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An Efficient Liveness Enforcing Supervisor for FMSs Based on Petri nets and the Theory of Regions

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***Abstract*— This paper presents an efficient method to obtain a maximally permissive deadlock prevention policy for a flexible manufacturing system based on theory of regions. The theory of regions policy can provides an optimal deadlock prevention controller which is based on the reachability graph. However, its disadvantage is the state explosion problem since all marking/transition-separation instances (MTSIs) have to be identified. For improving the shortcoming, this paper proposes a new concept of the crucial marking/transition-separation instances (CMTSIs). Experimental results indicate that the proposed control policy in this paper is more efficient than the closely related approaches in the literature.**

***Keywords*—Petri nets, theory of regions, deadlock prevention**

I. INTRODUCTION

In existing literature, there are mainly two PN analysis techniques used to deal with the deadlock prevention in FMS PN models. One is structural analysis and the other one is reachability graph analysis. In the former, a relationship between the behavioral properties of the system model, i.e., liveness, boundedness, and its structure is captured. Some of the deadlock prevention control policies [1]-[6] are then obtained based on the characterization of the liveness in terms of Petri net. Especially, the deadlock prevention problems [1]-[6] are solved using the concept of siphons. In our prior work [3], we have proposed a control policy for improving the drawback of the conservative policy. However, our control policy needed many control places and arcs on the original net. Therefore, a much more complex Petri net than the original one can be formed. For reducing the number of control places, we proposed an iterative approach by adding two kinds of control places to the original model prevent siphons from being unmarked [4]. However, this way is with some drawbacks in it. Therefore, we employed the theory of the elementary siphons [7] to improve our control policy [4]. Since all minimal siphons should be controlled, the deadlock prevent policy [5] is very time-consuming when the system is large. In [9], the authors use an algebraic polynomial kernel for analysis and deadlock prevention for augmented marked graphs. Recently, Li and Zhou [7] proposed a deadlock prevention policy using so-called elementary siphons. The main point of the concept is to show that all siphons in a net are not needed to be controlled. They classify the strict minimal siphons (SMSs) in a Petri nets into two categories: elementary siphons and redundant siphons. The authors reduce the number of siphons to be controlled much smaller, particularly in a large-size Petri net. It also illustrates that the number of elementary siphons is bounded by the smaller of place count and transition count in a Petri net. It has been shown that the set of elementary siphons is not unique, which means there are multiple sets of elementary siphons in a Petri net. Experimental studies show that different sets of elementary siphons are adopted. And also the different results will be obtained. In the latter, the reachability graph of a PN model is used to obtain the live system behavior. A deadlock prevention policy based on the exclusion of deadlock states from the reachability has investigated in [8]. However, the implementation of the deadlock prevention policy is not addressed in the adding some new net elements. The paper focuses on the method of the reachability graph techniques. On the other hand, a general method has been proposed to deal with deadlock problems in [9]. The advantage of method is not confined to a certain class of FMS. And it provides an optimal deadlock prevention controller. It is found that this method is based on the reachability graph skill which is another branch analysis technique. One can realize that the method in [9] adopts the theory of regions. And the theory of regions is not applied to Petri nets initially. However, the theory is employed in a transition system (TS) [10]. [10] shows that a state-based representation with arcs labeled with symbols from an alphabet of events (a transition system, TS) can be mapped into a Petri net model. Before that, [11] also has shown that for an *ETS* (elementary transition system) there exists a Petri net with minimum transition count (one transition for each label) with a reachability graph isomorphic to the original transition system. The concept of the regions was introduced in [12] and developed in [13−16]. The detailed information of the theory of regions can be found [17]. In short, the theory of regions is a formal synthesis technique to derive Petri nets from automaton-based models. Preciously, one can synthesis the new net elements, namely places with initial marking and related arcs. One knows that the reachability graph analysis of the Petri net model of FMS is based on the definition of the theory of regions. In [9], the authors proposed two points of the deadlock-zone (DZ) and the deadlock-free zone (DFZ). For prevention the deadlock problems, they defined and solved the event-state-separation-problems (ESSPs) using the theory of regions [17]. From their experimental results, one can found that the ESSPs play an important role to solve the deadlock problem under the policy of the theory of regions. However, many redundant control places are still found under the ESSPs equations [9]. In [18], the authors also proposed a new interpretation of the theory of regions and define the M_F (*forbidden marking*), M_D (*dangerous marking*), M_L (*legal marking*), and Ω (the set of *marking/transition-separation instance*, MTSI). In summary, an optimal Petri net controller synthesis method for FMSs is proposed based on the theory of regions [18]. Note that the properties of MTSI are the same as the ESSP. However, the problem of the redundant control places cannot be completely avoided in a large FMS. Besides, the major shortcoming of the theory of regions is the state explosion problem. The reason is that the reachability graph of a Petri net model has to be generated if one wants to find all marking/transition-separation instances. For overcoming the problem, Li *et al.* [19] try to adopt the combined algorithm (i.e., siphon and the theory of regions) to solve some S^3PR cases. The authors claimed that their performance of the control policy can still obtain the maximally permissive liveness-enforcing net supervisor (LENS) [19]. Besides, the most attractive advantage of this combined approach is that the number of separation instances is significantly reduced after some siphons are controlled. However, the algorithm for seeking the MTSI does not seem to be efficient enough.

In other hand, an iterative control policy of liveness enforcement for Petri nets based on theory of regions is proposed in [20]. The authors claimed that it required less computational cost to obtain the additional controllers in every iterative process way [20]. At each iterative process, a bad marking met first in the reachability graph of a PN model is identified to put into its control policy. Then and a relative control place is added to reduce some dead markings until the system is deadlock free. However, the drawback is pointed out by [21]. It reveals that the method needs to compute the reachability graph not only one times. Even the number of controlled permissive markings by [20] are small than the prior control policies [9] and [18]. In summary, the control policies [9] and [18] based on the theory of regions are more efficient than [20].

In this correspondence, the authors propose a more efficient deadlock prevention policy for the FMSs. The proposed policy hits the reachability graph on the dead marking. In addition, the authors further present a new *crucial marking/transition-separation instance* (CMTSI) based on controller synthesis. The advantage of the proposed approach is for improving the performance of the MTSI [18] and simplifying the problem of the redundant control places and the state explosion. In this paper, four stages are involved in the deadlock prevention policy. First, a whole reachability graph is needed to be constructed. Second, the dead states have to be recognized from the reachability graph. Next, an algorithm is used to find the *crucial* MTSI. Finally, the necessary control places can be obtained.

The rest of this paper is organized as follows. Section II presents the basic definitions and properties of Petri nets and the theory of regions that are related to this paper. Section III presents our deadlock prevention algorithm and two examples. Section IV gives the results to compare with existing methods. Conclusions are presented in Section V.

II. PRELIMINARIES

A. Petri nets [22]

A Petri net is a 5-tuple $PN = (P, T, F, W, M_0)$ where P is a finite set of places; T is a finite set of transitions, with $P \cup T \neq \varnothing$ and $P \cap T = \varnothing$; $F \subseteq (P \times T) \cup (T \times P)$ is the set of all directed arcs, where $P \times T \rightarrow N$ is the input function that defines the set of directed arcs from P to T, and $T \times P \rightarrow N$ is the output function that defines the set of directed arcs from T to P, where $N = \{0, 1, 2, \ldots\}$, W: $F \rightarrow N$ is the weight function. M_0: $P \rightarrow N$ is the initial marking. The set of input (resp., output) transitions of a place p is denoted by $\bullet p$ (resp., $p\bullet$). Similarly, the set of input (resp., output) places of a transition t is denoted by $\bullet t$ (resp., $t\bullet$). A Petri net structure (P, T, F, W) without any specific initial marking is denoted by N. A Petri net with the given initial marking is denoted by (N, M_0).

A transition t is said to be enabled or fired, if each input place $p \in \bullet t$ is marked with at least $w(p, t)$ tokens, where $w(p, t)$ is the weight of the arc from p to t. A transition may fire if it is enabled. A firing of an enabled transition t removes $w(p, t)$ tokens from each input place $p \in \bullet t$, and adds $w(t, p)$ tokens to each output place $p \in t\bullet$, where $w(t, p)$ is the weight of the arc from t to p. This process is denoted by $M [t > M'$. The marking M of a Petri net indicates the number of tokens in each place which is the current state of the modeled system. When a marking M' can be reached from a marking M by executing a firing sequence of transitions $\sigma = t_0 t_1 t_2 \ldots t_k$, this process is then denoted by $M [\sigma > M'$. The set of all reachable markings for a Petri net with initial marking M_0 is denoted by $R(N, M_0)$.

A transition t is said to be *live* if for any $M \in R(N, M_0)$, there exists a sequence of transitions fire able from M which contains t. A Petri net G is said to be *live* if all the transitions are live. A Petri net N contains a *deadlock* if there is a marking $M \in R(N, M_0)$ at which no transition is enabled. Such a marking is called a dead marking. Deadlock situations are as a result of inappropriate resource allocation policies or exhaustive use of some or all resources. Liveness of a Petri net means that for each marking $M \in R(N, M_0)$ reachable from M_0, it is finally possible to fire any transition t, $\forall t \in T$, in the Petri net through some firing sequence. This means that a live Petri net guaranties *deadlock-free* operation, no matter what firing sequence is chosen, i.e., if a Petri net is live, and then it has no deadlock. A Petri net $R(N, M_0)$ is said to be *reversible*, if for each marking $M \in R(N, M_0)$, M_0 is reachable from M. Thus, in a reversible net it is always possible to go back to initial marking (state) M_0. Many systems are required to return from the failure states to the preceding correct states. Thus reversibility property is important to manufacturing system error recovery. This property also guaranties cyclic behavior for all repetitive manufacturing systems. Moreover, if a net contains a deadlock, then it is not reversible [23]. A marking M' is said to be a *home state*, if for each marking $M \in R(N, M_0)$, M' is reachable from M. Reversibility is a special case of the home state property, i.e., if the home state $M' = M_0$, then the net is reversible.

B. The Theory of Regions and Supervisory Control Problem [18]

Consider any place p of the net $R(N, M_0)$ we look for. Because $R(N, M_0)$ is pure, p can be fully characterized by its corresponding incidence vector $[N](p, \cdot)\cdot\vec{\Gamma}_M$. For any transition t that is firable at any marking M, i.e., t is the label of an outgoing arc of the node M in G

$$M'(p) = M(p) + [N](p, \cdot)\cdot\vec{\Gamma}_M,\ \forall(M, M') \in G \text{ and } M[\ t > M' \tag{1}$$

Consider now any non-oriented cycle γ of the reachability graph. Applying the state equation to node in γ and summing them up gives the following cycle equation:

$$\sum_{t\in T}[N](p,t)\cdot\vec{\gamma}(t) = 0, \forall\gamma\in C \tag{2}$$

Where γ is any non-oriented cycle of R_C, $\gamma(t)$ denotes the algebraic sum of all occurrences of t in γ, and C is the set of non-oriented cycles of graph R_C.

According to the definition of G, there exists a non oriented path Γ_M from the initial state M_0 to M. Applying (1) along the path leads to $M(p) = M_0(p) + [N](p,\cdot)\cdot\vec{\Gamma}_M$, where $\vec{\Gamma}_M$ is the counting vector of the path Γ_M defined similar as $\vec{\gamma}$. There may exist several paths from M_0 to M. Under the cycle equations, the product $[N](p, \cdot)\cdot\vec{\Gamma}_M$ is the same for all these paths. As a result, the path Γ_M can be arbitrarily chosen. The reachability of any marking M in G implies that

$$M(p) = M_0(p) + [N](p,\cdot)\cdot\vec{\Gamma}_M \geq 0, \forall M\in R_C \tag{3}$$

The above equation (3) will be called the reachability condition.

It is now clear that the cycle equations and the reachability conditions hold for any place p of the net $R(N, M_0)$. For each pair (M, t) such that M is a reachable marking of G and t is a transition not firable at M, t, should be prevented from happening by some place p. Since the net is pure, t is prevented from happening at M by a place p iff

$$M = M_0(p) + [N](p,\cdot)\vec{\Gamma}_M + [N](p,t) \leq -1 \tag{4}$$

The above equation (4) is called the event separation condition of (M, t). The set of all possible pair (M, t) where M is a reachable marking and t is not firable at M will be called the set of event separation instances. Furthermore, the set of (M, t) is called Marking/Transitions-Separation Instances, MTSI.

III. CONTROLLER SYNTHESIS

In this paper, we focus on the forbidden state problem and propose a more efficient way to improve the MTSI. Under our policy, a maximally permissive deadlock prevention policy for FMSs can be obtained. And also the efficiency of the MTSI [21] can be improved.

A. The Crucial Set of Marking/Transition-Separation Instances (CMTSI)

For promoting the performance of the pioneering work [18], this paper defines three new parameters in this subsection. One is *Crucial Set of Marking/Transition-Separation Instances* (CMTSI, Ω_C), another is *dead marking* (M_d) and the other is *illegal marking* (M_I). The more detailed information is explained as follows.

Definition 4: The *dead marking* $M_d = \{M \in R(N, M_0) \mid \sim\exists\ M[t > M' \wedge (M' \in R(N, M_0))\}$.

For convenience, all markings of a reachability graph should be divided into two groups; legal markings (M_L) and illegal markings (M_I).

Definition 5: The *illegal marking* $M_I = \{M \in R(N, M_0) \mid \exists\ (M_I \supseteq M_d) \wedge (M_I \cap M_d = M_d) \wedge (M_I \cap M_L = \varnothing)\}$.

An *illegal marking* (M_I) means that the marking should be moved into a dead marking (M_d). It hints that the illegal marking is no ways to go back to a legal one. As a result, $M_I \supseteq M_d$ is obtained. It also hints that $M_I \cap M_d = M_d$.

Based on the definitions, one can find that once a legal marking is fired into an illegal marking through bad transitions, the legal marking should be leaded into the dead marking finally. Obviously, $M_L \subseteq R(N, M_0) - M_I$. In the following, the formal definition of CMTSI should be discussed.

Definition 6: The crucial set of marking/transition-separation instance (CMTSI) is $\Omega_C = \{(M, t) \in R(N, M_0) \mid \exists\ (M[\ t > M') \wedge (M[\ t' > M'') \wedge (M \in M_L) \wedge (M' \in M_d) \wedge (M'' \in M_L)\}$.

According to the definitions above, one can ensure that the all markings of M_L and the all forbad state transitions in Ω_C. As a result, the number of CMTSIs depends on the number of dead markings when the CMTSIs exist in this control system. It means that one dead marking (M_d) has its relative CMTSI if the CMTSI exists in the reachability graph.

B. Deadlock Prevention Policy

The proposed deadlock prevention algorithm:

1. Given a deadlock PN system model.

2. Generate the reachability graph and count the number of the dead states J.

3. Compute all the number of the CMTSIs. Such that $\Omega_{CK} = \{(M, t) \in R(N, M_0) \mid \exists\ (M[\ t > M') \wedge (M[\ t' > M'') \wedge (M \in M_L) \wedge (M' \in M_d) \wedge (M'' \in M_L)\}$, where $K = 1, 2, \ldots, J$.

If $\Omega_C \neq \varnothing$, go to step 5.

4. Identify all MTSIs ($\Omega = \{(M, t) \mid M[t > M' \wedge M \in M_L \wedge M' \notin M_L\}$).

5. Generate the event separation condition equations of (M, t).

$$M = M_0(p_C) + [N](p_C,\cdot)\vec{\Gamma}_M + [N](p_C, t_I) \leq -1$$

6. List the sets of the non-oriented cycles of the reachability graph.

$$\sum_{t \in T} [N](p_C, t) \cdot \vec{\gamma}(t) = 0, \forall \gamma \in C$$

7. Generate all reachability conditions of the reachability graph.

$$M(p_C) = M_0(p_C) + [N](p_C, \cdot) \cdot \vec{\Gamma}_M \geq 0, \forall M \in R_C$$

8. Obtain the control places and with its associated arcs, i.e., $M_0(p_C)$ and $[N](p_C, \cdot)$.

Theorem 1: Our deadlock prevention policy is more efficient than the past work [18].

Proof: Both our deadlock prevention policy and [18] utilize the concept of theory of regions to design control places. Then all of the marking/transition-separation instances can be controlled by the two control policies. It can be found that $\Omega_C \subseteq \Omega$. Therefore, it means that Ω_C can efficiently reduce the redundant control places. The author can conclude that our deadlock prevention policy is more efficient than the pioneering work [18]

In the following subsection, the authors would like to make a comparison with [9] [18] to prove our algorithm is more efficient than the two.

C. *Examples*

In this subsection, two examples are utilized to test our deadlock prevention policy. The first one is taken from [23]. In this example, a simple deadlock net model of the FMS is considered to demonstrate our policy how to make the deadlock nets live. The second one [26] is used to make a comparison with the deadlock prevention policy [18]-[19].

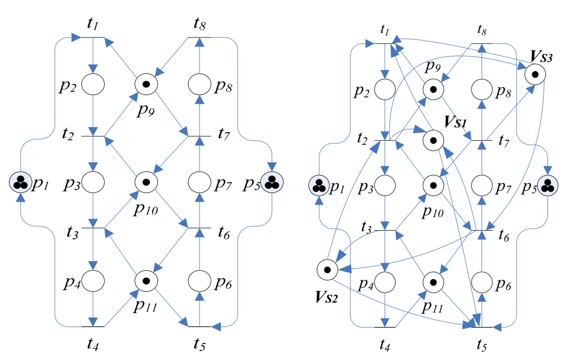

Fig 1(a). Petri net model for the two production sequences. (b) The Controlled net (N_{1H}, M_0)

Example I: An FMS [9] [25] (i.e., shown in Fig. 1) with two machine tools *M1* and *M2*, each of which can process one part at a time and one robot *R*, which can hold one part at a time. Parts enter into the FMS through input/output buffers *I1/O1* and *I2/O2*. In the situation, we consider that there are no parts in the system, and the production sequences are as follows.

P1: *M1* → Robot → *M2*
P2: *M2* → Robot → *M1*

Fig. 1(a) shows the PN model of the system. This system is an S^3PR, denoted by (N_1, M_0). It is worthy to notice that the theory of regions can be applied in any PN models.

First, the reachability graph of the PN model is needed to be constructed (i.e., Fig. 2). Here, it states that there are two CMTSIs in the Petri net system. According to the definition 6, the two CMTSIs are {(M_2, t_1)} and {(M_7, t_5)}.

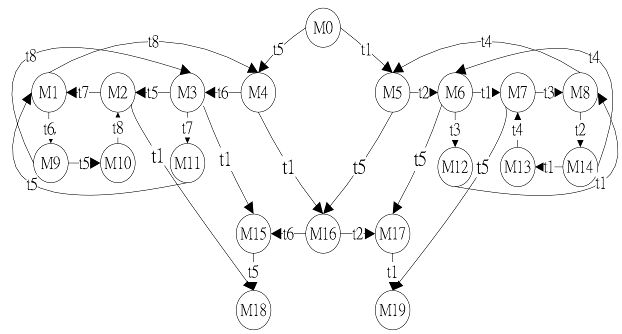

Fig. 2. The reachability graph of Fig. 1(a).

Considering Ω_{C1} = {(M_2, t_1)} and Ω_{C2} = {(M_7, t_5)}, one can obtain two event separation condition equations as follows.

$M_{18} = M_0 + t_1 + 2t_5 + t_6 \leq -1$

$M_{19} = M_0 + 2t_1 + t_2 + t_5 \leq -1$

However, there are 6 sets MTSIs are {(M_2, t_1)}, {(M_3, t_1)}, {(M_4, t_1)}, {(M_5, t_5)}, {(M_6, t_5)} and {(M_7, t_5)} in this example [9] [25]. Furthermore, six sets event separation condition equations need to be solved by [9] [25].

Once the two dead markings are removed from reachability graph, the three illegal markings (M_9, M_{10} and M_{11}) should be disappeared from the reachability graph simultaneously. It explains that the CMTSI method is able to improve the efficiency of the MTSI method.

In Fig. 1, one can point that there are two cycles in the system Petri net model. Based on the two cycles, one can obtain the two cycle equations as follows.

$t_1 + t_2 + t_3 + t_4 = 0$

$t_5 + t_6 + t_7 + t_8 = 0$

Then all reachability conditions of the reachability graph can be listed.

Since there are five illegal markings M_{15}, M_{16}, M_{17}, M_{18}, and M_{19} (including the dead marking, M_{18} and M_{19}), the total number of reachability markings is equal to 15 (i.e., 20 – 5 = 15). As a result, the number of reachability condition equations is equal to the number of legal markings. After calculating the inequalities $M_{19} = M_0 + 2t_1 + t_2 + t_5 \leq -1$ and the all legal reachability equations, one can obtain two control places C_{p1} and C_{p2}. The detailed information of the control places C_{p1} and C_{p2} as follows. $M_0(C_{P1}) = 1$, $t_1 = t_5 = -1$, $t_2 = t_6 = 1$, $t_3 = t_4 = t_7$

$= t_8 = 0$; $M_0(C_{P2})=1$, $t_2 = t_5 = -1$, $t_3 = t_6 = 1$, $t_1 = t_4 = t_7 = t_8 = 0$. By the same way, using inequalities $M_{18} = M_0 + t_1 + 2t_5 + t_6 \leq -1$ and the all legal reachability equations again, one can find the other two control places C_{p3} and C_{p4}. Here, $M_0(C_{P3}) = 1$, $t_1 = t_6 = -1$, $t_2 = t_7 = 1$, $t_3 = t_4 = t_5 = t_8 = 0$; $M_0(C_{P4}) = 1$, $t_1 = t_5 = -1$, $t_2 = t_6 = 1$, $t_3 = t_4 = t_7 = t_8 = 0$. It is worthy to notice that C_{P1} and C_{P4} are the same. As a result, the system net can be controlled with the three control places C_{P1}, C_{P2} and C_{P3}. The controlled system net (N_{1H}, M_0) is obtained and is shown in Fig. 1(b). And its reachability graph is shown in Fig. 3.

In Fig. 3, it is obviously that no any dead markings is existed the reachability graph of the controlled system net (N_{1H}, M_0). More preciously, the illegal markings are also disappeared from the reachability graph of the controlled system net (N_{1H}, M_0). The phenomenon explains that the dead marking is the key element of the dead PN system model. Once the dead marking is controlled (i.e., removed), the relative to illegal markings should be removed with the dead marking together.

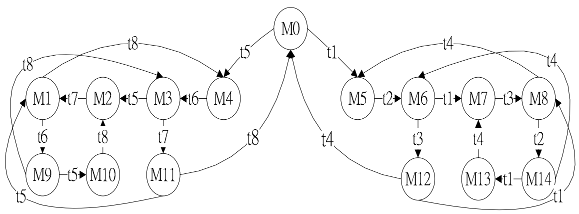

Fig. 3. The reachability graph of Controlled net (N_{1H}, M_0).

Example II: This example is taken from [26] and is used in [9] and [19]. The FMS consists of four machines *M1*, *M2*, *M3*, and *M4*, and two robots *R1* and *R2*, which can produce two part-types. The PN model of the system, denoted as (N_2, M_0), is shown in Fig. 4.

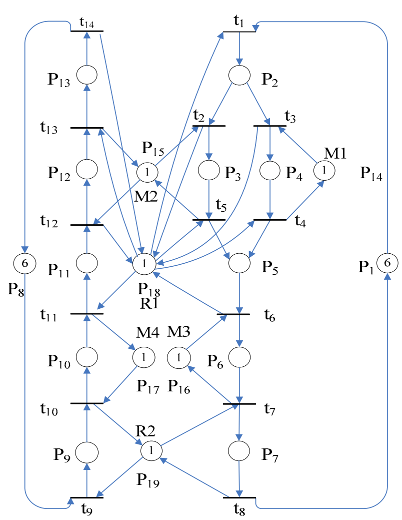

Fig. 4. The Petri nets model of example II [26]

To better understand the past control policies, the authors would like to introduce the deadlock prevention method of [19] in briefly. In [19], they solved the example by elementary siphons controlled policy (ESCP) and the theory of region. The detailed information of the two-stage policy is described in below. First, three sets elementary siphons $S_1 = \{P_2, P_5, P_{13}, P_{15}, P_{18}\}$, $S_2 = \{P_5, P_{13}, P_{14}, P_{15}, P_{18}\}$ and $S_3 = \{P_2, P_7, P_{11}, P_{13}, P_{16}, P_{17}, P_{18}, P_{19}\}$ can be found by ESCP. Hence, three control places V_{S4}, V_{S5}, and V_{S6} are needed to coordinate the three sets of the elementary siphons, respectively. .

After adding the three control places, the controlled net can be constructed, called (N_{2L1}, M_0). One can know that the controlled net (N_{2L1}, M_0) is still with a dead marking. In fact, the dead marking is M_{57} (i.e., Fig. 5) which belongs to the 210 reachable markings (i.e., the reachable markings M_1-M_{210} are denoted in [19]). For solving the deadlock marking, they utilized the concept of theory of regions to control the deadlock one. In this example, eight sets of MTSIs equations are needed to be handled. They are $\{(M_{43}, t_9)\}$, $\{(M_{44}, t_9)\}$, $\{(M_{47}, t_9)\}$, $\{(M_{48}, t_9)\}$, $\{(M_{49}, t_9)\}$, $\{(M_{53}, t_4)\}$, $\{(M_{59}, t_1)\}$, and $\{(M_{74}, t_2)\}$.

It is worthy to mention that the eight MTSIs exist because the illegal markings M_{54}, M_{55}, M_{56}, M_{57}, and M_{60} belong to the dead zone. And, considering the same example under our control policy, there is only one MTSI (i.e., $\{(M_{44}, t_9)\}$) needed to be prevented.

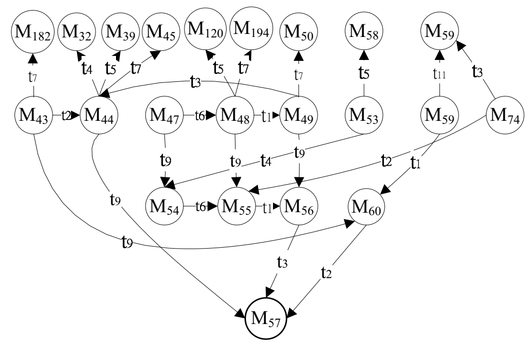

Fig. 5. A partial reachability graph of the net (N_{2L1}, M_0).

More detailed, the markings M_{44}, M_{56}, and M_{60} are possible to be led to the marking M_{57}. In addition, both markings M_{56} and M_{60} belong to the illegal markings. However, the marking M_{44} is the only one marking which is able to go back to the other legal markings in the system net (N_{2L1}, M_0) by itself. As a result, the authors can conclude that $\Omega_{C3} = \{(M_{44}, t_9)\}$ is the only CMTSI. Because the complete reachability graph of the controlled net (N_{2L1}, M_0) is too large to put it in this paper, the authors only show the partial reachability graph in Fig. 5. And, since there are five illegal and dead markings (i.e., M_{54}, M_{55}, M_{56}, M_{57} and M_{60}), the total number of reachability condition equations is equal to 205 (210 – 5 = 205).

By solving the CMTSI, cycle equations, and reachability equations, one can obtain three control places C_{p1}, C_{p2}, and C_{p3}. The three control places with their associated arcs are as follows. $M_0(C_{p1}) = 3$, $t_2 = t_4 = t_9 = -1$, $t_6 = t_{11} = 1$, $M_0(C_{P2}) = 3$, $t_2 = t_6 = t_9 = -1$, $t_5 = t_7 = t_{11} = 1$ and $M_0(C_{P3}) = 4$, $t_1 = t_9 = -1$, $t_7 = t_{11} = 1$. Finally, the controlled net (N_{2H}, M_0) is live and reversible when the net (N_{2L1}, M_0) is added with three controlled places C_{P1}, C_{P2} and C_{P3}. From the two experimental results, one can realize that the numbers of the event separation condition equations are reduced if the CMTSIs are identified.

In the next section, the advantage of the computation effort should be presented.

IV. Comparison with the existing methods

This section presents a numerical experiment to compare the proposed approaches with this contribution. The three approaches have proposed in [19] [25]. The approach (i.e., [25] and [19]) is proposed by Li *et al*., both called *algorithm L* here. Our control policy is called *algorithm H* in this paper. All results derived are summarized in Table I. Table I explains that the algorithm *L* need 6 MTSIs to be solved in example I, and need 8 MTSIs in example II. However, the algorithm *H* needs two MTSIs in example I and only need one MTSI in example II. It hints *algorithm H* is more efficient than *algorithm L* in the example I and II.

TABLE I. Comparison of the Controlled Systems

Example	# of Places	# of Resource Places	MTSI L, H	Control Places L, H
I	11	3	6, 2	3, 3
II	19	6	8, 1	6, 6

V. Conclusions

The main contribution of this paper is proposed an efficient deadlock prevention policy for FMSs, where the deadlock markings are found in its reachability graphs. The ideas underlying this policy are based on the fact that too many sets of inequalities (i.e., MTSIs) have to be solved due to a large number of marking/transition-separation instances in the original Petri net model. Considering the computation cost, this paper proposed a new definition of the MTSIs, called crucial marking/transition-separation instances (CMTSI), which is the key of MTSIs. The advantage of the proposed policy is able to reduce the computation cost due to few MTSIs involved in. Several deadlock systems are successfully controlled by our proposed policy.

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