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New Edge Detector Using 2D Gamma Distribution

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***Abstract*—Edge detection is an important field in image processing. It can be used in many applications such as feature detection and extraction. An edge is detected as the discontinuity in intensity level of a point. So edges are mostly detected using either the first derivatives, called gradient, or the second derivatives, called Laplacien. In this paper, we propose a new edge detector using the gradient of Gamma distribution. The results were very good compared with the well-known Sobel gradient and Canny gradient results.**

***Index Terms*—Edge Detection, Gamma Distribution, Gradient.**

I. Introduction

Edge detection is an important field in image processing. It can be used in many applications such as feature detection and extraction. An edge is detected as the discontinuity in intensity level of a point. The edge information of an image could be used to identify objects in the image, or on the other hand, enhancing the image by sharpening it. Edges are mostly detected using either the first derivatives, called gradient, or the second derivatives, called laplacien. Laplacien is more sensitive to noise since it uses more information because the nature of the second derivatives.

In this paper, we propose a new edge detector using the gradient of Gamma distribution. We generated many masks with different values of Gamma parameters and took the maximum result from the convolution of these masks with the input image, to reduce the sensitivity to noise and produce thinner edges. The results were very good compared with the well-known Sobel gradient [2] and Canny gradient [1] results.

The rest of the paper is organized as follows: Section 2 discusses the Gamma distribution while Section 3 discusses how it is used for edge detection. The results are displayed in Section 4 and compared with Sobel and Canny results. Then the paper is concluded in Section 5.

II. Overview of the 2D Gamma Distribution

The probability density function for Gamma distribution, as in [3], is:

$$G1D(x) = \frac{x^{\alpha-1}e^{-x/\theta}}{\Gamma(\alpha)\theta^{\alpha}}$$

Where α and θ are the parameters for shape and scale, respectively, and both are larger than zero. Where $\Gamma(\alpha)$ the Gamma function, as in [4], is:

$$\Gamma(n) = (n-1)!$$

So the 2D Gamma distribution will be:

$$\begin{aligned} G2D(x,y) &= G1D(x)\cdot G1D(y) \\ &= (\frac{x^{\alpha-1}e^{-x/\theta}}{\Gamma(\alpha)\theta^{\alpha}})(\frac{y^{\alpha-1}e^{-y/\theta}}{\Gamma(\alpha)\theta^{\alpha}}) \\ &= \frac{x^{\alpha-1}y^{\alpha-1}e^{(-x-y)/\theta}}{(\Gamma(\alpha)\theta^{\alpha})^2} \end{aligned}$$

Figures 1 to 8 illustrate the Gamma distribution with different values for the parameters. Notice that the curve is not symmetrical. The value of α is the same in both Figures 1 and 2 but θ is larger in the second one, resulting in almost the same shape but larger (expanded a larger area). Notice that from Figure 3 to Figure 6 the peak is shifting to the right as α gets larger (it gets more symmetrical). Figures 7 and 8 view the Gamma distribution in 2D.

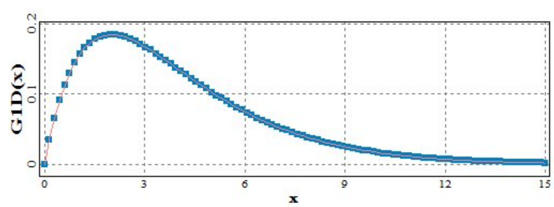

Fig. 1. $G1D(x)$ when $\alpha = \theta = 2$

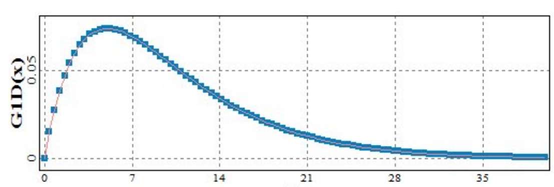

Fig. 2. $G1D(x)$ when $\alpha = 2$ and $\theta = 5$

So as you can see in these figures, the Gamma distribution is not symmetrical. By changing the Gamma parameters, we get different shapes covering more cases. Tables III to V are examples of some generated masks using Gamma distribution.

On the other hand, Sobel masks, shown in Table I, are symmetrical; the absolute values of M_x components are symmetrical with regard to y-axis, and the absolute values of

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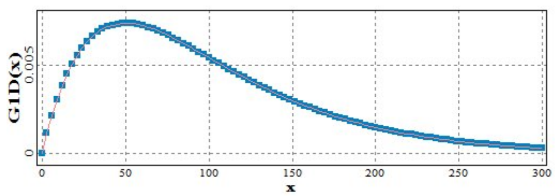


Fig. 3. $G1D(x)$ when $\alpha = 2$ and $\theta = 50$

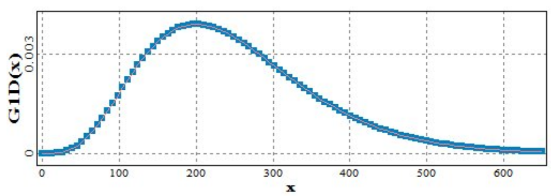


Fig. 4. $G1D(x)$ when $\alpha = 5$ and $\theta = 50$

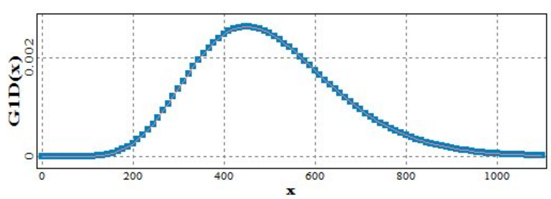


Fig. 5. $G1D(x)$ when $\alpha = 10$ and $\theta = 50$

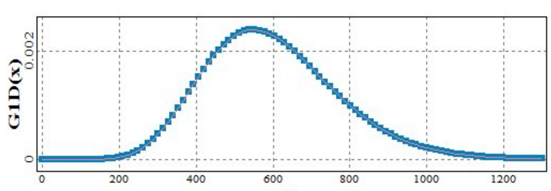


Fig. 6. $G1D(x)$ when $\alpha = 12$ and $\theta = 50$

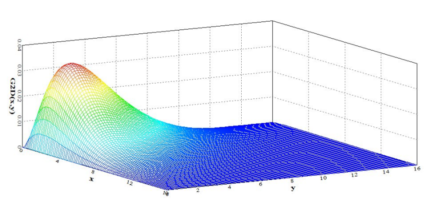


Fig. 7. $G2D(x, y)$ when $\alpha = \theta = 2$

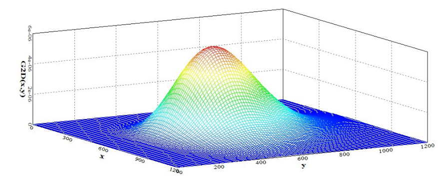


Fig. 8. $G2D(x, y)$ when $\alpha = 12$ and $\theta = 50$

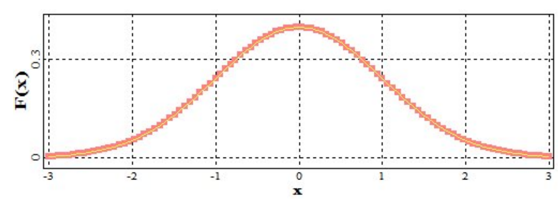


Fig. 9. Gaussian Function when $\mu = 0$ and $\sigma = 1$

TABLE I
SOBEL MASKS

M_x

-1	0	1
-2	0	2
-1	0	1

M_y

1	2	1
0	0	0
-1	-2	-1

M_y components are symmetrical with regard to x-axis. Sobel gradients produce high response for vertical and horizontal edges, highly sensitive to noise and detect them as edges, and produce thick edges. The advantage of this approach is that it is relatively computationally inexpensive.

Another well-known edge detector is Canny edge detector. It is based on the Gaussian function which is also symmetrical. Figure 9 displays the Gaussian function curve. So as a result, Gamma distribution is a more general method for edge detection.

III. EDGE DETECTION USING THE 2D GAMMA DISTRIBUTION

To detect the edges in an image $f(x, y)$, we use the first derivatives - the gradient. First, the two gradient masks, M_x and M_y, are created. Then the result of convolving these two masks with the image f is used to compute the gradient of the image f as expressed in the following equation:

$$| \nabla f |= \sqrt{{F_x}^2 + {F_y}^2}$$

Where F_x and F_y are the result of the convolution of the two masks, expressed in the following equations:

$$F_x(x, y) = (f * M_x)(x, y)$$

$$F_y(x, y) = (f * M_y)(x, y)$$

Where * is the convolution symbol and f is the input image.

TABLE II
GAMMA GRADIENT MASKS

M_x

$\frac{\partial}{\partial x}G2D(x-d, y-d)$	$\frac{\partial}{\partial x}G2D(x-d, y)$	$\frac{\partial}{\partial x}G2D(x-d, y+d)$
$\frac{\partial}{\partial x}G2D(x, y-d)$	$\frac{\partial}{\partial x}G2D(x, y)$	$\frac{\partial}{\partial x}G2D(x, y+d)$
$\frac{\partial}{\partial x}G2D(x+d, y-d)$	$\frac{\partial}{\partial x}G2D(x+d, y)$	$\frac{\partial}{\partial x}G2D(x+d, y+d)$

M_y

$\frac{\partial}{\partial y}G2D(x-d, y-d)$	$\frac{\partial}{\partial y}G2D(x-d, y)$	$\frac{\partial}{\partial y}G2D(x-d, y+d)$
$\frac{\partial}{\partial y}G2D(x, y-d)$	$\frac{\partial}{\partial y}G2D(x, y)$	$\frac{\partial}{\partial y}G2D(x, y+d)$
$\frac{\partial}{\partial y}G2D(x+d, y-d)$	$\frac{\partial}{\partial y}G2D(x+d, y)$	$\frac{\partial}{\partial y}G2D(x+d, y+d)$

TABLE III
M_x AND M_y MASKS WHEN $d = 1$, $\theta = 50$, AND $\alpha = 2$

M_x

-0.360513	0.092859	0.267654
-0.332793	0.0857192	0.247074
-0.306694	0.0789965	0.227697

M_y

-0.360513	-0.332793	-0.306694
0.092859	0.0857192	0.0789965
0.267654	0.247074	0.227697

To create the gradient masks M_x and M_y, we use the first derivatives for x and y of the 2D Gamma distribution. The first derivatives are computed as follows:

$$\frac{\partial}{\partial x}G2D(x, y) = \frac{(e^{(-x-y)/\theta})(x^{\alpha-2}y^{\alpha-1})(\alpha - 1 - \frac{x}{\theta})}{\Gamma(\alpha)\theta^{\alpha}}$$

$$\frac{\partial}{\partial y}G2D(x, y) = \frac{(e^{(-x-y)/\theta})(x^{\alpha-1}y^{\alpha-2})(\alpha - 1 - \frac{y}{\theta})}{\Gamma(\alpha)\theta^{\alpha}}$$

Table II shows the M_x and M_y masks. Where d is the distance, which is a user defined value. And these mask values need to be normalized before it is used because the summation of the gradient mask must be zero. One way to achieve this, is to divide the positive values by the summation of all the positive values in the mask, also dividing the negative values by the summation of all the negative values in the mask after multiplying it by -1 so they remain negative.

When $d = 1$, $\theta = 50$, and $\alpha = 2$, we get the normalized masks in Table III. While if $d = 1$, $\theta = 50$, and $\alpha = 8$, we get the normalized masks in Table IV. When $d = 1$, $\theta = 80$, and $\alpha = 2$, we get the normalized masks in Tables V.

TABLE IV
M_x AND M_y MASKS WHEN $d = 1$, $\theta = 50$, AND $\alpha = 8$

M_x

-0.00149872	7.12911e-007	0.001498
-0.00143174	6.81055e-007	0.00143106
-0.99707	0.000474288	0.996595

M_y

-0.00149872	-0.00143174	-0.99707
7.12911e-007	6.81055e-007	0.000474288
0.001498	0.00143106	0.996595

TABLE V
M_x AND M_y MASKS WHEN $d = 1$, $\theta = 80$, AND $\alpha = 2$

M_x

-0.350203	0.0892027	0.261001
-0.333123	0.0848521	0.248271
-0.316673	0.080662	0.236011

M_y

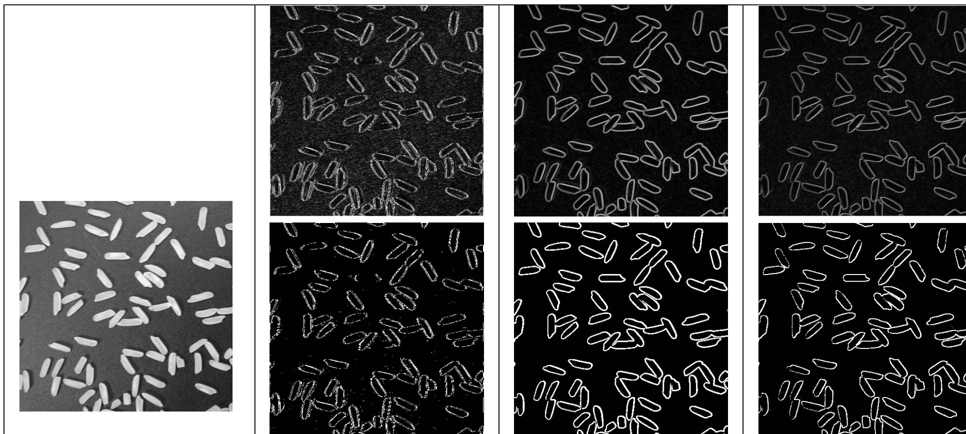
-0.350203	-0.333123	-0.316673
0.0892027	0.0848521	0.080662
0.261001	0.248271	0.236011

So in our method, we generated masks with different values of the parameters: from $\alpha = 2$ to $\alpha = 12$, and from $\theta = 50$ to $\theta = 80$, yielding 300 different mask sets (M_x and M_y) are generated. Then after the convolution of one set of masks, the gradient is calculated. Then the final gradient is assigned to the maximum gradient value calculated from these different masks. This way, the method becomes less sensitive to noise and produces thinner edges because it takes for each pixel the largest gradient value from the most suitable mask for that pixel.

IV. EXPERIMENTAL RESULTS

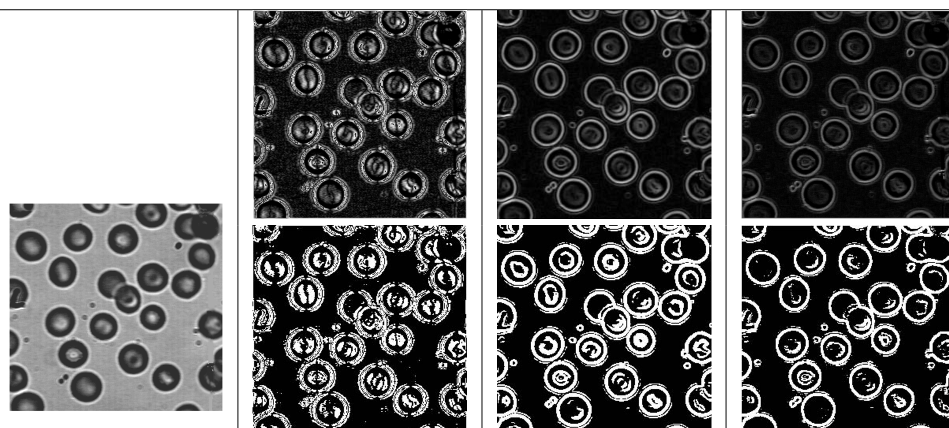
The Gamma edge detector was implemented as described in the previous section and tested on three images. The Gamma, Sobel, and Canny gradients are obtained for these images. These gradients are illustrated in Tables VI, VII, and VIII. For each gradient, a segmented image was obtained using a suitable threshold. By visually comparing the results, we notice that the Gamma gradients are much better than Sobel gradients and the lines are thinner. By comparing the Gamma gradients with the Canny gradients, the Gamma gradients are similar to Canny gradients but they have thinner edges. This is especially clear in Table VIII.

TABLE VI
GRADIENTS OF RICE IMAGE



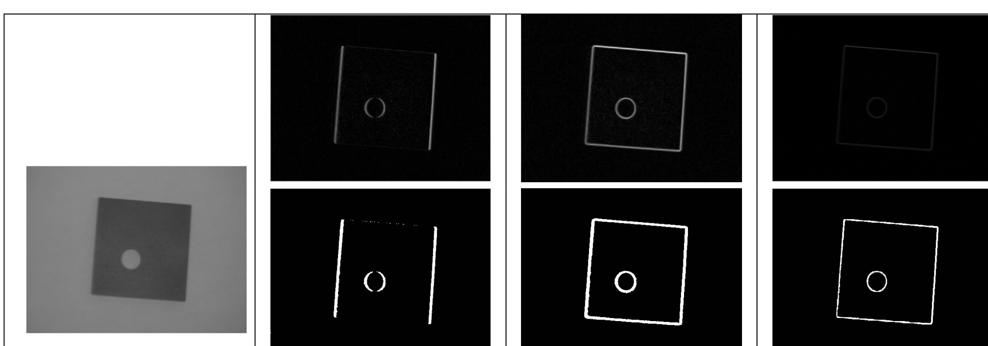
Original Image	Sobel Gradient	Canny Gradient	Gamma Gradient

TABLE VII
GRADIENTS OF BLOOD CELLS IMAGE



Original Image	Sobel Gradient	Canny Gradient	Gamma Gradient

TABLE VIII
GRADIENTS OF AN IMAGE OF A SQUARE WITH A HOLE



Original Image	Sobel Gradient	Canny Gradient	Gamma Gradient

V. Summary and Future Work

In this paper, we presented a new method for detecting edges of an image. We used the gradient of Gamma distribution since Gamma distribution is a general method for edge detection. Many masks were generated using different values of Gamma parameters. Then for each pixel, the maximum result was taken as the gradient for this pixel. The results were very good compared with Sobel and Canny gradient results. They were less sensitive to noise and the edges were thinner.

Further work can be using the laplacien of the Gamma distribution or improving the efficiency of the algorithm used to calculate the Gamma gradient.

References

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