

A Cellular Automata Based Model for Traffic in Congested City

Sukanta Das¹

Meghnath Saha²

Biplab K Sikdar³

¹Dept. of Information Tech., Bengal Engg. and Sc. University, Shibpur, Howrah, WB, India 711103, sukanta@it.becs.ac.in

² Dept. of Computer Sc. & Engg., Murshidabad College of Engg. & Tech., Baharampur, Murshidabad, India, meghnathsahaindia@yahoo.com

³ Dept. of Computer Sc. & Tech., Bengal Engg. and Sc. University, Shibpur, Howrah, WB, India 711103 biplab@cs.becs.ac.in

Abstract—This work targets modeling traffic flow in roads of a congested city. The Cellular Automata (CA) have been configured for such modeling. The urban traffic models [2], in general, concentrate on low traffic density. The inabilities of traditional models to address the issues arise out of heavy traffic in crowded cities of third world countries are properly addressed in the proposed elementary CA based model. The effectiveness of the model is verified considering the traffic in Kolkata. It is established that the model can fairly reproduce various traffic conditions as found empirically. Further, the proposed model succeeds the limitations of traffic flow models in congestion and also reproduces the empirical results.

Keywords: City traffic, Cellular Automata (CA), Rule 226

I. Introduction

A traffic system can be considered as a non-equilibrium many-body system being able to exchange energy and particles (vehicles, for example) with the surroundings. The behavior of this kind of systems is not governed by the general principles, – that is, of thermodynamics and statistical mechanics, as closed systems in equilibrium [6]. The units of traffic systems are self-driven, moving with the aid of their own energy sources and distinct from the driven particles whose movements are influenced by external forces such as pressure, gravitation, and electrical forces.

Modeling traffic flow are the interest of traffic science and engineering. The traffic science deals with the dynamical phase. Such a non-equilibrium systems are theoretically not as well understood as the equilibrium systems [3]. The other important issues involved with non-equilibrium traffic system are: fluctuations near steady states, how steady states are reached when the initial state of the system is not steady, can it be self-organized, are the phase transition points characterized with long-range correlations and scaling exponents of power laws, etc.

The growth of traffic volumes demands new and efficient solutions in order to accommodate the changing requirements. More efficient usage of the existing road infrastructure is related to the management and controlling of traffic flows, without compromising the safety and convenience

This work is supported by the Sponsored Cellular Automata Research Projects, Bengal Engineering and Science University, Shibpur, West Bengal, India-711103.

of the people. The controlling and management of traffic covers the whole road network, and events & circumstances constituting the traffic situation.

The modeling of traffic flow phenomena was dominated by two theoretical approaches. The so called car-following models are based on the fact that the behavior of a driver is determined by the leading vehicle. This assumption leads to dynamical velocity equations which in general depend on the distance to the leading vehicles and on the velocity difference between the leading and following vehicle. The alternative approach does not treat the individual cars but describes the dynamics of traffic networks in terms of macroscopic variables.

The major problem of present car-following models is that these are difficult to treat in computer simulations. On the other hand, the macroscopic approaches lead to some difficulties even though the large networks can be treated in principle. Further, the present macroscopic models handle a large number of parameters that don't have counterpart in empirical investigation and the information obtained in macroscopic models can not trace individual cars.

In order to address the above issues, cellular automata (CA) based traffic models have been reported [10]. The CA models are microscopic models which are by design well suited for large scale computer simulations. A comparison of the simulations with empirical data shows that the very simple approaches give meaningful results. The CA models for urban traffic [2], in general, concentrate on low traffic density. The work presented in [4], [10], [6] are mostly devoted to the study of highways. All these can not properly address the issues arise out of heavy traffic in this crowded cities.

In this work, we target modeling of city traffic in narrow roads and apply the model to analyze the traffic in different roads of Kolkata. We show that the inabilities of traditional models can be properly addressed in the proposed elementary CA based model that denies the necessity of probabilistic CA.

II. Traditional cellular automata model

This section introduces the preliminaries of cellular automata (*CA*) and the basics of *CA* models so far been reported for traffic flow.

A. Cellular automata basics

The concept of Cellular Automaton (*CA*) was initiated in the early 1950s by J. von Neumann and Stan Ulam. After the introduction, a number of researchers proposed simplified structure of *CA* amenable to characterization.

A *CA* is the discrete spatially-extended dynamical system that has been studied extensively as models of physical systems. It evolves in discrete space and time. In its simplest form, as it is proposed by Stephen Wolfram [12], a *CA* consists of a lattice of cells, each of which stores a discrete variable at time t that refers to the present state of the *CA*. The next state of the cell at $(t+1)$ is affected by its state and the states of its *neighbors* at time t . In the current work, we concentrate on 3-neighborhood (self, left and right neighbors) 1-dimensional *CA*, where a *CA* cell is having two states - 0 or 1. Such a *CA* is referred to as the elementary *CA*. The next state of a *CA* cell is

$$S_i^{t+1} = f(S_{i-1}^t, S_i^t, S_{i+1}^t) \quad (1)$$

where f is the next state function; S_{i-1}^t , S_i^t and S_{i+1}^t are the present states of the left neighbor, self and right neighbor of the i^{th} *CA* cell at time t . The f can be expressed as a lookup table as shown in *Table I*. The decimal equivalent of the 8 outputs is called 'Rule' \mathcal{R}_i . In a two-state 3-neighborhood *CA*, there can be a total of 2^8 (256) rules. Two such rules are 184 and 226 (*Table I*).

If the left most and right most cells of an n -cell *CA* are the neighbors of each other, the *CA* is *periodic boundary*. In our proposed model, we adopt the periodic boundary *CA*. Most of the *CA* based traffic models, so far been proposed, are of probabilistic in nature. The next state of the cell of such a *CA* is updated based on the present states of its neighbors as well as a predetermined probability. In the current work, the traffic model developed is based on the elementary (deterministic) *CA*.

B. 1-dimensional traffic models

In one dimensional models, a road is considered as a line of sites each of these can be either occupied by a vehicle/car or empty at time t . All cars on road travel in the same direction (say to the right). Their positions are updated synchronously. During motion, each car can be at rest or jump to the nearest neighboring site, along the direction of motion, only if its destination site is empty. This means the driver is short-sighted and do not know whether the car in front can move or is stuck by another car. Therefore, the

state of each site s_i at $(t+1)$ depends on its state at t (present state) as well as the states of its two nearest neighbors s_{i-1} and s_{i+1} . It is summarized in the following table,

Present car position :	111	110	101	100	011	010	001	000
Next car position :	1	0	1	1	1	0	0	0

where car present within the site is represented by 1. The 0 represents the site is empty. This corresponds to the *CA* rule 184 (*Table I*) if a site is considered as a *CA* cell.

The above mentioned dynamics captures interesting features of car movement. Let consider an instance

...0010000100000010000100...

It is a *free* traffic regime in which all the cars are able to move and points to low car density ρ in the system. The average velocity $\langle v \rangle$, defined as number of movements divided by the number of cars, is then $\langle v_f \rangle = 1$, where the subscript f indicates a free state. On the other hand, in a high density configuration such as

...01100011101001101001111010...

only 7 out of 14 cars can move and $\langle v \rangle = 1/2$. This represents a partially jammed regime.

In this model, the car occupancy of adjacent site is highly correlated and the vehicles cannot move until a hole (vacant site) is found. The car distribution tries to adjust itself to a situation where there is no spacing between the consecutive cars. Therefore, it is expected that the number of moving cars simply equals to the number of empty cells in high density [8]. Thus, the number of movements is $L(1 - \rho)$; L is the length of a road. The average velocity in jammed phase is

$$\langle v \rangle = \frac{1 - \rho}{\rho} \quad (2)$$

A more effective version of the above *CA* model is reported in [10]. It assumes the i^{th} car may have several possible velocities $u_i = 0, 1, 2, \dots, v_{max}$, and d_i is the distance, along the road, separating cars i and $i + 1$. The updating rule is

- The cars accelerate when possible: $u_i \rightarrow u'_i = u_i + 1$, if $u_i < u_{max}$.
- The cars slow down when required: $u'_i \rightarrow u''_i = d_i - 1$, if $u'_i \geq d_i$.
- The cars have a random behavior: $u''_i \rightarrow u'''_i = u''_i - 1$, with probability p_i if $u''_i > 0$.
- The cars move u'''_i sites ahead.

This rule captures some important behaviors of real traffic on a highway. The velocity fluctuations are due to non-deterministic behavior of the drivers, and *stop-and-go* waves observed in high density traffic regime (that is, some

TABLE I
LOOK-UP TABLE FOR RULE 184 IND 226

Present state :	111	110	101	100	011	010	001	000	Rule
	(7)	(6)	(5)	(4)	(3)	(2)	(1)	(0)	
(i) Next State :	1	0	1	1	1	0	0	0	184
(ii) Next State :	1	1	1	0	0	0	1	0	226

cars get stop for no specific reasons). The *NS* model [10] is a probabilistic *CA* based model, which in the case of $v_{max} = 1$ and the deterministic limit (that is, randomization probability $p_i = 0$) is equivalent to *CA* rule 184 in Wolfram notation.

A more realistic *CA* model of traffic flow was done by so-called velocity-dependent-randomization (*VDR*) model [1]. It extended the set of update rules of *NS* model. A velocity-dependent randomization $p_i(v)$ was introduced. The simplest version was

$$p_i(v) = \begin{cases} p_0, & \text{for } v = 0 \\ p, & \text{for } v > 0 \end{cases}$$

III. The Proposed CA Based Model

This section identifies the properties of our target city traffic and then propose a *CA* based model for the traffic flow.

A. City traffic in narrow roads

In a congested city, the roads are hardly free during most of the day time. A portion of the traffic network remains congested. The spacings between consecutive vehicles, moving in a particular direction, are very small. If not (red signal) obstructed, the vehicles move continuously in a particular direction. The overtaking is not entertained. However, the velocity of a car may vary in different segments of a road. If one or more vehicles are ahead of the vehicle c , it moves with a uniform velocity. On the other hand, if c finds a gap, it accelerates and covers the distance. The properties of such a traffic system can be summarized as follows:

1. Car density (ρ) is not very low. It is always above some threshold (ρ_{Th}). That is, $\rho \geq \rho_{Th}$.
2. The spacing between two consecutive vehicles is small.
3. The vehicles flow continuously in a particular direction.
4. A vehicle can cover a distance immediately if it finds a gap.

It can be noted that since we study the congested city traffic in narrow roads where continuous movement of vehicles is a general feature, the *NS* and *VDR* models can not be appropriate for modeling such a system [7].

B. CA model of city traffic

The properties of city traffic described in the earlier subsection define the occupancy of a cell/site at time $t + 1$. It depends on the occupancies of nearest neighbors at time t . The roads and the movements of vehicles can be described by an n -cell *CA* (cellular automata), where a *CA* cell i corresponds to road site. The current state of a cell is 1 implies that the corresponding site i is occupied by a vehicle; 0 implies empty site.

In a narrow road, the vehicles move in the same direction and the sites are updated synchronously during the motion (*i*) if a site s_i & its left neighbor s_{i-1} are occupied, the site s_i remains occupied in the next time instant, (*ii*) if s_i is empty but its right neighbor s_{i+1} is occupied, then s_i is filled up in the next time instant/step, and (*iii*) if the s_{i-1} is empty & s_i is occupied, the vehicle at s_i moves towards s_{i+1} and the site s_i becomes empty. Such phase transitions of sites can be appropriately modeled by a *CA* chosen for the road. The *CA* rule 226 (*Table II*) can exactly map such a flow of traffic described in (*i*), (*ii*), (*iii*). Therefore, an n -cell *CA*, each cell configured with rule 226, can demonstrate to the traffic flow in roads of a congested city.

To formalize the above properties of city traffic, we define car density ρ as the number of cars per kilometer. ρ_{Th} is specified from simulation of real life traffic data. We consider the length of each site is 2.5m which is slightly larger than the average length of a vehicle/car. Each car may take any velocity from v_{min} to v_{max} . Since cars even flow in jammed condition, $v_{min} > 0$. The acceleration rule adopted is quite similar as it is proposed in [5], [9]. If a car finds a number of empty sites in front of it, the car speed can rapidly be increased to its limit. Therefore, v_{max} depends on the maximum number of empty sites in front of a car. Hence, the v_{max} in turn depends on ρ_{Th} .

To illustrate the traffic scenario, let us consider that half of a road segment is occupied by vehicles, that is, the segment is partially jammed. *Table III* shows the flow of traffic in such a condition. We assume that the immediate next sites at both ends of the segment are empty. In *time step 1* (car position), we mark two cars (hat and underline) as test cars. It can be observed that the car with hat symbol moves one

TABLE II
RULE 226

Present car position :	111	110	101	100	011	010	001	000	<i>Rule</i>
	(7)	(6)	(5)	(4)	(3)	(2)	(1)	(0)	
Next car position :	1	1	1	0	0	0	1	0	226

TABLE III
FLOW OF VEHICLES IN PARTIALLY JAMMED CONDITION

time step 1 (car position) :	...	0	0	1	1	$\hat{1}$	1	0	0	1	0	1	<u>1</u>	0	0	0	1	1	0	...
time step 2 (car position) :	...	0	1	0	1	1	$\hat{1}$	0	1	0	1	0	1	0	0	<u>1</u>	0	1	0	...
time step 3 (car position) :	...	1	0	1	0	1	1	$\hat{1}$	0	1	0	1	0	0	1	0	<u>1</u>	0	0	...
time step 4 (car position) :	...	0	1	0	1	0	1	1	$\hat{1}$	0	1	0	0	1	0	1	0	0	0	...

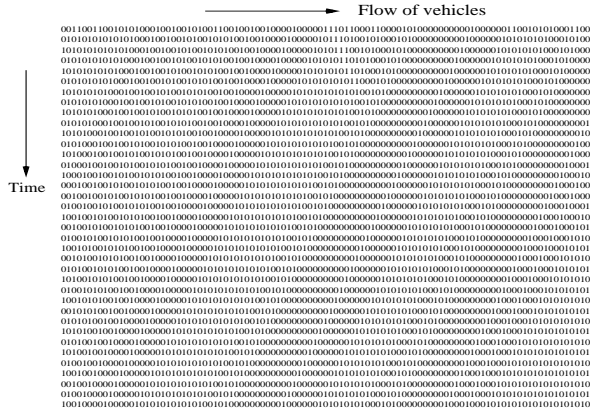


Fig. 1. Traffic flow in a small road segment (partially jammed traffic situation). 0 represents empty site and 1 represents presence of a car.

cell in each time step as either the car is guided by another one ahead of it or it finds only single empty site to move. On the other hand, the underlined car covers 3 sites in one time step as it finds 3 empty sites in front of it. However, the car moves only one site in *time step 3*, and finally leaves the segment in the next time step.

The above example clearly states that the proposed CA model for traffic allows the continuous movement of cars, and it also allows velocity variations based on the traffic condition. Fig.1 depicts a more detailed scenario in a small road segment with 100 sites and $\rho = 35$ cars in 100 sites – that is, 140 cars/km.

IV. Simulation Results

This section evaluates the effectiveness of proposed CA model to analyze the traffic flow in narrow roads of a congested city. Since we have verified our model for the traffic

system in Kolkata, a brief overview on the current state of traffic of Kolkata is provided. Fig.2 is the map showing the roads of Kolkata (<http://www.kolkatatrafficpolice.org>). The total length of highways, arterial and other major roads, in Kolkata metropolitan area is about 700 km [11]. A number of road crossings are the rotaries. A rotary is a ring on which several lanes are attached to. We deal with the entrance and exit of cars in rotary as it is dealt in [2]. However, due to unavailability of traffic data at junctions, we consider all the junctions as rotaries for the current work. We consider that the distance between two such rotaries (junction) is 1 km. Through the junction a car may leave or arrive in the road under consideration.

The vehicles are of different types and sizes varying from motorcycles, private cars to public buses. Inter-car spacing in jammed situation is very less. We consider the length of each site is 2.5 m, which is slightly larger than the average length of a vehicle. Kolkata is so congested that in 5% of total arterial road length, the travel speed is limited by 5 km/h [11]. It is also observed that in most free time of a day there are at least 4 cars in any 100 meter road segment. This information leads us to define ρ_{Th} . Therefore, we get

$$v_{min} = 5 \text{ km/h}$$

$$\rho_{Th} = 1 \text{ car in 11 cells} = 37 \text{ cars/km}$$

$$\Delta t = 1.8s$$

where Δt is a time step. It is required to cross one 2.5 m long cell with velocity v_{min} . The v_{max} is obtained from Δt and ρ_{Th} direct us to obtain v_{max} . If a car finds a number of free sites in front of it, the car immediately accelerates and covers those free cells with in a single time step. Therefore, when density is ρ_{Th} , a car will get 10 free sites in average in front of it. Hence

$$v_{max} = \frac{10 \times 2.5 \times 3600}{1.8} = 50 \text{ km/h}$$

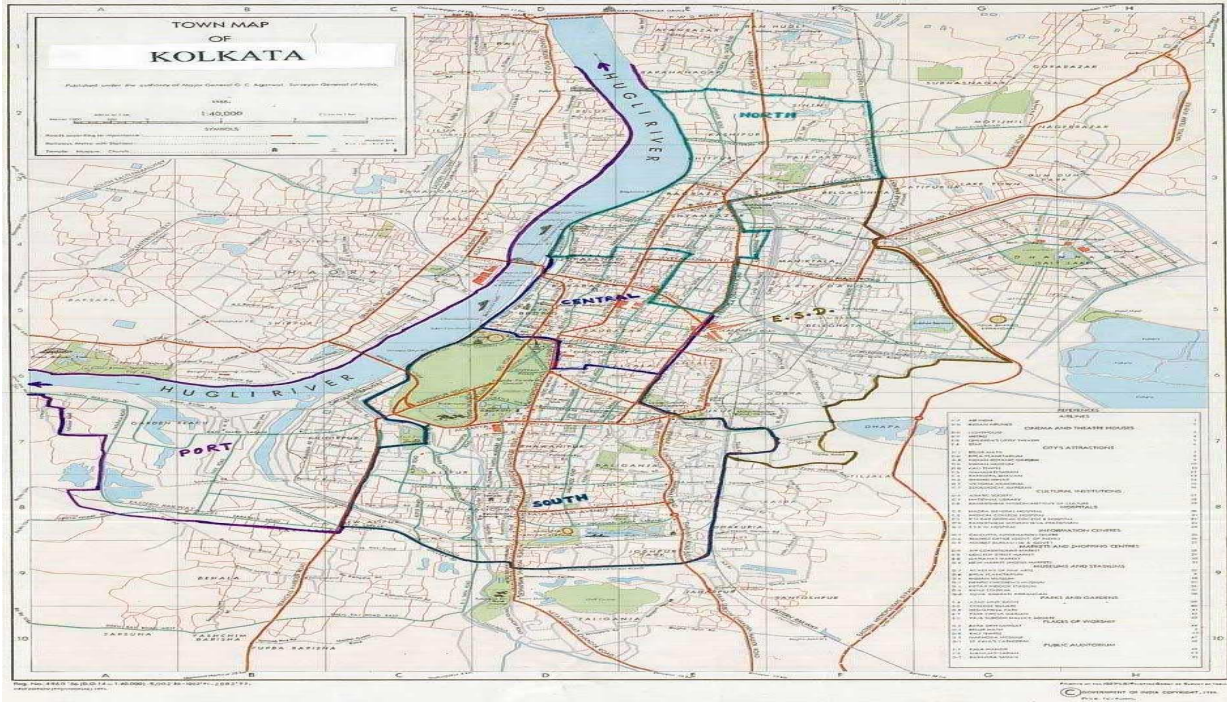


Fig. 2. Map of Kolkata

which is very much realistic in congested city roads.

A. Flow diagrams

The *Fundamental diagram* and the *Dynamical flow diagram* are widely used to understand the traffic flow in a network. The fundamental diagram describes the connection between traffic density ρ and flow rate q on a road. On the other hand, the dynamical flow diagram reflects the dependency of average car velocity on the traffic density. The flow rate (or flux) q is the expected number of vehicles passing through a site on the road in a time interval, and is expressed as the number of vehicles per hour.

Fig.3 depicts the fundamental diagram derived from the proposed model. It shows that the model can fairly reproduce the traffic data that is found empirically.

Fig.4 represents the dynamical flow diagram. The car density vs arrival time is shown in Fig.5. It is derived from the proposed test car run over a specific road segment.

B. Congestion and travel time

The main dynamical phases of traffic flow are the *free flow phase* and the *congested phase*. In free flow phase, the vehicles can move freely and can attain the speed limit. However, in this case also, we assume that the car density is above ρ_{Th} . The congested phase of traffic is much more complicated. It is observed that in our model, the traffic are

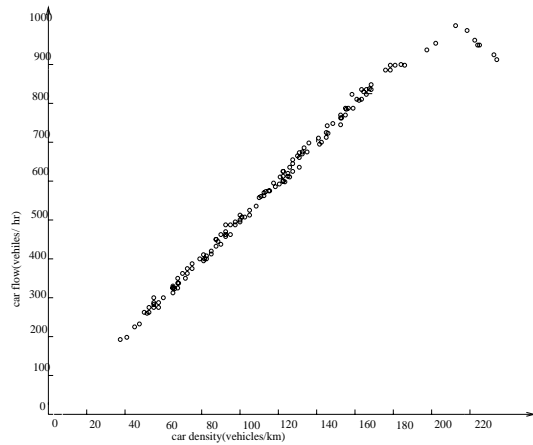


Fig. 3. Fundamental diagram

self-organized ($\dots 0101010 \dots$) in most cases of congested phase. If the cars gather or enter in a road segment almost simultaneously, due to some reasons (for example, in a junction cars from more than one path may move in one direction), then also cars move to the free road segment (if any) in the same direction, and are finally self-organized with the pattern $\dots 0101010 \dots$. This indicates, the cars maintain at

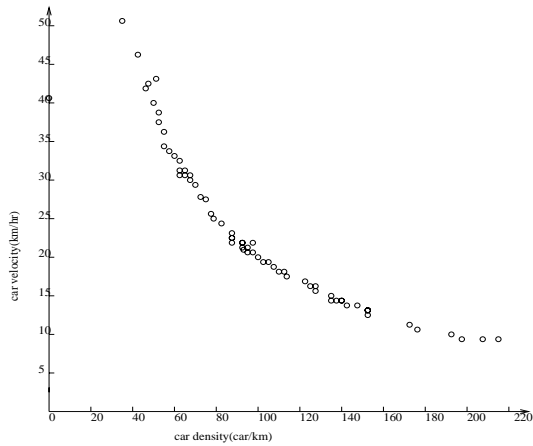


Fig. 4. Dynamicflow diagram

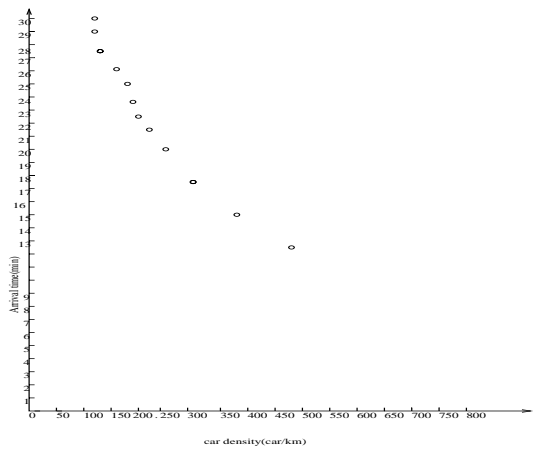


Fig. 5. Car velocity vs arrival time diagram

least one cell distance from their leaders. This efficiently facilitates the continuous movement of cars in the road.

To understand the travel time, we introduce a test car that crosses a part of the whole road with various traffic densities. Here, travel time is the time taken by test car during the rush hour while crossing a specific segment of a road. Practically, the vehicles arrive in the road in different time in different positions. The destinations and total traveled distance of those cars also vary. The test car also travels various parts of the road with different densities. In test run, however, we do not inject, as well as do not remove vehicles from the junction of a road for simplicity. We measure the time required to reach the destination of the test car and its average velocity. The results are noted in Fig.5.

V. Conclusion

This paper has reported an alternative traffic model based on rule 226. It simulates the traffic of a congested city like Kolkata with the continuous movement of vehicles. In reality, our model reveals that (a) cars are always moving either slow or fast, (b) roads are never free from cars, (c) cars run away as fast as they find clearances of road segments. The model can fairly reproduce various traffic data as found empirically. It can further be improved with the introduction of probabilistic CA.

References

- [1] R. Barlovic, L. Santen, A. Schadschneider, and M. Schreckenberg. Metastable states in cellular automata for traffic flow. *Eur. Physical Journal B*, 5(3):793–800, October 1998.
- [2] Bastien Chopard, Re Dupuis, and Pascal Luthi. A cellular automata model for urban traffic and its applications to the city of genova. *Proceedings of Traffic and Granular Flow*, 1997.
- [3] D. Chowdhury, L. Santen, and A. Schadschneider. Statistical Physics of Vehicular Traffic and Some Related Systems. *Physics Reports*, 329(4-6):199–329, May 2000.
- [4] M. Schreckenberg D. E. Wolf and editors A. Bachem. *Traffic and Granular Flow*. World Scientific, 1996.
- [5] M. Fukui and Y. Ishibashi. Traffic Flow in 1D Cellular Automaton Model Including Cars Moving with High Speed. *Journal of the Physical Society of Japan*, 65(6):1868–1870, 1996.
- [6] Dirk Helbing. Traffic and related self-driven many-particle systems. *Reviews of Modern Physics*, 73(4):1067+, December 2001.
- [7] W. Knospe, L. Santen, A. Schadschneider, and M. Schreckenberg. Empirical test for cellular automaton models of traffic flow. *Phys. Rev. E*, 70(1):016115 – 016139, July 2004.
- [8] S. Yukawa M. Kikuchi and S. Tadaki. Dynamical phase transition in one-dimensional traffic flow model with blockage. *J. Phys. Soc. Jpn*, 63(10):3609–3618, 1994.
- [9] Dan Mao, Bing-Hong Wang, Lei Wang, Pak-Ming Hui, and Chin-Kun Hu. Traffic Flow CA Model in Which Only the Cars Following the Trail of the Ahead Car Can Be Delayed. *International Journal of Nonlinear Sciences and Numerical Simulation*, 4:239–250, 2003.
- [10] K. Nagel and M. Schreckenberg. A cellular automata model for free-way traffic. *Journal de Physique I*, 2:2221 – 2229, 1992.
- [11] Sandip Chakroborty and Sudip K. Roy. Traffic Accident Characteristics of Kolkata. In *Traffic and Communications Bulletin for Asia and the Pacific*, No. 74, pages 75–86. United Nations, New York, 2005.
- [12] S. Wolfram. *Cellular Automata and Complexity — Collected Papers*. Addison Wesley, 1994.