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Fast Optimal Multimodal Thresholding Based on Between-Class Variance Using a Mixture of Gamma Distributions

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Abstract

Images segmentation is an important issue for many applications as pattern recognition and computer vision. Thresholding is an important and fast technique used in most applications. Gaussian Otsu's method is a thresholding technique based on between class variance. Gamma distribution models data more than Gaussian distribution. In this paper, we developed a new formula using Otsu's method for estimating the optimal threshold values based on Gamma distribution. Our method applied on bimodal and multimodal images. Also It uses an iteratively rather than sequentially to decrease the number of operations. Further, using Gamma distribution give satisfying thresholding results in low-high contrast images where modes are symmetric or non-symmetric. For our results, we compared it with the original Gaussian Otsu's method.

Keywords: *Gamma distribution, Thresholding, Between-Class Variance.*

1. Introduction

Image segmentation is an important issue in many image-processing applications for separating objects of interest and displaying them obviously. Many segmentation methods have been proposed and applied in many different applications [1, 2, 3, 4]. In thresholding techniques only, there are more than 40 thresholding methods [5]. However, many studies focus on thresholding techniques for segmentation. Thresholding based on between-class variance (BCV) separates the object from the background in most images. As well as many studies were invented to improve BCV methods [13, 8]. In [14] the obtaining threshold value is based in Genetic Algorithm (GA) by finding out the valley bottom between two peaks in the histogram of image. Also, paper [15] discusses the impact of class probability and class variance to find the threshold value. However, all these studies are based on Gaussian distribution to find the threshold value, which gives limited results restricted by symmetric modes. Our contribution in this paper is to improve the Otsu's method [6, 7] in the case of non-symmetrical histogram by using Gamma distribution. We applied our method for estimating the optimal threshold values on bimodal and multimodal images. In addition, using iteratively algorithm rather than sequential.

A similar study was an improvement in minimum cross entropy thresholding (MCET) method by using Gamma distribution [10, 12]. It is shows that used Gamma distribution to improve both bimodal and multimodal thresholding methods give good results which encourage us to use Gamma distribution to improve another method (Otsu's method) and develop our method. Furthermore, Gamma distribution has the ability to deal with low-high contrast images where modes (intensity value distribution in the histogram) are symmetric or non-symmetric. While Gaussian distribution that has been used in old Otsu's method is suitable more for symmetric mode, but it gives limited results in non-symmetric mode.

This paper is organized as follows: Section 2 describes Gamma distribution concept. In section 3, we present our new formula to find the optimal threshold value on bimodal and multimodal images. Section 4 provides experimental results of the new formula on bimodal and multimodal images. Finally, conclusion and future work is presented in section 5.

2. Gamma Distribution

Histogram represents statistical information of image pixels. It describes pixels intensity distribution in an image by graphing the number of pixels intensity

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at each gray level. Essentially, we can say there are two types of shapes of a gray level (mode): symmetric and non-symmetric. For the symmetric mode Gaussian distribution works well to estimate the threshold value. On the other side, for the non-symmetric mode it is better to use Gamma distribution to describe it. The Gamma distribution defines as [9]:

$$f(x,\mu,N) = \frac{2q}{\mu}\frac{N^N}{\Gamma(N)}\left[\frac{qx}{\mu}\right]^{2N-1} e^{-N\left(\frac{qx}{\mu}\right)^2} \tag{1}$$

Where $q = \Gamma(N+0.5)/(\sqrt{N}\Gamma(N))$, x is the intensity of the pixel, μ is the mean value of the distribution and N is the shape of distribution. In our method Gamma distribution is used to estimate the mean values of image modes and then find the optimal threshold value. However, we used Gamma distribution because it has the ability to represent both symmetric and non-symmetric mode rather than the limited Gaussian distribution. If an histogram is divided into two classes $C_1 = \{0,1,..., t\}$ and $C_2 = \{t+1, t+2,...,255\}$. Where t is the threshold value which separates the two classes, the mean values will be as follows [9]

$$\begin{aligned}\mu_1(t) &= \sqrt{\frac{\sum_{i=0}^{t-1} h(i) i^2 q^2}{\sum_{i=0}^{t-1} h(i)}} = \frac{\sqrt{\sum_{i=0}^{t-1} h(i) i^2 q^2}}{\sqrt{\sum_{i=0}^{t-1} h(i)}} \\ &= \frac{\sqrt{\sum_{i=0}^{t-1} h(i) i^2 q^2}}{\sqrt{\omega_1(t)}} = \frac{\mu_{1a}(t)}{\mu_{0a}(t)}\end{aligned} \tag{2}$$

$$\begin{aligned}\mu_2(t) &= \sqrt{\frac{\sum_{i=t}^{255} h(i) i^2 q^2}{\sum_{i=t}^{255} h(i)}} = \frac{\sqrt{\sum_{i=t}^{255} h(i) i^2 q^2}}{\sqrt{\sum_{i=t}^{255} h(i)}} \\ &= \frac{\sqrt{\sum_{i=t}^{255} h(i) i^2 q^2}}{\sqrt{\omega_1(t)}} = \frac{\mu_{1b}(t)}{\mu_{0b}(t)}\end{aligned} \tag{3}$$

Where: $\omega_1(t) = \sum_{i=0}^{t-1} h(i) \quad ; \quad \omega_2(t) = \sum_{i=t}^{255} h(i)$

$h(i), i = 0..255$ is the histogram of image.

3. New Formula for Optimal Thresholding

The objective of Otsu's method is to find threshold value that maximizes the between-class variance ($\sigma_B^2(t)$) of the histogram as [6, 7]:

$$t^* = Arg \underset{0\le t<255}{Max} \{\sigma_B^2(t)\} \tag{4}$$

We consider $h(i), i = 0..255$ be the normalized bimodal histogram of the original image. We assume that the histogram of an image can be seen as a combination of Gamma distributions as shown in Eq. (2, 3). However, from Eq. (4) the between-class variance defines as:

$$\begin{aligned}\eta(t) = \sigma_B^2(t) &= \omega_1(t)(\mu_1(t) - \mu_T)^2 \\ &+ \omega_2(t)(\mu_2(t) - \mu_T)^2\end{aligned} \tag{5}$$

Where $\mu_T(t)$ the total mean of image is, $\mu_1(t)$ is the mean for the first class. And $\mu_2(t)$ is the mean for the second class. Therefore, we calculate the first derivative of Eq. (5) and setting it to zero to get the optimal threshold value as:

$$\begin{aligned}&\omega_1'(t)(\mu_1(t) - \mu_T)^2 \\ &+ 2\omega_1(t)(\mu_1(t) - \mu_T)(\mu_1(t) - \mu_T)' \\ &+ \omega_2'(t)(\mu_2(t) - \mu_T)^2 \\ &+ 2\omega_2(t)(\mu_2(t) - \mu_T)(\mu_2(t) - \mu_T)' = 0\end{aligned} \tag{6}$$

By substituting terms in Eq. (6) we have:

$$A(t) + B(t)t^2 = 0 \tag{7}$$

Where:

$$\begin{aligned}A(t) &= (\mu_1(t) - \mu_T)\left[(\mu_1(t) - \mu_T) - \frac{\omega_1(t)\mu_1(t)}{\mu_{0a}^2(t)}\right] \\ &- (\mu_2(t) - \mu_T)\left[(\mu_2(t) - \mu_T) - \frac{\omega_2(t)\mu_2(t)}{\mu_{0b}^2(t)}\right]\end{aligned} \tag{8}$$

$$\begin{aligned}B(t) = &\frac{\omega_1(t)(\mu_1(t) - \mu_T)q^2}{\mu_{0a}(t)\mu_{1a}(t)} \\ &- \frac{\omega_2(t)(\mu_2(t) - \mu_T)q^2}{\mu_{0b}(t)\mu_{1b}(t)}\end{aligned} \tag{9}$$

From the Eq. (7), we get the optimal threshold value as:

$$\therefore t = +\sqrt{\frac{-A(t)}{B(t)}} \tag{10}$$

This formula allows us to use an iterative thresholding algorithm that makes the new threshold value converge to the optimal threshold value and decrease the processing time of calculation.

In the same way for the multimodal thresholding (with some additional details) where the image contains more than one object (more than one mode). Anyway, for this case the resulted image will be segmented to M modes according to a set of threshold values $T = \{t_1, t_2, t_3, ..., t_{M-1}\}$, where $t_0 = 0 < t_1 < t_2 < ..... < t_{M-1} < t_M = 255$. The means of each mode (classes) or the k[th] mode defined as:

$$\mu_k(t) = \sqrt{\frac{\sum_{i=t_{k-1}}^{t_k} h(i) i^2 q^2}{\sum_{i=t_{k-1}}^{t_k} h(i)}} \quad , k = 1,2,..., M \tag{11}$$

And the optimal threshold value

$T^* = \{t_1^*, t_2^*, t_3^*, ..., t_{M-1}^*\}$ for each class will be estimated using the bimodal thresholding mentioned before.

Also we use *K*-mean algorithm [11] to estimate the initial threshold values for each mode in the images.

4. Experimental Results

We applied the method on real Synthetic Aperture Radar (SAR) images, and explained the result for both bimodal thresholding and multi-modal thresholding. The original test images, their histograms, and the thresholded images by the original Gaussian Otsu's method compared with our Gamma method are presented in each figure.

- Bi-modal Thresholding: In Fig. 1, we show the results of applying our bimodal thresholding methods. We obtained t=45 by using Gaussian Otsu's method which represented by blue line, and t=37 by using our Gamma method which represented by green line. Comparing the results Fig. 1(c, d) it is obviously that our method provides a good result. Similarly, we did on Fig. 2.

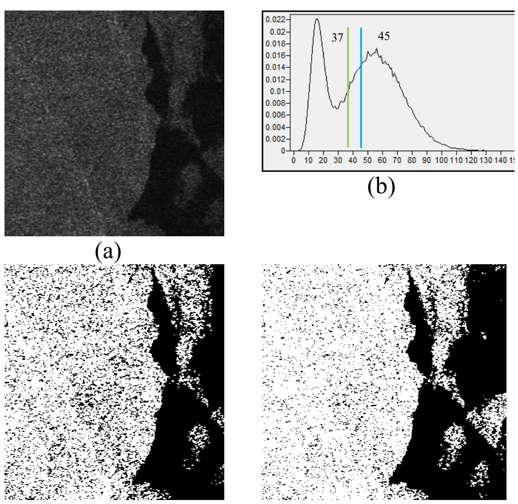


Figure 1: (a) the original image, (b) the histogram of original image with two estimated threshold values using Gaussian (blue line) and Gamma (green line), (c) Otsu Gaussian thresholding method (t=45) (d) our Gamma thresholding method (t=37).

As seen, our developed method was compared with the original Gaussian Otsu's method. In most of resultant images, Gamma gives more accurate outcomes than Gaussian.

However, the original Gaussian Otsu's method suffers from some weakness when treated low contrast images or where the object is small. Our method also suffers from the same problem. We can see the results in Fig. 2 and actually both methods give similar results. There are many papers invented to solve this problem [8] but it is out of scope this work.

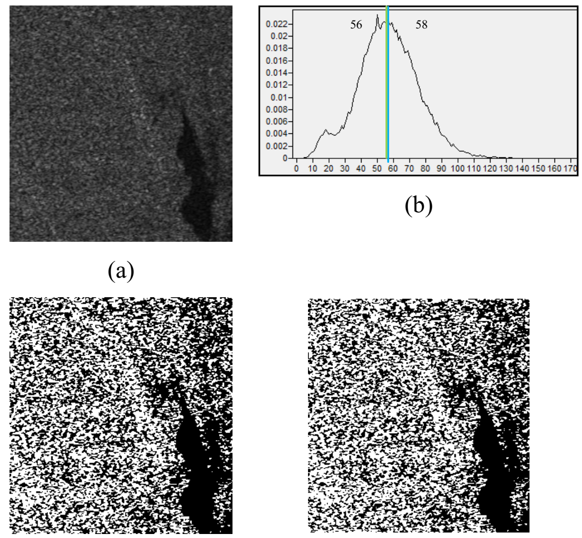
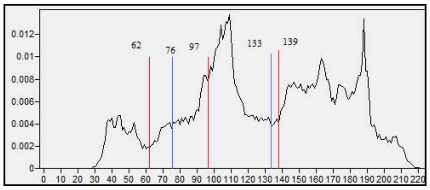


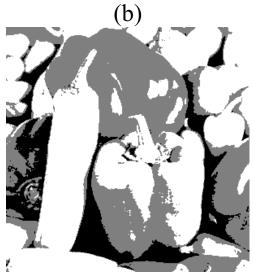
Figure 2: a) the original image, (b) the histogram of original image with two estimated thresholds using Gaussian (blue line) and Gamma (green line), (c) Otsu Gaussian thresholding method (t=58) (d) our Gamma thresholding method (t=56).

-Multi-modal Thresholding: In the multimodal thresholding that shown in Fig. 3, we applied our multimodal method on real image when the number of mode M=3, and M=4. We obtained t1=76, and t2=133 for M=3. For M=4, We obtained t1=62, t2=97, and t3=139.

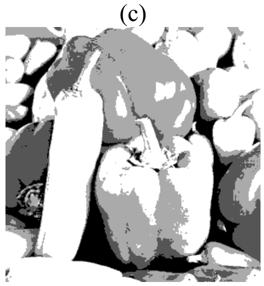
(a)



(b)



(c)



(d)

Figure 3: (a) the original image, (b) the histogram of original image with line of threshold values, (c) Gamma thresholding method with M=3 presented in blue line (t1=76, t2=133) (d) Gamma thresholding method with M=4 presented in red line (t1=62, t2=97, and t3=139).

5. Conclusion and Future Work

We have proposed a new method for image thresholding using BCV based on Gamma distribution that solves the problem of non-symmetric histogram of images. We have apply it iteratively to force the threshold value converge to be optimal value and reduce the computational as well. In addition, we have provided experimental results that show our method efficiency "compared to the original Otsu’s method". However, Otsu’s method has some flaws when it comes to deal with low contrast images as in [8] but it is not a subject in this paper. The weakness of the Otsu’s method when applied on images which contain a small object is affected also in our method and doesn’t produce a good result than Otsu’s method. In the future work, we will extend the proposed method to solve the problem of detecting a small object in the images.

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