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A Probabilistic Fuzzy Logic System: learning in the stochastic environment with incomplete dynamics

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***Abstract*—A completely new type of fuzzy logic system will be developed from the existing fuzzy structure and applied to modeling and control of complex processes under incomplete dynamics in the manufacturing industry. Using a unique three-dimensional membership function (fuzz grade, time and probability), the probabilistic processing features can be added into the existing fuzzy configuration to construct a probabilistic fuzzy inference engine. Thus, this developed probabilistic fuzzy logic system (PFLS) is able to learn uncertain information in both fuzzy and stochastic nature. The proposed PFLS will be very suitable to modeling of the complex stochastic process with incomplete dynamics. All the existing learning theories and methods can be directly applied to the proposed PFLS to enhance its learning performance. Integrated into the fuzzy-PID structure, it will turn into a probabilistic fuzzy logic controller for the stochastic control. Successful application of the proposed PLFS to the selected industrial process will have a great impact on both academia and industry.**

I. INTRODUCTION

When the system becomes more and more complex, it will be more and more difficult to maintain a consistent performance due to increasing uncertainties. These uncertainties come from various sources, such as, measurement error accumulated, unknown interactions between subsystems, and missing dynamics, etc. To model and control the uncertain dynamics well, the features of uncertainty from the data should be properly captured [1-4]. Though uncertainties come from various sources, they share some common features, either in fuzzy nature or in stochastic nature.

1) Vagueness - the fuzziness of data and knowledge may result from the following reasons:

- For decision making, modeling or control applications, the knowledge expressed by words may show inconsistent vagueness in information processing, which result in the fuzzy difficulties in the processing.
- The vagueness of data comes from the low resolution of sensor or poor measurement techniques. A linguistic description of vague data will be benefit to the robustness of modeling or control system.

2) Stochastic nature - this is the nature of the universe. A proper structure to capture the randomness and attenuate its side-effect is an important issue. One important source comes from missing dimension of data and makes the system analysis more difficult.

- The full dimension of data is infinite and the neglected dimension of state variable for modeling and control may result in an unknown stochastic dynamic of the systems.
- The full dimension of data is unknown. The missing dimension can be seen as stochastic features (such as random noise) to the full dimension of dynamics.

The kernel of uncertainty modeling and learning is to construct a nonlinear mapping between the data domain and the feature domain. The fuzzy logic system (FLS) is able to handle fuzzy uncertainty by mapping uncertain data or knowledge to a linguistic domain [6], and further improve its performance with neural computation [7-9]. However, its linguistic expression cannot handle the stochastic uncertainty, and both Mamdani and Takagi-Sugeno (T-S) type inference engine may not be suitable to work under stochastic environment with incomplete dynamics.

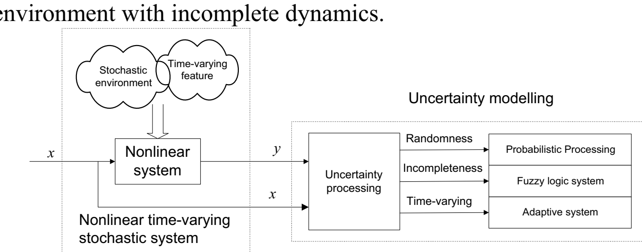

Figure 1: Modeling philosophy of uncertainties

The stochastic modeling method [14] is to generate the mapping to the probability space after analyzing the historical data. The Kalman filter has been widely used for state estimation with random noises [5]. However, it may not be able to process the incomplete information and cognitive uncertainties. Stochastic control theory [10] usually work well when a standard stochastic model is available, however this assumption may not be valid for incomplete or linguistic information processing. The robust modeling and control theory [11] can strictly guarantee the system performance by the assumption of relatively accurate mathematic model of uncertainty, but the scarcity of data will make it difficult in many real applications.

In summary, all the existing modeling and control theory emphasize one aspect of uncertainties. Mature mathematical theory can guarantee the efficiency of the uncertainty extraction and expression on one aspect of the system, and usually loses other uncertain features due to the limitation of

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data. It is impossible for the existing approaches to model incomplete dynamics with both fuzzy and stochastic uncertainty.

It would be valuable if the statistical analysis method and the fuzzy system can be integrated into a unified platform to process uncertainties that contain both fuzzy and stochastic features. The random set, fuzzy random set and fuzzy random variables are important concepts in the fuzzy theory [17-21]. So far, most of the previous works were investigating the relationships between randomness and fuzziness. It is still difficult to apply these concepts to the engineering applications. A few research attempted to develop the stochastic fuzzy system for modeling and controlling a dynamic system perturbed with stochastic uncertainty [15, 16]. Using a Takagi-Sugeno (T-S) type fuzzy system, the stochastic uncertainty was expressed by the state equation only in its consequent part, not from the antecedent. This may not properly reflect the stochastic and fuzzy nature of the process.

By the end of last decade, the type-2 FLS [12, 13] was proposed, which employs two fuzzy membership functions (MFs), to improve the ability of handling the uncertainty of linguistic expression. An interval type-2 FLS was developed later to reduce its computational complexity. Since two fuzzy membership functions (MFs) are used as the primary and secondary description of the same variable, the type-2 FLS is actually to improve approximation accuracy. This may limit its application potential to a multi-domain process that contains uncertainties in different physical domains, such as fuzzy and stochastic uncertainties.

Recently, the probabilistic fuzzy logic system (PFLS) [22, 23] was developed, based on the concept of the type-2 FLS, by changing the secondary MF into the probabilistic density function (PDF) for describing the stochastic variable. This inherent 3-domain (3D) nature system was applied to modeling of the process with both fuzzy and stochastic uncertainties.

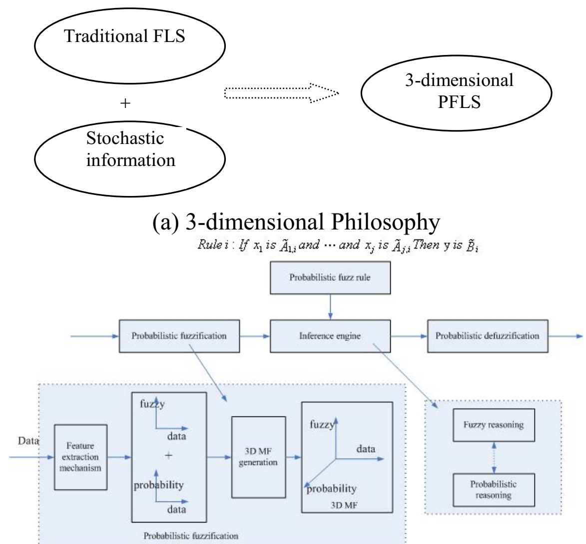
In this paper, a brief review and simple tutorial is given for the development of the PFLS for modeling and learning when there exist both fuzzy and stochastic uncertainties. The PFLS uses a 3-dimensional MF in the probabilistic fuzzy set, where the extra third dimension of the MF is used to express the stochastic feature. The probabilistic fuzzy logic operation is introduced to provide a unified framework to inference the uncertainty mixed with incompleteness and randomness. The PFLS can be further integrated with the neural network to improve its performance in time-varying situation. The future challenges and unsolved problems are also discussed in the paper.

II. PROBABILISTIC FUZZY LOGIC SYSTEM

The objective of the paper is to introduce a 3-dimensional (3D) probabilistic fuzzy logic system (PFLS) that is inherently capable of processing fuzzy and stochastic information simultaneously. The developed 3D-PFLS will be applied to modeling and control applications. After that, systematic on-line design and tuning approaches should be studied for modeling and control applications.

2.1 Configuration design

Since the well developed traditional FLC provides a basic fuzzy processing platform, it is natural to obtain a 3D philosophy from the integration of the traditional FLC and probabilistic information processing as shown in Figure 2a. Based on this 3D philosophy, the 3D-PFLS can be primitively obtained to have similar functions to the traditional FLS. A potential configuration of 3D-PFLS has probabilistic fuzzification, rule base, inference engine and probabilistic defuzzification, as shown in Figure 2b.



(a) 3-dimensional Philosophy

(b) A potential configuration

Figure 2: A Conceptual development of a 3D-PFLS

Generally, the knowledge used to construct rules in a FLS is uncertain. The PFLS is effective to handle the stochastic modeling problem using a proper probabilistic modeling method. The possible functional framework of the PFLS is given in Figure 3. The j^{th} rule of a PFLS can be obtained from human knowledge or data clustering algorithms and expressed as follows:

Rule j: If x_1 is $\widetilde{A}_{1,j}$ and … and x_i is $\widetilde{A}_{i,j}$ and … and x_n is $\widetilde{A}_{n,j}$

Then y is $\widetilde{B}_j$

where $\widetilde{A}_{i,j}(i=1,2,\cdots,n)(j=1,2,\cdots,\bar{J})$ and $\widetilde{B}_i$ are probabilistic fuzzy sets. Similar to the ordinary FLS, the PFLS still has operations of fuzzification, inference engine and defuzzification as shown in Figure 3. Different to the ordinary FLS, the PFLS uses the probabilistic fuzzy set (PFS) instead of the ordinary fuzzy set, which is described by a three-dimensional membership function as in Figure 4. The PFS is able to express the information with stochastic uncertainties.

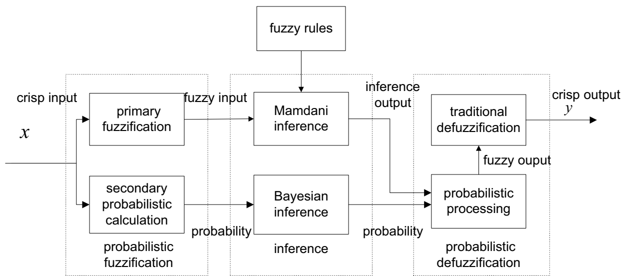
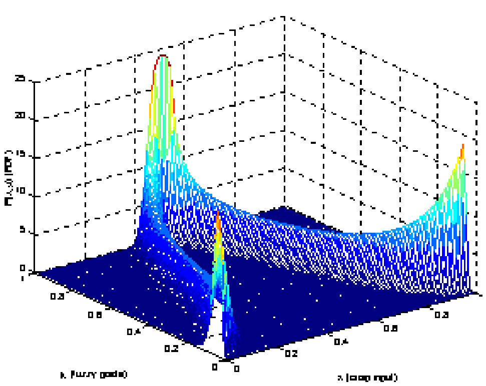


Figure 3: Functional framework of the PFLS

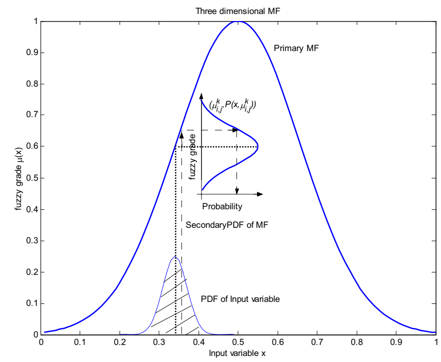
2.2. Probabilistic fuzzification

The significant difference of PFLS to FLS is that the fuzzification procedure is based on probabilistic fuzzy sets instead of ordinary fuzzy sets. Inspired by type-2 FLS, the primary MF will be fuzzy and the secondary MF will be designed as probabilistic density function (PDF). Formation of these two MFs will establish a 3D-MF to express both fuzzy and stochastic information. As the fuzzy and stochastic features are heavily coupled, the following issues need to study.

- It is difficult to extract the fuzzy and stochastic features simultaneously because they are strongly coupled. One possible mechanism is to extract the stochastic features first, and then the fuzzy feature [22]. The other is the inverse operation. It is still not clear how these two extractions affect the performance.
- Formulation of a 3D MF from the two-dimensional MF and PDF needs to study. A possible solution is to develop the primary MF in terms of stochastic parameters that are described in the secondary PDF, for example, a Gaussian type MF with the parameters as center and width described in PDF. Three different cases should be considered and compared: static center and stochastic deviation; static deviation and stochastic center; and stochastic center and stochastic deviation.



a) A appearance in three dimensions



b) Projection view of the primary MF and secondary PDF

Figure 4: The three dimensional MF

2.3. Probabilistic Inference engine

In the earlier version of PFLS [22], the Mamdani inference engine was modified to include the probabilistic reasoning using Bayesian inference, which works well under complete dynamics. The probabilistic fuzzy relation set is computed by the Mamdani reasoning method:

$$\mu_{\tilde{R}_j} = \mu_{\tilde{A}_{1,j}} \circ \mu_{\tilde{A}_{2,j}} \circ \cdots \circ \mu_{\tilde{A}_{n,j}} \circ \mu_{\tilde{B}_j} \tag{1}$$

where $\circ$ denotes the t-norm operation, $\mu_{\tilde{R}_j}, \mu_{\tilde{A}_{i,j}} (i = 1, \cdots, n)$ and $\mu_{\tilde{B}_j}$ is the fuzzy grade. The probabilistic information is processed with the Bayesian inference method.

$$P(E_{\tilde{R}_j}) = P(E_{\tilde{A}_{1,j}}) \cdot P(E_{\tilde{A}_{2,j}}) \cdot \cdots \cdot P(E_{\tilde{A}_{n,j}}) \cdot P(E_{\tilde{B}_j}) \tag{2}$$

where $P(E_{\tilde{R}_j})$, $P(E_{\tilde{A}_{i,j}})$ and $P(E_{\tilde{B}_j})$ denote the corresponding probability of the fuzzy grade $\mu_{\tilde{R}_j}, \mu_{\tilde{A}_{i,j}}$ and $\mu_{\tilde{B}_j}$, with $E_{\tilde{A}_{i,j}}$ and $E_{\tilde{B}_j}$ as independent events

Since the secondary PDF is obtained from data learning, which always contains incomplete dynamics, the Bayesian inference may not be able to provide reliable reasoning under incomplete PDF.

The Dempster-Shafer theory has been a powerful tool for the reasoning of incomplete stochastic information [29]. It is possible to develop the inference engine by using the Dempster-Shafer theory to handle the incomplete stochastic information as follows

$$m_{i,j}(\Psi) = \frac{\sum_{A \cap B = \Psi} m_i(A) \cdot m_j(B)}{1 - \sum_{A \cap B = \Phi} m_i(A) \cdot m_j(B)} \tag{3}$$

where $m_i(A)$ and $m_j(B)$ denotes the probability assignment of the i^{th} and j^{th} fuzzy relations respectively; $m_{i,j}(\Psi)$ denotes the combined probability.

The most critical problem is how to integrate fuzzy reasoning and probabilistic reasoning in a more rigorous and logic way. The exponential growth of rules for multi-variables is always an important issue to address.

2.4 Probabilistic Defuzzification

One task of probabilistic defuzzification is to develop the final crisp output of PFLS. The existing work has integrated the expectation and centroid computation together for stochastic and fuzzy information to develop the crisp output, where the probabilistic defuzzification improves the traditional defuzzification method with the probabilistic processing method. The traditional defuzzification computes the centroid output y_C with the inference set $\mu_{\tilde{R}_j}$ as :

$$y_C = \frac{\sum_{j=1}^{\bar{J}} y_j \mu_{\tilde{R}}(x, y_j)}{\sum_{j=1}^{\bar{J}} \mu_{\tilde{R}}(x, y_j)} \tag{4}$$

$$\mu_{\tilde{R}} = Max(\mu_{\tilde{R}_1}, \cdots, \mu_{\tilde{R}_j}, \cdots, \mu_{\tilde{R}\bar{J}})$$

where y_j is the crisp consequent, y_C and $\mu_{\tilde{R}}$ are random variables. The crisp output y of the PFLS is the mathematical expectation of the traditional defuzzification:

$$y = Ex(y_C) \tag{5}$$

This kind of defuzzification results in a great amount of computation [22]. It will be interesting to develop a more simple computational structure for probabilistic defuzzification.

It would also be a challenge to develop a new defuzzfication mechanism that provides a 3D output in probabilistic distribution, which will be useful for the decision making system to provide rules in probability.

III. PROBABILISTIC AND FUZZY DESIGN FOR MODELING

After After the basic design of 3D PFLS, it is required to generate the rule base and 3D MF for the on-line modeling applications. Basically, the modeling design procedure is similar to the traditional FLS. The most challenge comes from the optimal rule base and 3D MF design for the stochastic domain. The modeling procedure includes three procedures: the PDF estimation; PDF based probabilistic fuzzy rule generation; neural system for learning of 3D MF.

3.1 PDF estimation

To extract the stochastic features from limited data, it is required to estimate the distribution functions for input data. Many statistical analysis methods [24-26] have been proved successfully for the distribution estimation, such as maximum likelihood methods, parzen window estimation, and SVM. These estimations can converge to an arbitrarily-complex PDF. However, to guarantee the convergence of estimation, the sample size is required increase exponentially with the dimensionality of the space. In practice, such a large number of samples are not normally available. It is required to use different statistical analysis tools to estimate the distribution function for each input variable with insufficient sample size.

3.2 PDF based rule base design

To study the nominal fuzzy relationship of input-output variables, the spatial distance is commonly used as the optimization criterion to develop the clustering centers for further fuzzy design. The optimization criterion for clustering can be given to minimize the weighted distance from the original data to clustering centers. The optimization of clustering criterions is designed on the deterministic computational mode [28] in most existing clustering methods. However, the deterministic design is not suitable to express the relation of data with stochastic nature. It is very interesting to study a PDF based clustering method for the construction of rule base when stochastic uncertainties is concerned.

The stochastic optimization criterion will be studied for the clustering processing. A possible solution is to develop a clustering center with PDF. Different to the deterministic criterion, a stochastic criterion will increase the likelihood evidence of clustering results in a stochastic environment

Furthermore, the stochastic iterative algorithm will be studied for the optimization process. It is difficult to explore the global optimal point by traditional deterministic iterative algorithms due to the stochastic variation in the process, and the stochastic nature-based iterative algorithm is superior to the deterministic one for the global optimization.

3.3 Neural system for learning of 3D MF

After the offline design of rule base, it is still not easy to obtain the optimal parameter online due to time-varying uncertainties. Thus, it is important to further adjust the parameter for the best performance according to the online data. Unfortunately, the limited data is suffered with lots of incomplete information, which make the optimization processing difficult. The traditional deterministic optimization (e.g. gradient decent algorithm) focused on the minimization of modeling performance index (e.g. mean square error). However, it will be difficult to find the proper direction to the global optimization points in stochastic environment. The reinforcement learning method, which is inherently designed with stochastic nature, might be more suitable for the optimization problem in unknown dynamic systems [30-32]. It is interesting to tune the 3D MF with the stochastic-based reinforcement learning method to guarantee the global optimization.

Most existed exploration strategies usually suffer from the difficulties to hold good balance between exploration and exploitation. The balance problem will be more difficult in a stochastic environment. Hence new ideas are necessary to explore more effective exploration strategies to achieve better performance in stochastic situations.

It is required to combine the neural construction with reinforcement learning mechanism. The neural network (NN) has the strength on learning, adaptation and parallelism. If the

PFLS can be implemented in NN, it will add the advantages of NN for time-varying dynamics learning into the probabilistic fuzzy logic [23]. The Functional configuration of this neural PLFS is depicted in Figure 5, which clearly shows the integration of fuzzy neural network (FNN) with the probabilistic processing for uncertainty modeling, where both fuzzy and probability information are processed together in a neural system. A key technique is how to take the most advantage of neural construction in reinforcement learning optimization.

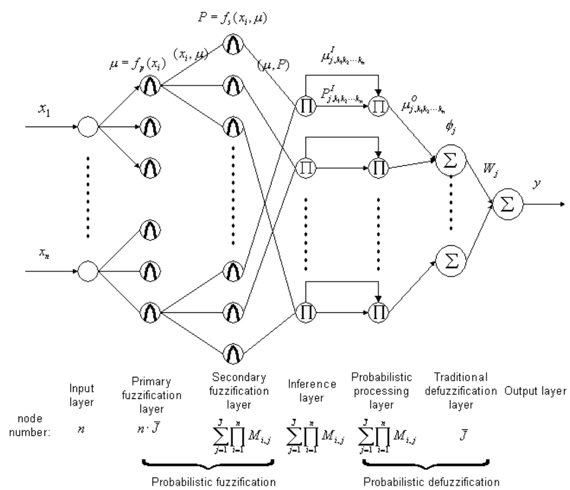


Figure 5: The structure of a neural PFLS

IV. SIMULATION RESEARCH

A nonlinear system is used to illustrate the modeling performance of a neural PFLS for a time-varying plant expressed as:

$$y(k)=\frac{y(k-1)y(k-2)[y(k-1)+2.5]}{1+y(k-1)^2+y(k-2)^2}+u(k-1)$$

where $y(k)$ is the output of the nonlinear system, and $u(k)$ is a sine signal with random noise $\varepsilon(t)$, which is descried by a PDF $N(\mu_\varepsilon,\sigma_\varepsilon^2(t))$ and $\sigma_\varepsilon(t)\in[0.01,0.02]$ as in Figure 6.

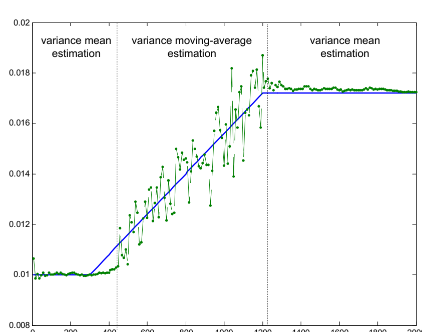


Figure 6: Noise variance: actual (solid) and estimated (dashed)

The neural PFLS is used to model the nonlinear system and the input variable is defined as $x(k)=[y(k-1),y(k-2),u(k-1)]^T$. The following steps can be followed to design the neural PFLS.

1) The input-output data is obtained and used to find an initial primary MF (center c, width σ) for each fuzzy set by the fuzzy c-mean method. The primary MF of the PFLS can be determined by the clustering algorithm.
2) A statistical experiment is performed to compute the probabilistic feature of the k^{th} sampling data *u(k)*. The expectation value and variance can be estimated [23].
3) Compute the three-dimensional MF. The MF for *u(k)* can be expressed as $\mu(u(k))=\exp\left(-\frac{(u(k)-c)^2}{2\xi^2}\right)$, where μ is the random membership grade, c and ξ is the parameter of the primary MF. Then, the neural PFLS can be constructed accordingly [23].

In the simulation, the neural PFLS with a hybrid adaptive algorithm is compared with the ordinary FNN. The neural structure of the PFLS is $\prod_{3\times17\times27\times560\times560\times180\times1}$. The structure of the ordinary FNN is $\prod_{3\times17\times27\times560\times560\times180\times1}$. In this simulation, the variance $\sigma_\varepsilon(t)$ of the random noise is time-varying and unavailable, therefore the uncertainty modeling problem would be difficult for traditional filter systems. The estimated variance in Figure 6 is used to construct the secondary PDF of the PFNN. Figure 7 shows that the modeling performance of the PFNN is better than that of a FNN in terms of the performance index defined as $J=\sum(\hat{y}(k)-y(k))^2$ with $\hat{y}(k)$ as the prediction of PFNN.

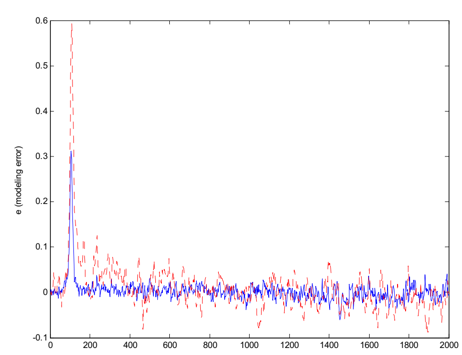


Figure 7: The performance of neural PFLS (solid) and FNN(dotted)

V. CONCLUSION

A probabilistic logic system is discussed in this paper to model the complex process with both stochastic and fuzzy uncertainties. The PFLS uses a three-dimensional MF to express the mixed stochastic and fuzzy uncertainties in a unified structure, and uses probabilistic fuzzy inference logic to inference under probabilistic-fuzzy environment. The PFLS can be integrated with the neural network for handling time-varying dynamics. In the future, the Dempster-Shafer theory can replace the Bayesian method to improve inference performance for the incomplete dynamics. Then PFLS developed will be more suitable to work in the complex system modeling with strong stochastic and incomplete dynamics.

One of interesting future work is the integration of PFLS with the traditional fuzzy-PID structure [33], which can formulate a PID type probabilistic fuzzy logic controller that is more suitable to work in the complex environment where both fuzzy and stochastic uncertainties exist.

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