 & -g\beta_1 r_w \frac{\sin\theta_s}{\theta_s} \end{bmatrix}$$
$$\tau = \begin{bmatrix} \tau_w & \tau_s \end{bmatrix}^T, \alpha_1 = J_w + (m_w + m_s + m_b) r_w^2$$
$$\beta_1 = m_b(\ell_s + r_s) + m_s r_s,$$
$$\gamma_1 = J_b + J_s + \ell_s^2 m_b + 2\ell_s m_b r_b + m_b r_b^2 + m_s r_s^2$$

### B. Derivation of an error system

In this paper, an error system is built as what is stabilizing the unicycle when the operator is running by fixed speed $\dot{\theta}_{wd}$. Torque $\tau$ in the case of running by fixed speed $\dot{\theta}_{wd}$ is as follows.

$$\tau = M\ddot{q}_d + N\dot{q}_d + Gq_d + u \tag{9}$$

$u$ is an input generated from an operator's controller. However, $q_d$ is taken as the following.

$$q_d = \begin{bmatrix} \dot{\theta}_{wd} t & 0 \end{bmatrix}^T$$

It will be set to (10), if (9) is substituted for (8) and a error is set to $e = q - q_d$.

$$M\ddot{e} + N\dot{e} + Ge = u \tag{10}$$

(10) is changed into state-dependent coefficient depiction.

$$\begin{bmatrix} \dot{e} \\ \ddot{e} \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}G & -M^{-1}N \end{bmatrix} \begin{bmatrix} e \\ \dot{e} \end{bmatrix} + \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} u \tag{11}$$

As mentioned above, the error system was called for.

### C. The model of a saddle on which man has ridden

LMI of (7) can be solved using lmisolver of Scilab etc. Therefore, if (7) can solve from the data obtained for the preceding chapter, man's stabilization controller in the unicycle operation considered as a 2 input 2 output system will be obtained. However, in lmisolver of Scilab used this time, it was difficult to determine appropriately the initial point estimate of $P$ which can solve LMI as the dimension of a system becomes high. Then, when the unicycle is carrying out uniform motion, only paying attention to a saddle, it considers solving an inverse problem in the case of a SISO model, assuming that the torque and the state of a wheel can be disregarded. Fig.15, then a system serve as (12) in a model when an operator rides on a saddle.

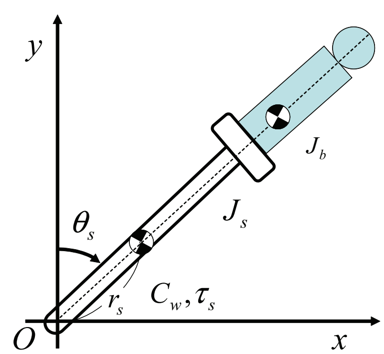


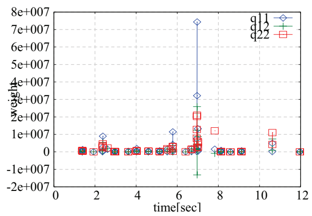

Fig. 15. Model of a saddle on which man has ridden

$$\begin{gathered} \begin{bmatrix} \dot{\theta}_s \\ \ddot{\theta}_s \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{\beta}{\alpha}\frac{\sin\theta_s}{\theta_s} & -\frac{C_w}{\alpha} \end{bmatrix} \begin{bmatrix} \theta_s \\ \dot{\theta}_s \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\alpha} \end{bmatrix} \tau_s \\ \alpha = J_b + J_s + m_b(\ell_s + r_b)^2 + m_s r_s^2 \\ \beta = g(m_b(\ell_s + r_b) + m_s r_s) \end{gathered} \tag{12}$$

## V. ANALYSIS OF HUMAN'S CONTROLLER

An inverse problem is solved using $F$ for which it asked from input-and-output relations. Since weighting matrix $R$ about an input assumes that it is an identity matrix, weighting matrix $Q$ of a state is obtained as a solution. This is solved for every acquisition time of a state and an input, and matrix $Q$ to a state per hour is derived.

(7) shows $Q$ for every time of Volunteer A and Volunteer B when a solution is obtained to figure 16 and figure 17, respectively. In addition, in the figure of a weighting matrix, blue, red, and green graph are the weight concerning an angle, the weight concerning angular velocity, and a non-diagonal clause of matrix $Q$, respectively.



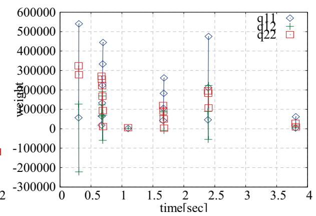
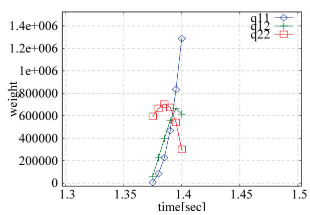




Fig. 16. Weighting matrix $Q$ (volunteer A)

Fig. 17. Weighting matrix $Q$ (volunteer B)

In Fig16, near 1.3 [sec] to 1.5 [sec] where the solution was obtained continuously, near 6.98 [sec] to 7.04 [sec], and near 9.1 [sec] to 9.15 [sec] The angle at the time and angular velocity are indicated to be the expanded figures from Fig.18 to Fig.26.



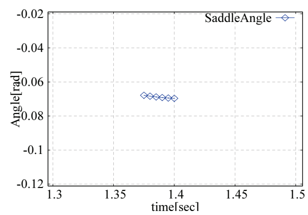




Fig. 18. Weighting matrix $Q$ (volunteer A:near 1.3 [sec] to 1.5 [sec])

Fig. 19. Angle of the saddle (volunteer A:near 1.3 [sec] to 1.5 [sec])

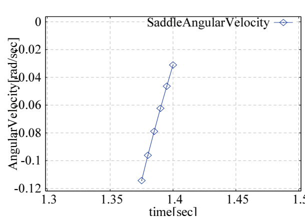



Fig. 20. Angular velocity of the saddle (volunteer A:near 1.3 [sec] to 1.5 [sec])

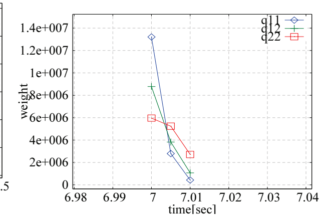



Fig. 21. Weighting matrix $Q$ (volunteer A:near 6.98[sec] to 7.04[sec])

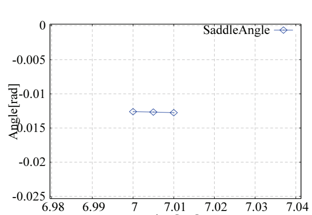



Fig. 22. Angle of the saddle (volunteer A:near 6.98[sec] to 7.04[sec])

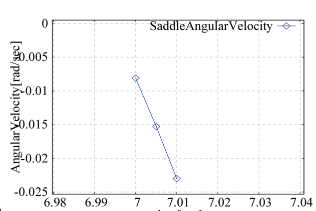
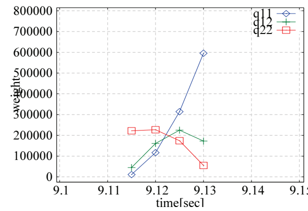



Fig. 23. Angular velocity of the saddle (volunteer A:near 6.98[sec] to 7.04[sec])



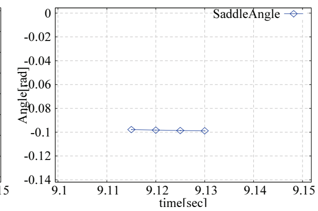




Fig. 24. Weighting matrix $Q$ (volunteer A:near 9.1[sec] to 9.15[sec])

Fig. 25. Angle of the saddle (volunteer A:near 9.1[sec] to 9.15[sec])

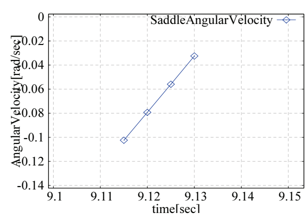
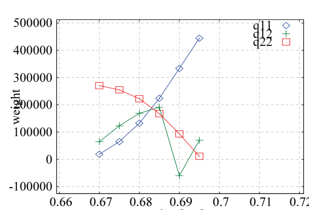



Fig. 26. Angular velocity of the saddle (volunteer A:near 9.1[sec] to 9.15[sec])

The dignity of the angle became large gradually and the size of a value is reversed as the weight concerning angular velocity will become small gradually, if Fig.20 from Fig.18 and Fig.24 to Fig.26 is seen. It turns out that the size of the angular velocity at this time is approaching 0. This has a tendency with the same said of the time when the solution was obtained continuously [ the others in Fig.16 ]. Uniquely, near 6.98 [sec] to 7.04 [sec] of figure 21 to figure 23, the weight of an angle becomes small and the way of the weight of angular velocity is large. At this time, the size of angular velocity is separated from 0, and it turns out that they are other tendencies and true contraries. The figure which expanded near 0.66 [sec] to 0.72 [sec] and near 2.36 [sec] to 2.44 [sec] in Fig.17, [ where the solution was obtained continuously ] The angle at the time and angular velocity are shown in Fig.32 from Fig.27.



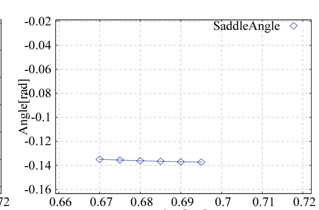




Fig. 27. Weighting matrix $Q$ (volunteer B:near 0.66[sec] to 0.72[sec])

Fig. 28. Angle of the saddle (volunteer B:near 0.66[sec] to 0.72[sec])

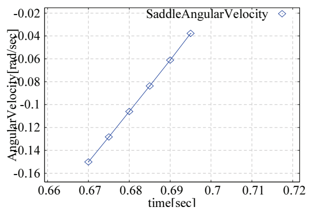



Fig. 29. Angular velocity of the saddle (volunteer B:near 0.66[sec] to 0.72[sec])

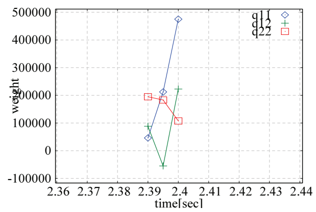



Fig. 30. Weighting matrix $Q$ (volunteer B:near 2.36[sec] to 2.44[sec])

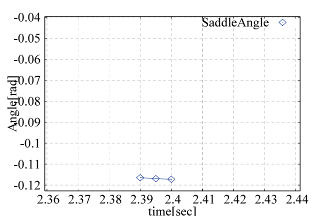



Fig. 31. Angle of the saddle (volunteer B:near 2.36[sec] to 2.44[sec])

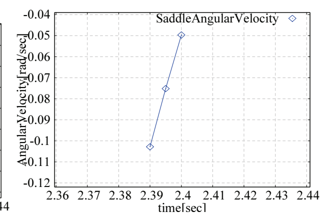



Fig. 32. Angular velocity of the saddle (volunteer B:near 2.36[sec] to 2.44[sec])

The dignity of the angle became large gradually and the size of a value is reversed as the dignity applied to angular velocity like Volunteer A will become small gradually, if Fig.29 from Fig.27 and Fig.30 to Fig.32 is seen. It turns out that the size of the angular velocity at this time is approaching 0. When human's controller controls angular velocity preferentially and angular velocity approaches 0 from the tendency seen by Volunteers A and B, weight to angular velocity is made light and it turns out that priority is given to control of an angle.

Break weighting matrix $Q$ by Euclid norm $\|Q\|(of\mathrm{Q})$, take out a diagonal ingredient here, and a horizontal axis as the weight corresponding to an angle, and weight corresponding to angular velocity for a vertical axis What was plotted is shown in Fig.33 and 34. If $Q$ is broken for $Q$ by Euclid norm $\|Q\|$, the specific gravity of the weight of each element can be known.

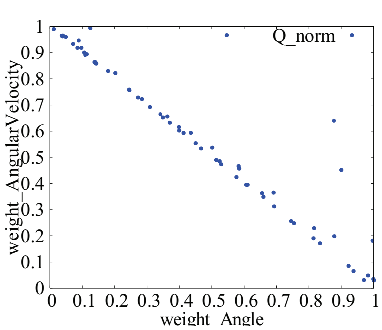



Fig. 33. Weighting matrix $Q$ divided by the norm (Volunteer A)

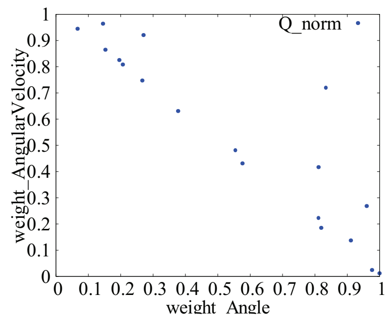



Fig. 34. Weighting matrix $Q$ divided by the norm (Volunteer B)

In Fig.33 and 34, the specific gravity of the weight of an angle and the specific gravity of the weight of angular velocity are linearly located in a line. Therefore, it turns out that the weight of angular velocity is small when the weight of an angle is large, and the weight of an angle is small when the weight of angular velocity is conversely large. As mentioned above, since the specific gravity of weight changes continuously to change of a state, it can be said that human's controller is that from which weight changes according to a state.

## VI. Conclusion

In this paper, the inverse regulator problem was solved from the measured data using the model of the saddle of a unicycle. From the weight procession obtained as the solution to, human's controller changes weight according to a state, and is stable. An inverse problem is solved like the case of a saddle about a unicycle from now on.

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