New Edge Detector Using 2D Gamma Distribution

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Abstract—Edge detection is an important field in image processing. It can be used in many applications such as feature detection and extraction. An edge is detected as the discontinuity in intensity level of a point. So edges are mostly detected using either the first derivatives, called gradient, or the second derivatives, called Laplacien. In this paper, we propose a new edge detector using the gradient of Gamma distribution. The results were very good compared with the well-known Sobel gradient and Canny gradient results.

Index Terms—Edge Detection, Gamma Distribution, Gradient.

I. INTRODUCTION

Edge detection is an important field in image processing. It can be used in many applications such as feature detection and extraction. An edge is detected as the discontinuity in intensity level of a point. So edges are mostly detected using either the first derivatives, called gradient, or the second derivatives, called Laplacien. In this paper, we propose a new edge detector using the gradient of Gamma distribution. The results were very good compared with the well-known Sobel gradient and Canny gradient results.

II. OVERVIEW OF THE 2D GAMMA DISTRIBUTION

The probability density function for Gamma distribution, as in [3], is:

\[ G1D(x) = \frac{x^{\alpha-1}e^{-x/\theta}}{\Gamma(\alpha)\theta^\alpha} \]

Where \( \alpha \) and \( \theta \) are the parameters for shape and scale, respectively, and both are larger than zero. Where \( \Gamma(\alpha) \) the Gamma function, as in [4], is:

\[ \Gamma(n) = (n-1)! \]

So the 2D Gamma distribution will be:

\[ G2D(x, y) = G1D(x) \cdot G1D(y) = \frac{x^{\alpha-1}y^{\alpha-1}e^{-x/\theta}}{\left(\Gamma(\alpha)\theta^\alpha\right)^2} \]

So as you can see in these figures, the Gamma distribution is not symmetrical. By changing the Gamma parameters, we get different shapes covering more cases. Tables III to V are examples of some generated masks using Gamma distribution.

On the other hand, Sobel masks, shown in Table I, are symmetrical; the absolute values of \( M_x \) components are symmetrical with regard to y-axis, and the absolute values of
My components are symmetrical with regard to x-axis. Sobel gradients produce high response for vertical and horizontal edges, highly sensitive to noise and detect them as edges, and produce thick edges. The advantage of this approach is that it is relatively computationally inexpensive.

Another well-known edge detector is Canny edge detector. It is based on the Gaussian function which is also symmetrical. Figure 9 displays the Gaussian function curve. So as a result, Gamma distribution is a more general method for edge detection.

### III. Edge Detection Using the 2D Gamma Distribution

To detect the edges in an image \( f(x, y) \), we use the first derivatives - the gradient. First, the two gradient masks, \( M_x \) and \( M_y \), are created. Then the result of convolving these two masks with the image \( f \) is used to compute the gradient of the image \( f \) as expressed in the following equation:

\[
|\nabla f| = \sqrt{F_x^2 + F_y^2}
\]

Where \( F_x \) and \( F_y \) are the result of the convolution of the two masks, expressed in the following equations:

\[
F_x(x, y) = (f \ast M_x)(x, y)
\]

\[
F_y(x, y) = (f \ast M_y)(x, y)
\]

Where \( \ast \) is the convolution symbol and \( f \) is the input image.

<table>
<thead>
<tr>
<th>( M_x )</th>
<th>( M_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>1</td>
</tr>
<tr>
<td>-2</td>
<td>2</td>
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<tr>
<td>-1</td>
<td>1</td>
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<td>0</td>
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<tr>
<td>-1</td>
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</table>
To create the gradient masks $M_x$ and $M_y$, we use the first derivatives for $x$ and $y$ of the 2D Gamma distribution. The first derivatives are computed as follows:

$$\frac{\partial}{\partial x} G_2D(x,y) = \frac{e^{(-x-y)/\theta}(x^{\alpha-1}y^{\alpha-2})(\alpha - 1 - \frac{y}{\beta})}{\Gamma(\alpha)\theta^\alpha}$$

$$\frac{\partial}{\partial y} G_2D(x,y) = \frac{e^{(-x-y)/\theta}(x^{\alpha-2}y^{\alpha-1})(\alpha - 1 - \frac{x}{\alpha})}{\Gamma(\alpha)\theta^\alpha}$$

Table II shows the $M_x$ and $M_y$ masks. Where $d$ is the distance, which is a user defined value. And these mask values need to be normalized before it is used because the summation of the gradient mask must be zero. One way to achieve this, is to divide the positive values by the summation of all the positive values in the mask, also dividing the negative values by the summation of all the negative values in the mask after multiplying it by -1 so they remain negative.

When $d = 1$, $\theta = 50$, and $\alpha = 2$, we get the normalized masks in Table III. While if $d = 1$, $\theta = 50$, and $\alpha = 8$, we get the normalized masks in Table IV. When $d = 1$, $\theta = 80$, and $\alpha = 2$, we get the normalized masks in Table V.

So in our method, we generated masks with different values of the parameters: from $\alpha = 2$ to $\alpha = 12$, and from $\theta = 50$ to $\theta = 80$, yielding 300 different mask sets ($M_x$ and $M_y$) are generated. Then after the convolution of one set of masks, the gradient is calculated. Then the final gradient is assigned to the maximum gradient value calculated from these different masks. This way, the method becomes less sensitive to noise and produces thinner edges because it takes for each pixel the largest gradient value from the most suitable mask for that pixel.

### IV. Experimental Results

The Gamma edge detector was implemented as described in the previous section and tested on three images. The Gamma, Sobel, and Canny gradients are obtained for these images. These gradients are illustrated in Tables VI, VII, and VIII. For each gradient, a segmented image was obtained using a suitable threshold. By visually comparing the results, we notice that the Gamma gradients are much better than Sobel gradients and the lines are thinner. By comparing the Gamma gradients with the Canny gradients, the Gamma gradients are similar to Canny gradients but they have thinner edges. This is especially clear in Table VIII.
### TABLE VI
**Gradients of rice image**

<table>
<thead>
<tr>
<th>Original Image</th>
<th>Sobel Gradient</th>
<th>Canny Gradient</th>
<th>Gamma Gradient</th>
</tr>
</thead>
</table>

### TABLE VII
**Gradients of blood cells image**

<table>
<thead>
<tr>
<th>Original Image</th>
<th>Sobel Gradient</th>
<th>Canny Gradient</th>
<th>Gamma Gradient</th>
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</table>

### TABLE VIII
**Gradients of an image of a square with a hole**

<table>
<thead>
<tr>
<th>Original Image</th>
<th>Sobel Gradient</th>
<th>Canny Gradient</th>
<th>Gamma Gradient</th>
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V. SUMMARY AND FUTURE WORK

In this paper, we presented a new method for detecting edges of an image. We used the gradient of Gamma distribution since Gamma distribution is a general method for edge detection. Many masks were generated using different values of Gamma parameters. Then for each pixel, the maximum result was taken as the gradient for this pixel. The results were very good compared with Sobel and Canny gradient results. They were less sensitive to noise and the edges were thinner.

Further work can be using the laplacian of the Gamma distribution or improving the efficiency of the algorithm used to calculate the Gamma gradient.

REFERENCES


Ali El-Zaart was a senior software developer at Department of Research and Development, Semiconductor Insight, Ottawa, Canada during 2000-2001. From 2001 to 2004, he was an assistant professor at the Department of Biomedical Technology, College of Applied Medical Sciences, King Saud University. Since 2004, he is an assistant professor at the Department of Computer Science, College of computer and information Sciences, King Saud University. He has published numerous articles and proceedings in the areas of image processing, remote sensing, and computer vision. He received a B.Sc. in computer science from the Lebanese University; Beirut, Lebanon in 1990, M.Sc. degree in computer science from the University of Sherbrooke, Sherbrooke, Canada in 1996, and Ph.D. degree in computer science from the University of Sherbrooke, Sherbrooke, Canada in 2001. His research interests include image processing, pattern recognition, remote sensing, and computer vision.

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