A Cellular Automata Based Model for Traffic in Congested City

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Abstract—This work targets modeling traffic flow in roads of a congested city. The Cellular Automata (CA) have been configured for such modeling. The urban traffic models \cite{2}, in general, concentrate on low traffic density. The inabilities of traditional models to address the issues arise out of heavy traffic in crowded cities of third world countries are properly addressed in the proposed elementary CA based model. The effectiveness of the model is verified considering the traffic in Kolkata. It is established that the model can fairly reproduce various traffic conditions as found empirically. Further, the proposed model succeeds the limitations of traffic flow models in congestion and also reproduces the empirical results.

Keywords: City traffic, Cellular Automata (CA), Rule 226

I. Introduction

A traffic system can be considered as a non-equilibrium many-body system being able to exchange energy and particles (vehicles, for example) with the surroundings. The behavior of this kind of systems is not governed by the general principles, that is, of thermodynamics and statistical mechanics, as closed systems in equilibrium \cite{6}. The units of traffic systems are self-driven, moving with the aid of their own energy sources and distinct from the driven particles whose movements are influenced by external forces such as pressure, gravitation, and electrical forces.

Modeling traffic flow are the interest of traffic science and engineering. The traffic science deals with the dynamical phase. Such a non-equilibrium systems are theoretically not as well understood as the equilibrium systems \cite{3}. The other important issues involved with non-equilibrium traffic system are: fluctuations near steady states, how steady states are reached when the initial state of the system is not steady, can it be self-organized, are the phase transition points characterized with long-range correlations and scaling exponents of power laws, etc.

The growth of traffic volumes demands new and efficient solutions in order to accommodate the changing requirements. More efficient usage of the existing road infrastructure is related to the management and controlling of traffic flows, without compromising the safety and convenience of the people. The controlling and management of traffic covers the whole road network, and events & circumstances constituting the traffic situation.

The modeling of traffic flow phenomena was dominated by two theoretical approaches. The so called car-following models are based on the fact that the behavior of a driver is determined by the leading vehicle. This assumption leads to dynamical velocity equations which in general depend on the distance to the leading vehicles and on the velocity difference between the leading and following vehicle. The alternative approach does not treat the individual cars but describes the dynamics of traffic networks in terms of macroscopic variables.

The major problem of present car-following models is that these are difficult to treat in computer simulations. On the other hand, the macroscopic approaches lead to some difficulties even though the large networks can be treated in principle. Further, the present macroscopic models handle a large number of parameters that don’t have counterpart in empirical investigation and the information obtained in macroscopic models can not trace individual cars.

In order to address the above issues, cellular automata (CA) based traffic models have been reported \cite{10}. The CA models are microscopic models which are by design well suited for large scale computer simulations. A comparison of the simulations with empirical data shows that the very simple approaches give meaningful results. The CA models for urban traffic \cite{2}, in general, concentrate on low traffic density. The work presented in \cite{4}, \cite{10}, \cite{6} are mostly devoted to the study of highways. All these can not properly address the issues arise out of heavy traffic in this crowded cities.

In this work, we target modeling of city traffic in narrow roads and apply the model to analyze the traffic in different roads of Kolkata. We show that the inabilities of traditional models can be properly addressed in the proposed elementary CA based model that denies the necessity of probabilistic CA.
II. Traditional cellular automata model

This section introduces the preliminaries of cellular automata (CA) and the basics of CA models so far been reported for traffic flow.

A. Cellular automata basics

The concept of Cellular Automaton (CA) was initiated in the early 1950s by J. von Neumann and Stan Ulam. After the introduction, a number of researchers proposed simplified structure of CA amenable to characterization.

A CA is the discrete spatially-extended dynamical system that has been studied extensively as models of physical systems. It evolves in discrete space and time. In its simplest form, as it is proposed by Stephen Wolfram [12], a CA consists of a lattice of cells, each of which stores a discrete value at a time step (say at time $t$). The next state of the cell at time $t+1$ is affected by its state and the states of its neighbors at time $t$. In the current work, we concentrate on 3-neighborhood (self, left and right neighbors) 1-dimensional CA, where a CA cell is having two states - 0 or 1. Such a CA is referred to as the elementary CA. The next state of a CA cell is

$$S_{i}^{t+1} = f(S_{i-1}^t, S_i^t, S_{i+1}^t)$$

(1)

where $f$ is the next state function; $S_{i-1}^t$, $S_i^t$ and $S_{i+1}^t$ are the present states of the left neighbor, self and right neighbor of the $i^{th}$ CA cell at time $t$. The $f$ can be expressed as a lookup table as shown in Table I. The decimal equivalent of the 8 outputs is called ‘Rule’ $R_i$. In a two-state 3-neighborhood CA, there can be a total of $2^8$ (256) rules. Two such rules are 184 and 226 (Table I).

If the left most and right most cells of an $n$-cell CA are the neighbors of each other, the CA is periodic boundary. In our proposed model, we adopt the periodic boundary CA. Most of the CA based traffic models, so far been proposed, are of probabilistic in nature. In the current work, the traffic model developed is based on the elementary (deterministic) CA.

B. 1-dimensional traffic models

In one dimensional models, a road is considered as a line of sites each of which can be either occupied by a vehicle/car or empty at time $t$. All cars on road travel in the same direction (say to the right). Their positions are updated synchronously. During motion, each car can be at rest or jump to the nearest neighboring site, along the direction of motion, only if its destination site is empty. This means the driver is short-sighted and do not know whether the car in front can move or is stuck by another car. Therefore, the state of each site $s_i$ at $(t+1)$ depends on its state at $t$ (present state) as well as the states of its two nearest neighbors $s_{i-1}$ and $s_{i+1}$. It is summarized in the following table.

<table>
<thead>
<tr>
<th>Current state</th>
<th>Next state</th>
</tr>
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<tbody>
<tr>
<td>000</td>
<td>111</td>
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<tr>
<td>001</td>
<td>110</td>
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<tr>
<td>010</td>
<td>111</td>
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<td>011</td>
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<tr>
<td>110</td>
<td>111</td>
</tr>
<tr>
<td>111</td>
<td>000</td>
</tr>
</tbody>
</table>

where car present within the site is represented by 1. The 0 represents the site is empty. This corresponds to the CA rule 184 (Table I) if a site is considered as a CA cell.

The above mentioned dynamics captures interesting features of car movement. Let consider an instance

$$\cdots0010000100000010000100\cdots$$

It is a free traffic regime in which all the cars are able to move and points to low car density $\rho$ in the system. The average velocity $<v>$, defined as number of movements divided by the number of cars, is then $<v_f> = 1$, where the subscript $f$ indicates a free state. On the other hand, in a high density configuration such as

$$\cdots01100011010011101110\cdots$$

only 7 out of 14 cars can move and $<v> = 1/2$. This represents a partially jammed regime.

In this model, the car occupancy of adjacent site is highly correlated and the vehicles cannot move until a hole (vacant site) is found. The car distribution tries to adjust itself to a situation where there is no spacing between the consecutive cars. Therefore, it is expected that the number of moving cars simply equals to the number of empty cells in high density [8]. Thus, the number of movements is $L(1 - \rho)$; $L$ is the length of a road. The average velocity in jammed phase is

$$<v> = \frac{1 - \rho}{\rho}$$

(2)

A more effective version of the above CA model is reported in [10]. It assumes the $i^{th}$ car may have several possible velocities $u_i = 0, 1, 2, \cdots, v_{max}$, and $d_i$ is the distance, along the road, separating cars $i$ and $i+1$. The updating rule is

- The cars accelerate when possible: $u_i \rightarrow u_i' = u_i + 1$, if $u_i < u_{max}$.
- The cars slow down when required: $u_i' \rightarrow u_i'' = d_i - 1$, if $u_i' > d_i$.
- The cars have a random behavior: $u_i'' \rightarrow u_i''' = u_i'' - 1$, with probability $p_i$, if $u_i'' > 0$.
- The cars move $u_i'''$ sites ahead.

This rule captures some important behaviors of real traffic on a highway. The velocity fluctuations are due to non-deterministic behavior of the drivers, and stop-and-go waves observed in high density traffic regime (that is, some
is a probabilistic CA based model, which in the case of $v_{max} = 1$ and the deterministic limit (that is, randomization probability $p_i = 0$) is equivalent to CA rule 184 in Wolfram notation.

A more realistic CA model of traffic flow was done by so-called velocity-dependent-randomization (VDR) model [1]. It extended the set of update rules of $NS$ model. A velocity-dependent randomization $p_i(v)$ was introduced. The simplest version was

$$p_i(v) = \begin{cases} p_0, & \text{for } v = 0 \\ p, & \text{for } v > 0 \end{cases}$$

### III. The Proposed CA Based Model

This section identifies the properties of our target city traffic and then propose a CA based model for the traffic flow.

#### A. City traffic in narrow roads

In a congested city, the roads are hardly free during most of the day time. A portion of the traffic network remains congested. The spacings between consecutive vehicles, moving in a particular direction, are very small. If not (red signal) obstructed, the vehicles move continuously in a particular direction. The overtaking is not entertained. However, the velocity of a car may vary in different segments of a road. If one or more vehicles are ahead of the vehicle $c$, it moves with a uniform velocity. On the other hand, if $c$ finds a gap, it accelerates and covers the distance. The properties of such a traffic system can be summarized as follows:

1. Car density ($\rho$) is not very low. It is always above some threshold ($\rho_{Th}$). That is, $\rho \geq \rho_{Th}$.
2. The spacing between two consecutive vehicles is small.
3. The vehicles flow continuously in a particular direction.
4. A vehicle can cover a distance immediately if it finds a gap.

It can be noted that since we study the congested city traffic in narrow roads where continuous movement of vehicles is a general feature, the NS and VDR models can not be appropriate for modeling such a system [7].

#### B. CA model of city traffic

The properties of city traffic described in the earlier subsection define the occupancy of a cell/site at time $t + 1$. It depends on the occupancies of nearest neighbors at time $t$. The roads and the movements of vehicles can be described by an $n$-cell CA (cellular automata), where a CA cell $i$ corresponds to road site. The current state of a cell is 1 implies that the corresponding site $i$ is occupied by a vehicle; 0 implies empty site.

In a narrow road, the vehicles move in the same direction and the sites are updated synchronously during the motion (i) if a site $s_i$ & its left neighbor $s_{i-1}$ are occupied, the site $s_i$ remains occupied in the next time instant, (ii) if $s_i$ is empty but its right neighbor $s_{i+1}$ is occupied, then $s_i$ is filled up in the next time instant/step, and (iii) if the $s_{i-1}$ is empty & $s_i$ is occupied, the vehicle at $s_i$ moves towards $s_{i+1}$ and the site $s_i$ becomes empty. Such phase transitions of sites can be appropriately modeled by a CA chosen for the road. The CA rule 226 (Table II) can exactly map such a flow of traffic described in (i), (ii), (iii). Therefore, an $n$-cell CA, each cell configured with rule 226, can demonstrate to the traffic flow in roads of a congested city.

To formalize the above properties of city traffic, we define car density $\rho$ as the number of cars per kilometer. $\rho_{Th}$ is specified from simulation of real life traffic data. We consider the length of each site is 2.5m which is slightly larger than the average length of a vehicle/car. Each car may take any velocity from $v_{min}$ to $v_{max}$. Since cars even flow in jammed condition, $v_{min} > 0$. The acceleration rule adopted is quite similar as it is proposed in [5], [9]. If a car finds a number of empty sites in front of it, the car speed can rapidly be increased to its limit. Therefore, $v_{max}$ depends on the maximum number of empty sites in front of a car. Hence, the $v_{max}$ in turn depends on $\rho_{Th}$.

To illustrate the traffic scenario, let us consider that half of a road segment is occupied by vehicles, that is, the segment is partially jammed. Table III shows the flow of traffic in such a condition. We assume that the immediate next sites at both ends of the segment are empty. In time step 1 (car position), we mark two cars (hat and underline) as test cars. It can be observed that the car with hat symbol moves one
cell in each time step as either the car is guided by another one ahead of it or it finds only single empty site to move. On the other hand, the underlined car covers 3 sites in one time step as it finds 3 empty sites in front of it. However, the car moves only one site in time step 3, and finally leaves the segment in the next time step.

The above example clearly states that the proposed CA model for traffic allows the continuous movement of cars, and it also allows velocity variations based on the traffic condition. Fig. 1 depicts a more detailed scenario in a small road segment with 100 sites and \( \rho = 35 \) cars in 100 sites – that is, 140 cars/km.

**IV. Simulation Results**

This section evaluates the effectiveness of proposed CA model to analyze the traffic flow in narrow roads of a congested city. Since we have verified our model for the traffic system in Kolkata, a brief overview on the current state of traffic of Kolkata is provided. Fig. 2 is the map showing the roads of Kolkata (http://www.kolkatatrafficpolice.org). The total length of highways, arterial and other major roads, in Kolkata metropolitan area is about 700 km [11]. A number of road crossings are the rotaries. A rotary is a ring on which several lanes are attached to. We deal with the entrance and exit of cars in rotary as it is dealt in [2]. However, due to unavailability of traffic data at junctions, we consider all the junctions as rotaries for the current work. We consider that the distance between two such rotaries (junction) is 1 km. Through the junction a car may leave or arrive in the road under consideration.

The vehicles are of different types and sizes varying from motorcycles, private cars to public buses. Inter-car spacing in jammed situation is very less. We consider the length of each site is 2.5 m, which is slightly larger than the average length of a vehicle. Kolkata is so congested that in 5% of total arterial road length, the travel speed is limited by 5 km/h [11]. It is also observed that in most free time of a day there are at least 4 cars in any 100 meter road segment. This information leads us to define \( \rho_{Th} \). Therefore, we get

\[
\begin{align*}
\nu_{min} & = 5 \text{ km/h} \\
\rho_{Th} & = 1 \text{ car in 11 cells} = 37 \text{ cars/km} \\
\Delta t & = 1.8 s
\end{align*}
\]

where \( \Delta t \) is a time step. It is required to cross one 2.5 m long cell with velocity \( \nu_{min} \). The \( \nu_{max} \) is obtained from \( \Delta t \) and \( \rho_{Th} \) direct us to obtain \( \nu_{max} \). If a car finds a number of free sites in front of it, the car immediately accelerates and covers those free cells with in a single time step. Therefore, when density is \( \rho_{Th} \), a car will get 10 free sites in average in front of it. Hence

\[
\nu_{max} = \frac{10 \times 2.5 \times 3600}{1.8} = 50 \text{ km/h}
\]
A. Flow diagrams

The Fundamental diagram and the Dynamical flow diagram are widely used to understand the traffic flow in a network. The fundamental diagram describes the connection between traffic density $\rho$ and flow rate $q$ on a road. On the other hand, the dynamical flow diagram reflects the dependency of average car velocity on the traffic density. The flow rate (or flux) $q$ is the expected number of vehicles passing through a site on the road in a time interval, and is expressed as the number of vehicles per hour.

Fig. 3 depicts the fundamental diagram derived from the proposed model. It shows that the model can fairly reproduce the traffic data that is found empirically.

Fig. 4 represents the dynamical flow diagram. The car density vs arrival time is shown in Fig. 5. It is derived from the proposed test car run over a specific road segment.

B. Congestion and travel time

The main dynamical phases of traffic flow are the free flow phase and the congested phase. In free flow phase, the vehicles can move freely and can attain the speed limit. However, in this case also, we assume that the car density is above $\rho_f$. The congested phase of traffic is much more complicated. It is observed that in our model, the traffic are self-organized $\cdots\text{0101010}\cdots$ in most cases of congested phase. If the cars gather or enter in a road segment almost simultaneously, due to some reasons (for example, in a junction cars from more than one path may move in one direction), then also cars move to the free road segment (if any) in the same direction, and are finally self-organized with the pattern $\cdots\text{0101010}\cdots$. This indicates, the cars maintain at

![Map of Kolkata](image-url)
least one cell distance from their leaders. This efficiently facilitates the continuous movement of cars in the road.

To understand the travel time, we introduce a test car that crosses a part of the whole road with various traffic densities. Here, travel time is the time taken by test car during the rush hour while crossing a specific segment of a road. Practically, the vehicles arrive in the road in different time in different positions. The destinations and total traveled distance of those cars also vary. The test car also travels various parts of the road with different densities. In test run, however, we do not inject, as well as do not remove vehicles from the junction of a road for simplicity. We measure the time required to reach the destination of the test car and its average velocity. The results are noted in Fig. 5.

V. Conclusion

This paper has reported an alternative traffic model based on rule 226. It simulates the traffic of a congested city like Kolkata with the continuous movement of vehicles. In reality, our model reveals that (a) cars are always moving either slow or fast, (b) roads are never free from cars, (c) cars run away as fast as they find clearances of road segments. The model can fairly reproduce various traffic data as found empirically. It can further be improved with the introduction of probabilistic CA.

References